Logic

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THE THEORY OF THOUGHT

A TREATISE ON

DEDUCTIVE LOGIC

[By
Noah K. Davis.]

Ὁ μὲν οὖν κατὰ τὸ πρᾶγμα θεωρῶν τὰ κοινὰ διαλεκτικός, ὁ δὲ τοῦτο φαινομένως ποιῶν σοφιστικός.—ARISTOTLE.

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PREFACE.

Little preface is needed. The treatise is not elementary in the sense of bringing the subject within the grasp of immature minds. This I believe to be impracticable, and no such profession is made. It is elementary in the sense that it begins at the beginning, supposing the reader to have no previous knowledge of the subject. Its extent is such that one who masters its contents will be in possession of the technical details of the science, acquainted with its established doctrines, and prepared to study with profit and interest advanced treatises. It is, in general, a reproduction of the old Logic. Whatever scorn the modern student may have for antiquity, he must know its doctrine respecting Thought if he would read intelligently the recent literature of the subject, even that in sympathy with him, for it is permeated with the terminology and the doctrines of the ancient logicians.

The treatise reverts to Aristotle, and is largely a restatement of his theory as colored by filtration through mediæval mind. "Since his day," says Kant, "Logic has taken no backward step, and also up to this time it has been able to take no step forward, and thus, to all appearance, is concluded and perfected." A fiery trial for ages has neither consumed it nor refined it, and it is likely to remain perpetually an accepted part of the sum of human knowledge. Hence, in treating the old Logic, I have aimed at clear, correct, and complete statement rather than at any modification. Of late years many innovations have been proposed, some of which are examined and criticised. Whenever in the treatise a new view is offered, it is distinctly indicated as such.

A great number and variety of examples, both for illustration and for praxis, mitigating somewhat the severity of the subject, seemed to me desirable. They have been collected from every available source; some are ancient, some modern, many are newly invented.

I have used with great freedom standard authors, keeping constantly at hand Arnauld, Whately, Hamilton, Mansel, Thomson, De Morgan, Mill, and Bain, and a dozen or more school-book writers, profited by their research, and adopted their views and phraseology whenever it seemed advantageous. Abundant references to them, together with this general acknowledgment, will, I hope, be deemed sufficient. I have not sought to embellish the margin with recondite matter, but have added many references to other accessible works, hoping to lead the reader into yet wider fields.

The treatise has been prepared with much pains. That there are no blunders in it would be too much to hope, but it is sent to its account with all its imperfections on its head. If, on the whole, it is a good book, it will live and be useful; if not, it will die, the sooner the better.

Noah K. Davis.

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LOGIC,

OR

THE THEORY OF THOUGHT.

PART FIRST.—INTRODUCTORY.

I. DEFINITION OF LOGIC.

§ 1. Logic is the science of the necessary forms of thought.

The word "Logic" is derived from the Greek λογική, an adjective qualifying ἐπιστήμη (science) or πραγμητεία (matter of study) understood. The meaning of λογική and of its original, λόγος, is ambiguous. The latter is equivalent to both the *ratio* and the *oratio* of the Latins, to thought and to speech. This ambiguity passed into the derivative, and has affected the views of many logicians as to the object-matter of the science, some holding that it treats of words or language rather than of thought.[1]

Aristotle did not designate by the term λογική the science whose doctrine he first fully developed. The terms Analytic, Apodeictic,

[1] See Hamilton's *Logic*, p. 3. It may be well to note at the outset that logicians are divided into three schools, according as they hold *words*, *things*, or *conceptions* to be the subject of Logic; and these are entitled, respectively, the Verbal, the Phenomenal, and the Conceptional Schools. The first is represented by many scholastics, by Hobbes, Whately, and De Morgan. The second numbers Bacon, Helvetius, Comte, J. S. Mill, and Bain among its chief expositors. At the head of the third is Kant, followed by Krug, Esser, and the recent German logicians generally, and by Hamilton and Mansel with their train of Scotch and English pupils; to whom may be added most French writers, following Arnauld. The present treatise takes the Kantian, or conceptualist, view. Logic treats of *thought*. But as thought is always about *things*, and is expressed in *words*, Logic cannot proceed in entire disregard of these, but should constantly keep them subordinate. See Cretiens's *Logical Method*, ch. v. Oxford, 1848.

and Topic (the latter equivalent to Dialectic, and including Sophistic) were special names by which he denoted parts or applications of Logic. He used no one term to designate the whole science. Plato used the term Dialectic to denote more than Logic proper includes, while Aristotle used it to denote less, and it is usually regarded as the most ancient synonym for Logic. With whom the designation Logic originated does not appear; but it is ancient, being used by Cicero, and is attributed by Boethius to the early Peripatetics.

§ 2. "A Science is a complement of cognitions, having, in point of form, the character of logical perfection; in point of matter, the character of real truth."[2] The logical perfection of a branch of knowledge is attained by systematically arranging and exhibiting its object-matter, clearly, distinctly, completely, and in harmonious connection. This implies classification. Again, the object-matter of a science must be real truth, otherwise it cannot be said to be known; what is unreal or false cannot be a constituent of a science.[3] Hence the definition may be conveniently abbreviated thus: A Science is a perfected system of real truths; or thus: Science is classified knowledge. Few branches of knowledge have reached this ideal perfection; if not the mathematics, none have done so. But since in many departments knowledge has far outgone its crude beginnings, and made great progress towards this ideal, such branches are properly called sciences.

"The distinction between science and art is, that science is a body of principles and deductions to explain some object-matter; an art is a body of precepts, with practical skill, for the completion of some work. A science teaches us to know, an art to do; the former declares that something exists, with the laws and causes which belong to its existence; the latter teaches how something must be produced."[4] In science *scimus ut sciamus;* in art *scimus ut producamus.* Science discovers laws; art gives rules. Περὶ γένεσιν τέχνη, περὶ τὸ ὂν ἐπιστήμη.[5] This distinction holds good, in reference to the extremes, as to pure speculative sciences and mere manual arts. But science often leads so directly into art, and art, except in its rudest forms, is so dependent on science, that usually they cannot be set clearly apart.

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[2] Hamilton's *Logic*, p. 335.

[3] Scientific knowledge (τὸ ἐπίστασθαι), except when of axiomatic principles (νοεῖν), requires a conviction of the truth of the given proposition, and a knowledge of its reason or cause.—Aristotle's *Anal. Post.* i, 2, 1.

[4] Thomson's *Outline of the Laws of Thought*, § 6.

[5] Aristotle.

Moreover, there is a body of practical sciences, e. g. Ethics, Economics, etc., that occupy intermediate ground, and yet are never called arts; others again, e. g. Rhetoric, Grammar, etc., are commonly viewed as arts.[6]

Some logicians have viewed Logic as an art, and entitled it The art of thinking (Arnauld[7]); The art of reasoning (Aldrich); The right use of reason (Watts), etc. Others pronounce it to be both, thus: *Ars artium, et scientia scientiarum* (Duns Scotus, 13th century);[8] The art and science of reasoning (Whately); The art of thinking, which means, of correct thinking, and the science of the conditions of correct thinking (Mill[9]). The extreme view of Logic as an art is that it teaches us *how to think*. This is evidently absurd. A course in Logic is about as needful for making men thinkers as a course in Ethics is to make them virtuous, or a course in Optics to make them see. A modified view is that Logic teaches us how to think correctly, or, negatively, how to avoid fallacy, or that it teaches how to test the validity of given arguments. If such is the scope and object of Logic, it may be set aside as of little or no value, consisting of a system of rules which the initiated never use and the uninitiated never miss. Such views have historically brought Logic into great discredit, just as Chemistry was brought into disrepute by the extravagant pretensions of the alchemists.[10]

But Logic is not primarily, nor even secondarily, an art. It is strictly a science; the science teaching how we do think and how we must think if we think correctly. It is the theory of reasoning; or, better, it is the theory of thought. The difference between Logic and an Art of Thinking is similar to that between Anatomy and Surgery. The value of Logic is such as belongs to pure science, which, in this day, needs no demonstration. It is something of profoundest interest to know what are the mental processes in the intellectual act of thinking, and of such matter the liberal mind asks primarily, Is it true? not, Is it useful? Knowledge is power, but we have to do

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[6] See Hamilton's *Metaphysics*, pp. 81–84.

[7] *L'Art de Penser*, 1662, that most admirable work, known commonly as the "Port-Royal Logic."

[8] See Hamilton's *Logic*, p. 25. [9] Ex. of Hamilton's Phil. vol. ii, p. 149.

[10] See Locke's contemptuous opinion of Logic, *Essay*, bk. iv, ch. xvii. Also Goethe's, in *Faust*, pt. i, speech of Mephistopheles to "der Schüler." It may be objected that this is merely the mocking gibe of Mephistopheles; but cf. in *Wahr. und Dicht.* pt. i, bk. iv.

with it here as knowledge, not as power. Where, however, one has mastered the science, there is a practical result in a special cultivation of his reasoning powers; and, moreover, whatever process one clearly understands, it is manifest he can more clearly and efficiently perform.[11]

The Greek Aristotelians, and after these the scholastic Aristotelians, subdivided Logic into what the latter called *Logica docens* and *Logica utens*. The former is explained as an abstract theory of thought—*quæ tradit præcepta;* the latter as a concrete practice, as an application of these rules to use—*quæ utitur præceptis*. Hamilton, following Kant, calls the former "General or Abstract Logic," the latter "Special or Concrete Logic." The former only is Logic; the latter, quite properly called "Applied Logic," and treating chiefly of the methods by which particular sciences should be logically developed, is no part whatever of the science of Logic, of Logic proper, and accordingly will be disregarded in the present treatise.[12]

§ 3. The object-matter of Logic is thought. Thus it is distinguished from other sciences, each of which has its own special object-matter. Astronomy treats of the stars; geology, of the earth's crust; zoology, of its fauna; botany, of its flora; mathematics, of quantity; theology, of God; philosophy, of principles; psychology, of mind; ethics, of morals, etc.; so Logic treats of thought. Thought denotes only the acts of the understanding, as distinguished from perception, memory, feeling, desire, volition, of whose exercises Logic takes no account. Thought may be simply defined as the cognition of one notion in or under another. Hence in this act we are said to comprehend or understand a thing. E. g., A book lies before me. I may be conscious of the impression the thing makes without cognizing what it is. This is mere perception. But if I cognize what it is, and say, "It is a book," I have brought it under a certain class or concept of things which we call "book." This is thought.[13] Now we think about all conceivable things, but all of these are to Logic perfectly indifferent except one, that is, thought itself. In Logic we think about thought. What thought involves, Logic evolves.[14]

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[11] See Hamilton's *Logic*, pp. 7, 8; and McCosh's *Logic*, pt. iii, § 80.

[12] Hamilton's *Logic*, p. 38, and p. 42.

[13] Id. pp. 9, 10.

[14] See Aristotle, *De Soph. Elench.* ix. Sciences and demonstrations, says he, are possibly infinite, and would require omniscience to treat them. The dialectician has to discover only the principles common to all spheres of thought.

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§ 4. We observe, then, that Logic does not at all concern itself with what things thought considers. It treats of thought regardless of its content. It is usual to express this by saying that Logic treats of the forms of thought abstractly, i. e. excluding its matter. The form of thought as distinguished from its matter may be exemplified thus: When I think that the book before me is a folio, the matter of this thought is "book;" and "folio," the form of it, is "a judgment." The forms of thought may be represented as empty shells, into which very various matter may enter as the content of thought; or as mere outlines, to which different substances may conform, like as a statue may be formally the same whether of wood, metal, or marble. So the science Morphology treats of the forms of plants and animals, and Crystalology, an abstract geometrical science, treats only of the forms of minerals. The matter and the form have, of course, no separate existence. No object is cogitable except under some form of thought; and no form of thought can have any existence in consciousness unless some object be thought under it. But by analytic abstraction we can consider these apart; we can consider either the object thought, or the manner of thinking it; we can distinguish the matter from the form of thought. Now it is the form of thought, abstracting its matter, that Logic considers. Modern logicians are fond of saying that all matter is extralogical. This might be understood to represent Logic as a science without a content, without matter of its own. But Logic, like every other science, has its own special content. Its object-matter is thought; all *other* matter is extralogical. Its object-matter is thought discharged of its matter; i. e., it is the form of thought.

Logic, then, is properly an abstract science, one abstracting from each and all the sciences, and considering only what is common to all; i. e. the formal thought to which all are subjected, and making that its object-matter. Hence Logic is in a similar and equal relation to all sciences, and fundamental to all. Now philosophy is the science of principles, and is therefore the fundamental science in the sense that its object-matter is the primary truths that underlie all knowledge. But philosophy proceeds logically or not at all. Hence Logic is fundamental even to philosophy in the sense that it exhibits the necessary processes of thought which bind philosophy as well as every other science. Moreover, Logic is itself bound to proceed logically, and can become a science only by conforming to those processes which it is its province to explicate and exhibit.

Let it not, however, be supposed that Logic treats of thought only as exercised and displayed in scientific pursuits. It treats of thought universally. Thought as exhibited in all kinds of literature and speech, in common conversation, in silent meditation; all our common every-day thinking, about the most trivial things and at every instant, is formally all of the same nature, proceeds in the same manner, is governed by the same laws, is logical if correct. Consequently, illustrations of the principles of Logic are to be drawn not merely from any of the sciences, but from any kind of knowledge, wherein anything whatever becomes an object of thought. Logic teaches or explains how any human mind rightly thinks at any time about anything.

§ 5. To define Logic as the science of the forms of thought would not be sufficient to set it entirely apart, would not discriminate clearly its character. Psychology is *inter alia* a science of formal thought, and needs to be distinguished from Logic. Psychology is an empirical science; it is evolved from experience. It is therefore an inductive, natural science, one *a posteriori*. It systematizes the conscious mental activities, and points out their laws. In dealing with the faculty of thought, it explains the modes in which we think, teaching how we do think, and refers for the test of its doctrine to the reflective consciousness of every individual.

Logic, on the other hand, if taken in its strictest sense, is not at all an empirical, but a speculative or theoretical, science. It accepts from Psychology, or obtains by the analysis of given products of thought, certain primary laws; from these it deduces secondary laws of thought, and thus proceeds to demonstrate the necessary processes of thought, those we must follow in thinking correctly. It is therefore a purely deductive science, one *a priori*. It teaches not how we do think, as a matter of fact, but how we must think, as a matter of necessity, if the thinking be consequent. It appeals, not to consciousness, but to demonstration, in support of its truthfulness.

Psychology, then, is the natural history of thought; Logic is the theory of thought. Psychology considers thought as an operation; Logic considers it as a product. Psychology treats of conceiving, judging, reasoning; Logic treats of concepts, judgments, reasonings. Psychology treats of thought as it is; Logic of thought as it must be. Psychology teaches how we do think, Logic teaches how we must think. The one treats the forms of thought merely as actual, the other proves them necessary. Like mathematics, Logic is purely de-

monstrative. Indeed, in respect of their demonstrative character, "Logic and Mathematics stand alone among the sciences, and their peculiar certainty flows from the same source. Both are conversant about the relations of certain *a priori* forms of intelligence—Mathematics about the necessary forms of imagination, Logic about the necessary forms of understanding. Both are thus demonstrative or absolutely certain sciences, each developing what is given as necessary in the mind itself." [15] Hence Kant, followed by Esser, who in turn is followed by Hamilton, defines Logic to be the science of the necessary forms of thought.[16]

Such is the definition of Pure Logic. It excludes Psychology. We have already seen that it excludes Applied Logic. If we adhere to it, we must reject also Modified Logic as not properly any part of the science. For Modified Logic considers thought "not as determined by its necessary and universal laws, but as contingently affected by the empirical conditions under which thought is actually exerted, showing what these conditions are, how they impede, and, in general, modify the act of thinking; and how, in fine, their influence may be counteracted." [17] Treatises on Concrete or Applied Logic, and on Modified Logic, may be valuable appendices to works on Logic, but they constitute no part of the pure science.[18]

§ 6. As an expressive synonym for Logic we have adopted the phrase "Theory of Thought." Theory is properly opposed to practice. Theory is mere knowledge; practice is the application of it.[19]

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[15] Hamilton's *Logic*, p. 31.

[16] It will be seen hereafter that the "necessary forms of thought" corresponds to the old logical phrase "second intentions." Hence an excellent definition of Logic, were not the phrase obscure, would be "the science of second intentions." See pt. ii, ch. iv, § 8.

[17] Hamilton's *Logic*, p. 43.

[18] On the title-page occurs the phrase "Deductive Logic," to indicate the absence of any treatment of Induction in the present work. The importance of Induction cannot be overestimated, but it calls for a distinct treatise. We often hear the phrase "Inductive Logic." But induction is not correctly, etymology apart, opposed to Deduction, and all Logic proper is Deductive Logic.

[19] With Plato θεωρεῖν is applied to a deep contemplation of the truth. By Aristotle it is always opposed to πράττειν, and to ποιεῖν, so that he makes philosophy *theoretical*, *practical*, and *artistical*. The Latins and Boethius rendered θεωρεῖν by *speculari*.—Trendelenburg's *Element. Log. Arist.* p. 76. See also, on theory and practice, Hamilton's *Metaphysics*, p. 120.

Theory denotes the most general laws to which certain facts can be reduced. It is a collection of the inferences drawn from facts and compressed into principles; it is a systematized explanation of facts demonstrably true. Logic is such a systematized collection of the laws that govern thinking, and it is occupied with demonstrating their validity from certain axioms. It is, therefore, properly called the Theory of Thought.

§ 7. It is evident that a work strictly limited by the definition of pure Logic would be very abstract and difficult. Being a discussion of forms, it could offer no examples; for since a pure abstract form cannot be realized in consciousness apart from matter, much less can it be expressed. Even in general expressions by algebraic symbols, the symbols themselves are a species of matter that is extralogical.

Again, if the treatise be kept strictly apart from Psychology, it will admit of no reference to actual thinking. It will tell us nothing of how the mind does actually proceed in its efforts to systematize its knowledge, nothing of the nature of the thinking act as giving rise to the logical product, nothing of the phenomena of illegitimate thinking. Thus our science would be shorn of its rays.

Consequently, few, if any, writers have allowed themselves to be rigidly bound by the definition. In the present treatise, while we make pure, abstract Logic its basis, while developing systematically the Theory of Thought, and keeping this prime object constantly in view, we shall freely transgress the limits of the definition whenever it seems desirable. We shall consider not merely how the mind must think, but how it does think. We shall give copious concrete illustrations, and analyze and exhibit actual exercises of thought, appealing to observation and to the experience of consciousness to corroborate the theory, just as the astronomer turns to the stars to observe the fulfilment of the laws of the *Mécanique Céleste*.

II. PRIMARY LAWS.

§ 1. In the study of Psychology we find by subjective analysis that there are certain modes of intelligence to which the mind is necessitated by virtue of its essential and original constitution. Among others are certain forms of thought determined or necessitated by the nature of the thinking subject itself. The chief of these necessary forms are, the concept, the judgment, the reasoning. By saying that these forms are necessary is meant that the mind cannot truly think except in them. But since they are native and necessary, they must be universal, both in the sense that they are found in every human mind, and in the sense that all the thoughts of each mind are always determined in them. For it cannot be that a form is necessary on some occasions and not on others. If so, it would be merely contingent, which contradicts our notion of necessity. Now, the forms being necessary and universal, we may view them as governed by necessary laws. These laws will be an expression of the general abstract principles common to the forms, and, as the result of a complete analysis, will be ultimate and axiomatic. When evolved and enunciated, they are known as Logical Principles, or as the Primary or Fundamental Laws of Thought.[1]

Again, if, preliminary to pure Logic, products of thought viewed objectively as embodied in language are subjected to a critical analysis, they are found to exhibit general or universal forms. In other words, if from the various manifestations of thought in speech and literature we abstract the matter and all differences characterizing them, we discover a residuum common to all, a mode, a manner, having certain forms that belong to all, that interpenetrate all. These forms, being universal, are considered as governed by laws; and these laws, when enunciated, are found to be the same as those obtained by subjective analysis. Thus the two processes are mutually corroborative.

This complement of laws is assumed by Logic as its *punctum saliens*, and it proceeds to demonstrate synthetically from them as

[1] For the history of these laws, see Hamilton's *Logic*, pp. 62–68.

axioms the secondary or special laws of the concept, the judgment, the reasoning. The whole of pure Logic is only an articulate development of these Primary Laws, and of the various modes in which they are applied.

To say that these self-evident laws are necessary, is to say that the contradictory of each is inconceivable. It is not that they are inviolable, not that the mind is constrained of necessity to obey them, as a planet is blindly constrained to obey the laws of gravitation, inertia, etc. They are violable in the sense that we may wilfully or unconsciously disregard them; but the result is fallacy, inconsequence; or, rather, the mental process is then suicidal or absolutely null and void. All consequent thinking must be legitimate; i. e., it necessarily conforms to these laws, advertently or inadvertently. They are the primary conditions of the possibility of valid thought.[2]

The reader must not be offended to find these axiomatic laws so obvious as to seem mere truisms. When stated, they appear to have been always known, being implied in every thought we have ever experienced or observed, though until stated we are as unconscious of them as we are of the laws that govern our breathing. Being the widest generalities, penetrating every science, and, indeed, governing every mental movement that comprehends anything, they seem of all things the most familiar and trite. Their very truth requires that they contain nothing new. Standing related to the axioms of geometry as these are related to elaborate propositions, they at first appear singularly meagre, barren of significance, and even frivolous. But if these laws are really the code by which all human thought is actually regulated, then their study is not futile; so far from being barren, they are the most wonderfully productive of principles; so far from being frivolous, they have the profoundest significance.

§ 2. The Primary Laws of thought are three. The first is the Law of Identity. It is the principle of all logical affirmation. It is variously enunciated: e. g., Whatever is, is, or *Omne ens est ens;*[3] Everything is equal to itself;[4] Every object of thought is conceived as itself;[5] A thing is what it is;[6] Conceptions which agree can be united in thought, or affirmed of the same subject at the same time.[7] The formula is $A=A$; or $A=a'+a''+a'''$ - - - -.

[2] See Hamilton's *Logic*, p. 56. [3] Scholastic form. [4] Hamilton's *Logic*, p. 57.
[5] Mansel's *Prolegomena Logica*, p. 167. [6] Bain's *Logic*, p. 16.
[7] Thomson's *Outline*, § 114.

The following are examples: "4=4;" "4=2×2;" "2+2=2×2;" "4=3+1," etc.; "According to Plato, The Idea is equal to itself;" "Man's a man for a' that;" "Saltpetre is nitrate of potash;" "Francis Bacon was Baron Verulam;" "Francis Bacon was the father of inductive philosophy;" "Man is rational and animal;" "Man is the last creation;" "Man is the only being that laughs;" "A habit is a habit;" "What I have written, I have written."

Hamilton extends this law to include the relation of partial identity or sameness in which a concept stands to each of its constituents, as expressed in the second formula. E. g., "Man is rational," i. e. my notion "Man" comprehends the notion "rational" as one of its constituents;—similarly, "Man is animal." We may go further,—to the part of a part. E. g., The notion animal comprehends the notion corporeal, and we may say, "Man is corporeal." In this extension of the law, the predicate is only a part of what is implicated in the subject.

To affirm that a thing is itself seems to be solemn trifling, and is ridiculed by Locke. *Nulla propositio est verior illa in qua idem prædicatur de seipso.*[8] When, however, we consider that every object of thought has definite characteristics by which it is marked off and distinguished from all others as being itself and nothing else, it is evident that every concept may be viewed in relation to these characteristics, and that these two several aspects must be affirmed of each other. The law then declares the necessity of thinking the concept and its constituent characters as the same. A better expression of it would perhaps be: A notion and its constituents are the same. This is a more general expression of the axiom: A whole is equal to the sum of its parts. In the predicate, the whole is contained explicitly which in the subject is contained implicitly.

It is obvious that this law enjoins self-consistency; or, rather, it is the necessity for self-consistency in thought that is formulated in this law. Whatever be the aspects of a thing, whatever be the modes of statement concerning it, they must be equivalent; the thought underlying each must be the same.

§ 3. The second is the Law of Contradiction. It is the principle of all logical negation. Enunciations are: The same attribute cannot be at the same time affirmed and denied of the same sub-

[8] Boethius. See Hamilton's *Logic*, p. 507.

ject;[9] No subject can have a predicate that contradicts it;[10] What is contradictory is unthinkable;[11] No object can be thought under contradictory attributes;[12] The same thing cannot be A and non-A: this room cannot be both hot and cold.[13] As this law enjoins the absence of contradiction as the indispensable condition of thought, it ought rather to be called the Law of Non-contradiction.[14] The formula is: A is not A=0; or, A is not non-A. Examples which, if taken literally, violate the law are: "Dotage is infancy in old age;" "His left hand is most dexterous;" "The blind see, the deaf hear, the dumb speak," etc.; "However unwilling the choice, he was compelled to volunteer;" "Since the war, all values have risen;" "Two kinds of individuals prepare extempore speeches, fops and fools;" "Nothing in this life is true;" "The decomposition of the elements;" "We want nothing but silence, and but little of that." Each of the foregoing examples is a logical paradox, a self-contradiction; each violates the law, and is a *felo de se.*

By a fundamental law of mind, which Bain calls the Law of Relativity, every notion has an opposite or counter notion, and only by virtue of the one can the other be conceived. To the straight line there is opposed the not-straight line, or crooked line; to good is opposed evil, and a knowledge of good is impossible to a mind not knowing evil. Hence the old scholastic maxim: *Contrariorum eadem est scientia.* Now these opposites cannot consist, their union is contradiction, and thorough-going consistency, as formulated in the Law of Contradiction, forbids it. Thus, when we affirm that this is a straight line, we must not also say that it is a crooked line; when we think an act good, we may not also think it evil. Our assertions, our thoughts, to be consistent, must not contradict each other. If they do, the thought is null, it destroys itself. Having made an assertion, we are to abide by that. Affirmations not self-consistent are unintelligible.

But the principle of contradiction carries us one step further. An affirmation being made, it not merely forbids us to affirm also its contradictory, but it authorizes us, or requires us, to pronounce the contradictory false; i. e., to deny, of an object of thought, its contradic-

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[9] Aristotle, who says this is by nature the principle of all other axioms.—*Metaph.* IV (Γ), lii.

[10] Kant's *Critique of Pure Reason.* See Meiklejohn's transl. p. 115.

[11] Hamilton's *Logic*, p. 58.

[12] Mansel's *Prolegomena Logica*, p. 167.

[13] Bain's *Logic*, p. 16.

[14] Krug's *Logik*, § 18; followed by Hamilton.

tory. Accordingly, the principle may be enunciated thus: Of two contradictories, one must be false. E. g., "This straight line is not crooked;" "This good act is not evil;" "No chastisement is joyous;" "Francis Bacon was not Roger Bacon;" "A dishonest man is not trustworthy." If all diamonds are precious, then to say that some or any diamonds are not precious is false. Whatever is repugnant, opposite, contradictory, to a notion must be denied of it.

The Laws of Identity and Contradiction are co-ordinate. Neither can be deduced as a second from the other as first. In every such attempt the evolved secondary is unavoidably presupposed, which is *petitio principii.*[15] The two have, however, been identified by many eminent philosophers, as Leibnitz, Wolf, Kant, Herbart. And Hamilton says, "The two laws are essentially one, differing only by a positive and negative form."[16] Perhaps the two may be fairly summed in the statement: All thought must be self-consistent.

§ 4. The third is the Law of Excluded Middle. Its logical significance is that it limits the thinkable in relation to affirmation; for it determines that of the two forms given in the first two laws, the one or the other must be affirmed as necessary. No middle ground, no third affirmation, being possible, one or the other must be true. Hence the names: *Lex exclusi medii aut tertii inter duo contradictoria; Principium contradictionis affirmativum.* We enunciate it thus: Of two contradictories, one must be true. Either a given judgment must be true, or its contradictory: there is no middle course.[17] Of two contradictories, one must exist in every subject.[18] The formula is: X is either A or non-A; one being sublated, the other must be posited.

A few examples will suffice: "Either it is true that God exists, or it is true that he does not exist;" "Man must be a free agent, unless his acts are necessitated;" "To be or not to be, that is the question;" "Infinite mercy offers salvation to all." In this last example the opposition is between bounds and no-bounds; bounds is denied in "infinite," and hence no-bounds must be affirmed, which is done in "offers salvation to all." The argument called *Reductio ad absurdum* is an application of this law. Of two alternatives it shows one to be absurd, hence the other must be true; for one proposition being false,

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[15] Shown in Hoffbauer's *Logik*, § 23.
[16] *Logic*, p. 59.
[17] Thomson's *Outline*, § 114.
[18] Mansel's *App. to Aldrich*, p. 241.

we are authorized or required by this law to pronounce its contradictory true.

The Laws of Contradiction and Excluded Middle may be conveniently united in one statement, to which might be given the name "Law of Duality." It is the principle of strict logical division and disjunction.[19] We may enunciate it thus: Of two contradictories, one must be true, the other false; Every predicate may be either affirmed or denied of every subject; Every assertion must be either true or false.[20] This compound form is often mistaken by logical writers for the Law of Excluded Middle. So Goclenius: *Oportet de omni re affirmare aut negare.*[21] Hamilton also. He gives for the Law of Excluded Middle: Of contradictory attributions, we can affirm only one of a thing; and if one be explicitly affirmed, the other is implicitly denied.[22] This is the compound; the latter member is the principle of contradiction. His subsequent exposition, however, is correct. Bain clearly makes the mistake.[23] So also Herbert Spencer. He says the principle of Excluded Middle is: The appearance of any positive mode of consciousness cannot occur without excluding a correlative negative mode; and the negative mode cannot occur without excluding the correlative positive mode.[24]

§ 5. The Laws of Identity, Contradiction, and Excluded Middle are mutually complementary. "The object which I conceive is by the Law of Identity discerned as being that which it is, and by the Law of Contradiction is distinguished from that which it is not. But these two correlatives must also be regarded as constituting between them the universe of all that is conceivable; for the distinction above made is not between two definite objects of thought, but between the object of which I think and all those of which I do not think. Non-A implies the exclusion of A only, and of nothing else, and thus denotes

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[19] The *Ἀξίωμα διαιρετικόν* of the Greeks.

[20] Mill questions the absolute truth of this axiom.—*Logic*, p. 205. He says that between the true and the false there is a third possibility, the unmeaning: e. g., "Acracadabra is a second intention," is neither true nor false. But is an unmeaning proposition any assertion at all? Its content is a vacuum. If unmeaning, it means nothing, says nothing. The third possibility, then, is nothing; or, there is nothing between the true and the false. See also *Examination of Hamilton*, ch. xxi.

[21] *Lex. Philosoph.* p. 136.

[22] *Logic*, p. 59.

[23] *Logic*, p. 17.

[24] *Fortnightly Review*, No. 5.

the universe of all conceivable objects with that one exception."[25] In other words, A and non-A divide the universe between them, admitting no intermediate or third possibility, which is declared by the Law of Excluded Middle.

By the Law of Identity, whatever is one is that one.
By the Law of Contradiction, whatever is one is not the other.
By that of Excluded Middle, whatever is not one is the other.

By Contradiction, no thing can be both A and non-A.
By Excluded Middle, every possible thing is either A or non-A.

By the former, two contradictories cannot both be true; i. e., one must be false.

By the latter, two contradictories cannot both be false; i. e., one must be true.

Many fruitless attempts have been made to reduce the three laws to one. So intimate is their relation that each supposes the other; but, like the sides of a triangle, they are not the same, not reducible to unity, each having equal right to be considered first, and each, if considered first, giving, in its own existence, the existence of the other two. Accordingly every attempt to deduce either one from the others has failed. They are complementary, co-ordinate, distinct, and inseparable.[26]

§ 6. Whatever violates either of these laws we feel to be impossible, not only in thought, but in existence. We cannot believe that anything can differ from itself, that anything can at once be and not be, that anything can neither be nor not be. We cannot but regard that as false and unreal which these laws condemn. They thus determine to us the sphere of impossibility, and that not merely in thought, but in reality; not only logically, but metaphysically. What is contradictory is inconceivable in thought and impossible in fact.

But, on the other hand, it does not hold that what is thought in conformity with these laws is therefore true in reality; that whatever is conceivable in thought is actual, or even possible, in fact. For the sphere of thought is far wider than the sphere of reality, and no inference is valid from the correctest thinking of an object to its actual existence. What is conceivable conforms to the laws of thought, and

[25] Mansel's *Prolegomena Logica*, p. 168. [26] Hamilton's *Logic*, pp. 70 and 506.

is said to be logically possible, i. e., possible in thought; and this is true of many things that are impossible in fact. Pure mathematics deals exclusively with mere logical possibilities. That the stars may fall on the earth is physically impossible; that revenge may be a duty is ethically impossible. But both are conceivable; they may be represented in thought; they are logically possible. I may think Waterloo a fiction, or Christianity a failure, but this conceivability is no evidence that they are so. While, then, these laws are the highest criterion of the non-reality of an object, they are no criterion at all of its reality; and they thus stand to existence in a negative, and not in a positive, relation. Says Kant, "The principle of contradiction is a universal but purely negative criterion of all truth."[27] And this holds equally of all the proximate and special applications of these laws; that is, of the whole of Logic. Our science, then, in its relation to other sciences, is not a positive criterion of truth; it can only be a negative criterion, being conversant with thoughts and not with things, with the possibility and not the reality of existence.

We have referred to the psychological Law of Relativity. Some eminent German philosophers have held that the human mind is competent to the cognition of the absolute, or that which has no relation, and have elaborated thereon extensive systems of philosophy. This Philosophy of the Absolute can proceed only upon a more or less complete denial of the primary laws of thought. Fichte and Schelling admit the Law of Identity, but deny the two others, "the empirical antagonism between the Ego and the Non-ego being merged in the identity of the absolute Ego." Hegel regards all the laws as valid, but only for the finite Understanding, they being inapplicable to the higher processes of the Reason. The eclecticism of Cousin attempts the cognition of the absolute without repudiating the laws of Logic. It is therefore at once involved in undeniable contradictions from which there is no escape.

§ 7. The principle of Sufficient Reason, or Determinant Reason, has been laid down as a fourth primary law of thought. It is enunciated thus: Every judgment must have a sufficient ground for its assertion. It was first distinctly enounced by Leibnitz, who made it, together with the principles of Identity and Contradiction, the basis of his Logic. Kant adopted it, regarding Contradiction as the crite-

[27] *Critique of Pure Reason*, p. 115.

rion of logical possibility, and Sufficient Reason as the criterion of logical reality. But logical possibility and logical reality are one. Hamilton, in his lectures, followed Fries and Krug in admitting the principle to this high position in Logic, but subsequently he gave it up, and pronounced it extralogical.[28] Mansel says, "The relation of this principle to the act of judgment is merely negative; it forbids us in certain cases to judge at all, and it does no more. . . . The only logical reason for a thought of any kind is its relation to some other thought; and this relation will in each case be determined by its own proper law," i. e., by one or more of the three given Primary Laws. "The principle of Sufficient Reason is therefore no law of thought, but only the statement that every act of thought must be governed by some law or other."[29] Adopting this view, we reject the principle, as forming no positive element of Logic.

§ 8. In connection with the Primary Laws, it is appropriate to state an important Postulate of Logic. It is this: Logic postulates to be allowed to state explicitly in language all that is implicitly contained in the thought.[30] According to Aristotle, the doctrine of the syllogism deals, not with the expression of reasoning in ordinary language, but with the internal reasoning of the mind itself. Logic, therefore, has always presented all the propositions of a syllogism, although in actual argument one or more of them is usually left unexpressed. But, since all speech is very elliptical and highly rhetorical, the postulate must be allowed to Logic in general, and must be further extended to include not only the accurate and complete rendition of the thought into language, however prolix and awkward its expression may be, for Logic disregards rhetorical elegance, but also the transmuting of metaphors, and, indeed, of all rhetorical forms, into the most literal and direct statement practicable, providing only that the thought itself be not changed. For as Logic deals only with the thought, it must be independent of the accidents of expression.

Hence when a logician deals with an abbreviated or figurative expression, one wherein "more is meant than meets the ear," for much thought is conveyed in hints, intimations, and metaphors, he at once asks, What is the full and true meaning of this? He then proceeds, and must be allowed to strip off all ornament, to supply all lacunæ,

[28] *Logic*, p. 62, note; and p. 251. [29] *Prolegomena Logica*, p. 182.
[30] Hamilton's *Logic*, p. 81.

and to exhibit the thought naked and entire. This is often difficult to do, thought being so subtle and evasive, and language so meagre and inaccurate. He must be allowed, too, to make changes in phraseology for mere convenience, provided always the thought is not thereby essentially modified. Such alternative and entirely similar propositions, having equal power and reach, are called "Equipollent Propositions," a term for which we shall have much use in the sequel. Mill states the matter thus: "Logic postulates to be allowed to assert the same meaning in any words which will express it; we require the liberty of exchanging a proposition for any other that is equipollent with it." The justice of the Postulate is self-evident on the ground that Logic deals not with words, but with thoughts.

PART SECOND.—OF CONCEPTS.

I. THE TERM.

§ 1. Thought viewed as a product of intellect exhibits three forms, the Concept, the Judgment, the Reasoning, which, when expressed in language, severally appear as the Term, the Proposition, the Syllogism. The three are not different in kind, for both concepts and reasonings may be reduced to judgments. A concept is the result of one or more prior acts of judgment, and may be analyzed into these again. A reasoning is a judgment of the relation of two things through their relations to a third. But each of these forms of thought calls for distinct consideration, and constitutes a general division of elementary Logic. Under Concepts, then, let us consider first their Origin, they and their constituent elements being comprised by the common title of the Nòtion, or the Term.

§ 2. An account of the genesis of concepts belongs more strictly to Psychology, but cannot be entirely omitted here. Three momenta may be distinguished, *viz.*, Abstraction, Generalization, Conception. First of Abstraction.

When the mind is attracted to some object, either an external thing as presented in sense, or a mental image presented in memory, it apprehends it only as possessed of a number of qualities. These qualities are apprehended as unlike each other and several, the mind experiencing what is called "the shock of difference." If attention is now fixed on one quality, as the color, or the weight, then while the other qualities consequently become obscure, or perhaps pass out of consciousness, this one on which attention is fixed is thereby drawn into vivid consciousness, becoming the chief, if not the exclusive, object of cognition. Thus by attention to this one quality the mind has been abstracted or drawn away from all others. In this psychological view Abstraction is the negative correlative of the positive act

of attention, the mind being denied to a plurality of qualities, in being concentrated on one. But this one quality may be considered as abstracted or drawn away from all others. In this logical view Abstraction is a positive act by which we cognize one quality apart. It is thus by abstraction that we obtain a clear and distinct notion of the qualities, attributes, properties, characters, features, etc., of an object, all of which terms are nearly synonymous, and are included in Logic under the one term, marks.

It may be at once noted that marks are of several sorts or kinds. They are,—

1st. Positive or negative; as *rational* is a positive, and *imperfect* a negative, mark of man.

2d. Essential, necessary, or accidental, contingent; as *rational* is an essential, and *learned* an accidental, mark of man.

3d. Original or derivative; as *rational* is an original, *calculating* a derivative, mark of man, this being a consequence of his rationality.

4th. Simple or complex; as *conscious* is a simple mark (i. e., one not susceptible of analysis) and *animal* a complex mark of man, the latter being compounded of *organized* and *sentient*. So *red* is a simple mark of one kind of rose, and *vegetable* a complex mark.

5th. Common or peculiar; as *mortal* is a mark common to man with other animals, *risible* a peculiar mark, found in no other being. A peculiar mark is called "a property," as belonging to this, and to no other, yet not considered essential; thus *mobile* is a property of body. A particular mark is one belonging to a single individual alone; as the mark set upon Cain.

The number of marks which may be discerned in any object is indefinitely great. It would be impossible to enumerate exhaustively all the marks which might be discerned in so simple an object as a grain of corn.

§ 3. But objects are presented to us in sense or in memory as many and complex. In our apprehending them, very many of their marks produce the shock of difference, or produce dissimilar impressions; but some give the shock of similarity, or produce similar impressions. The repetition of an impression is precisely what excites attention, or determines the direction of reflective consciousness. When some objects are found to agree in certain respects, while others wholly disagree, consciousness is concentrated naturally on those which partially agree; and then, in them, on those marks in or through which they

agree. So far this is mere abstraction. To give a crude illustration: We observe a number of animals; our attention is attracted to a horse, an ox, a goat, a dog, etc., differing greatly from the birds, reptiles, etc., that may be present, but agreeing in some respects. We then observe more particularly that each has a *hairy hide*, and is *four-footed*, in which marks they agree.

Similar marks are those which stand in similar relation to our organs and faculties of cognition. When the similarity is complete, the effects which they produce in us are indiscernible. But what we cannot distinguish is to us virtually the same; i. e., they are subjectively to us identical, as if they were objectively identical. The same, accordingly, we consider them to be, though really in different objects. This act, to think the similar the same, is the essence of all Generalization; so we may say that to generalize is to think the similar the same. It is a fiction of thought,[1] but one without which our limited powers would be unable to grasp the multiplicity of objects presented to us. We think that each of the animals named in the above example has the same mark, e. g., *four-footed*. This mark is now applicable to either of the objects. A plurality has been reduced to unity in thought. Such a generalized mark is a simple general abstract notion.

We may observe by anticipation that generalization is classification. They are but different aspects of the same operation. By thinking a mark as common to several individuals, we thereby group them, we constitute a class containing them. Thus the animals above named belong to the class or group *quadruped*.

Also we remark that when we speak of observing a number of animals together, we have already thought them as one group. Their common marks have already been generalized, and thereby we have already constituted the total of the objects considered into the class *animal*.

Now let it be noted that, having affirmed the mark *four-footed* of some of these objects, thereby constituting a class, we, in the same act, under the shock of dissimilarity, deny this mark of the rest. The birds, reptiles, etc., are thereby constituted into a negative class; i. e.,

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[1] Overlooking the fictitious character of this act, and thinking the similar to be really the same, gave rise to the erroneous doctrine of the Realists of mediæval times, that a universal objectively exists independently of things, and is common to all things of one kind. See Whately's chapter on Realism, *Logic*, p. 305 sq.; Thomson's *Outline*, § 62; and Ueberweg's *History of Philosophy*, § 91 sq.

one characterized by the negative mark, *non-four-footed*. These two groups are dissimilar in that one possesses the mark *four-footed*, and the other does not. The sum of the two groups, the A and non-A, equals the total of the universe, *animal*. Further, if we contemplate the special group *quadruped*, we again experience the shock of similarity and of dissimilarity. The ox and the goat each have horns; being similar, we generalize and call them *horned quadrupeds*. The horse and the dog have no horns; being similar in this negative respect, we generalize them into a negative group of *non-horned quadrupeds*. But, at the same time, the two groups being dissimilar in respect of having and not having horns, we think them different or diverse. They are thus specialized, or classified into two co-ordinate kinds, the *horned* and the *non-horned*, subordinate to *quadruped*, which is their sum. This, then, is Specialization, the necessary correlative to generalization, and we may say that to specialize is to think the dissimilar diverse, or different, or distinct. It also is classification. It is not a process distinct from generalization, for neither occurs without the other; they mutually imply each other.

§ 4. The third moment of thought is Conception, its product the Concept. To conceive (*con-capere*) is to grasp together.[2] When a number of marks have been abstracted, they may be collected by thought into one notion, and this referred to substance constitutes a concept. A concept, then, is a union of marks in one notion; or, a concept is a bundle of marks thought of as inhering in some thing.

Every thing presented to the mind has an indefinite plurality of marks. Observation can make many known to us, but our knowledge, though constantly increasing in fulness and complexity, can never exhaust them. Moreover, the limited powers of the mind cannot take in at once all those marks whose presence is known. A representation becomes confused when we attempt to grasp or comprehend in one more than a very few of them. Giving up the attempt, we form a concept of the thing, embracing comparatively few of its ascertained marks, making a selection of those which are most distinctive and essential. The concept or notion, therefore, involves the representation of a part only of the marks of which an individual object is the sum, and consequently affords only a one-sided and in-

[2] So the German *begreifen*, cognate to our words "grip," "grab," "group," etc. Hence *Begriff* is the German for "concept."

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adequate knowledge of the thing. This inadequacy of concepts is a further consequence of the limited powers of mind, which must accept a small part as though it were the whole of a thing.

For example, I take the marks, citizen of Athens, teacher of Plato, son of Sophroniscus, husband of Xantippe, famous, virtuous, inquisitive, moralist, martyr of philosophy,—these and perhaps others,—and constitute my notion of Socrates. I may know much more about him, but practically this, or some such limited group of marks, comprises all that I use in representing him.

"Every object," says Drobisch, "is thought as a determinate object only through the marks appertaining to it, by means of which it is comparable, in respect to its nature, with other things, and is distinguishable from them. Without these marks it is only an indeterminate something, a thing or being without further determination. These marks, on the other hand, have no independent being in and for themselves, but can be separated only in thought from the object in which they exist. In the concept of the object, then, there is the thought of an independent but indeterminate something united with determinate but dependent marks. The concept of the object is the union of the two."

In the concept as now described we may consider that the constituting marks are not generalized. A notion of this sort is complex, but not general. We may say that such is the concept of an individual, and to this form of concept Esser's definition applies: A concept is the representation of a thing through its distinctive marks. It should be observed, however, that such a concept of an individual is potentially general, potentially a class notion. Thus there might be several persons having all the marks attributed above to Socrates, and forming my notion of him. Should it be found so, I must seek and add to my concept some particular mark, and thus secure the singular character of my concept. Hence every concept of an individual should comprise at least one particular mark by which it is distinguished from all other things.

When a concept is constituted of common marks, marks which have been generalized, the notion is then complex and general, and contains under it the several things to which the marks are common; i. e., it is a class notion. For example, I take the following various marks, which I have abstracted and generalized, which I have thought of as common to a number of individual things: self-luminous, bright, sparkling, celestial, very distant, relatively fixed, etc.; and,

making a unity of this plurality, I form my concept of *star*. This complex notion is applicable to, or contains under it, a host of distinct things, for each of many individuals has all these marks. The notion is therefore general, and the word "star," which has been adopted to stand for or express this bundle of marks, is the common name of many individuals. To this form of concept Mansel's definition applies: A concept is a combination or reduction to unity in thought of the similar qualities or marks of the objects thought upon, which are thereby constituted into a class.

We may now more adequately define thought as the act in which the mind knows things by means of concepts. To think is to conceive,—is to form concepts.

§ 5. It is obvious that the concept expresses merely the relation of similarity between the things it denotes, implying, of course, that there are also differences. But a mere relation cannot be realized in consciousness; for to suppose we can cognize a relation of things, and yet not the things related, is contradictory and absurd. Or, an act of cognition necessarily implies an object cognized; but a relation stripped of its terms cannot be an object, since there is nothing in it opposite to the thinking subject. A concept, therefore, is not cognizable in itself; that is, it affords no absolute or irrespective object of knowledge. A concept can be realized in consciousness only by applying it to one or more of the objects related as similar in those respects. When we attempt to represent by an image any abstract generality as an absolute object, we find it impossible. We can thus realize it only as attached to some individual and determinate object. Its whole generality is found to consist in this: that though we must realize it in thought as comprised in some individual of the class, we may do it under any. The generality of a concept, then, is potential, not actual.[2]

For example: I have the general notion *triangle*, a figure of three sides. This term is applicable to several species, among others to the equilateral and to the scalene. Now should I attempt to realize *triangle* in its generality, it must be at the same time both equilateral and scalene. But herein is a contradiction; the image must have its sides all equal and yet unequal. Hence such an image is impossible. Still, while the image cannot be both equilateral and scalene, it must

[2] See Hamilton's *Logic*, pp. 91, 96.

be one or the other. I image, then, or else draw with my pencil, an equilateral triangle; and by disregarding the equality of its sides, and all particular marks characterizing this individual figure, I can contemplate alone the notion *trilateral figure* which it comprises. Thus only is the concept triangle realized in consciousness.

§ 6. It must not be understood that the three momenta now described are actually separate and successive in the mind. They are not in reality distinct and independent acts, but are only so distinguished and stated in order to enable us to comprehend and speak of the several aspects of an actually indivisible operation. It is merely a logical analysis. For instance, the generalization of a mark cannot occur without a classification; that is, without a grouping of several objects in one class, which is essentially conception; and, again, abstraction is analysis, which implies that there was already by the mind a synthesis, though it may have been very obscure, of marks, from among which one is drawn into clear consciousness; but a synthesis of marks, however obscure, is conception.

Moreover, a mark and a concept are commutable. Every mark is potentially a concept, and every concept potentially a mark. A concept is expressed by a substantive or substantive phrase; a mark by an adjective or adjective phrase; and the transmutation of these grammatical forms, corresponding to the change in the aspect of thought, is a familiar fact. Thus: "Man is an animal, or is animal." Here *animal* is used first as a concept, next as a mark. The distinction consists in the use made of the notion by thought. If used denotatively, the notion is a concept; if used connotatively, the notion is a mark. Thus: "Man is an animal" means that man is one of the kinds of things denoted by the class *animal;* but "Man is animal" means that man has the attributes connoted by the mark *animal.*

Let it be now observed that here, and throughout this treatise, the word "notion" is used generically; it means either a concept or a mark. The two are so freely convertible, so constantly changing the one into the other in thought, that we need a common term to express either. For this the word "notion" is most suitable.

§ 7. In this connection may be noticed another very subtile but very common play of thought. A mark, which is strictly only the quality or attribute of something, is, this relation being obscured by abstraction, often thought of as though it were itself a thing. In-

stead of being referred to its original substance, it is, as it were, completely severed therefrom by thought, and established in an independent existence. Marks so treated are specifically called "abstractions," and are expressed by abstract terms, of which a great number have the termination "-ness." Thus: *blue* is a mark of the sky, of the ocean, of sapphire, etc.; but *blueness* is thought of as something independent of these things, and spoken of as though it had a real existence by itself. Again, Aristides is *just*, but *justice* is extolled apart from any person. In the one case the mark is thought as concrete (*con-crescere*=to grow together), as inherent in something; in the other case it is thought as entirely abstract. These are proper opposites. Human is concrete; humanity, abstract. A concrete term is the name of an object; an abstract term the name of an attribute. An abstract term, then, is the name of a mark thought as a thing.[3]

The uncounted multitude of such terms in every refined language shows what familiar use is made by human thought of this fiction. The concrete and the abstract are different regions of thought, and the difference should be clearly and constantly observed, as a confusing of one with the other often leads to the grossest absurdities. Plato, in the *Sophistes*, not recognizing the factitious and fictitious nature of abstracts, argues that things may exist which are incorporeal; for justice and wisdom, says he, are incorporeal; and justice and wisdom must be something. By "something" he means a thing capable of existing in and by itself, and not merely as the quality of some other thing. From this source grew the Platonic doctrine of Ideas, teaching that abstracts are independent entities. The Aristotelian doctrine of substantial forms and second substances, and all the idle speculations respecting τὸ ὄν, τὸ ἕν, τὸ ὅμοιον, and similar abstractions, have the same origin. Many of the gross blunders of modern metaphysics are attributable to this confusion of the abstract and concrete. "If the student of philosophy would always, or at least in cases of importance, adopt the rule of throwing the abstract language in which it is so frequently couched into a concrete form, he would find it a powerful aid in dealing with the obscurities and perplexities of metaphysical speculation. He would then see clearly the character of the immense mass of nothings which constitute what passes for philosophy."[4]

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[3] See Mill's *Logic*, bk i, ch. ii, § 4.

[4] Bailey's *Letters on the Mind*, vol. ii, p. 159. See remarks of Bain, *Logic*, p. 52 sq.

§ 8. It is important to consider the relation of language to our concepts. A concept would immediately fall back into the infinitude and confusion from which it has been called out were there not some means by which to render it permanent. This is accomplished by words. The concept is fixed and ratified in a verbal sign, by means of which it can at any time be recalled into consciousness. Language, then, is the attribution of signs to our cognitions of things. It is the register of thought. Many thoughts are valuable either not at all or only for the moment, and are dismissed. Any one of high value and needed for further use is preserved by a sign; we give it a name. "*Nomina sunt notionum notæ.*"

The name of a general notion is a common noun. Every common noun, therefore, expresses a fasciculus of attributes belonging to each of several objects. It stands for a product of thought, and is a factitious unit to be used in further thought. We have already remarked that a concept is expressed by a substantive noun, a mark by an adjective noun, and that an abstract noun is the name of a mark considered as a thing. We may add that a verb is the name of an action,[5] and also that many notions are registered in phrases instead of single words; e. g., we have no single word to express our notion of "a rainy day." A singular noun applies to only one object, like a proper name, but then it is singular only in its present application; as "a song," "this world," "my horse," "the king." It is evident that the singular meaning is obtained by adding some limiting word. The indefinite article means "some or any one of the class;" as in "Give us a song." The definite article, together with demonstratives, possessives, etc., indicates a particular individual, yet designates it as belonging to a class; as "The king comes," "Cæsar's army." All such names are connotative, they imply attributes or marks, and when used to denominate a subject they carry these marks into the subject and attribute them to it. A proper name strictly is non-connotative.[6] It denotes an individual, but does not indicate or imply any attributes of that individual. It is not the name of a quality or qualities; it is but an unmeaning mark or sign which we connect in our minds with an object, so that when this sign meets our eyes or ears we may

[5] J. C. Scaliger traced the distinction between the noun and the verb to a difference of time; for the noun represents a permanent thing, the verb a temporary and transitory state.

[6] See Mill's *Logic*, bk. i, ch. ii, § 5, for an able discussion of the distinction between connotative and non-connotative terms.

think of that individual; it does not of itself connote or imply any quality of the individual, nor convey any information respecting it. This is true of the proper name considered as the name or sign of an individual object presented to mere intuition. But if it stands for or expresses my notion of an individual, it is evidently a complement of marks, and connotes an indefinite plurality, as in the example given above (§ 4) of the notion Socrates. When Euclides, having heard of the fame of Socrates, went from Megara to Athens to see him, and some one pointed him out, saying, That person is Socrates, then certainly the proper name, thus attributed to the person, connoted and carried with it the marks which constituted Euclides' notion of Socrates, and identified this concept with that person.

While language is not absolutely necessary to thought, for the thought must have been prior to its name, it is necessary to any considerable progress. Without it we could never rise above the very lowest degrees in the scale of thought. A sign is necessary to give stability to our intellectual progress, to establish each step in advance as a new starting-point for our advance, to another beyond. Without language there could be no knowledge realized of the essential properties of things, and all ascent from the sphere of sense to the sphere of moral and religious intelligence is without it impossible, or possible only to a very low degree.[7]

In thinking without language, it follows from what was said in § 5 that at every step in the process each motion must be realized in consciousness by the image of an example. It is obvious that this is a clumsy and very restricted procedure. By the device of language the mind is emancipated from the necessity of continuous realization. Instead of this intuitive thinking, or, as I would prefer to call it, this thinking by example, we may think by signs, either perceived or imaged, which is called symbolic thinking.[8] As Berkeley remarks,[9] "It is not necessary, even in the strictest reasonings, that significant names, which stand for notions, should, every time they are used, excite in the understanding the ideas they are made to stand for. In reading and discoursing, names are used, for the most part, as symbolic letters are in algebra, in which, though a particular quantity be marked by each letter, yet, to proceed right, it is not requisite that

[7] See Hamilton's admirable exposition of these points, *Logic*, pp. 98, 99.

[8] Εστὶ μὲν οὖν τὰ ἐν τῇ φωνῇ τῶν ἐν τῇ ψύχῃ παθημάτων σύμβολα.—Aristotle, *De Int.* ch. i.

[9] See *Minute Philosopher*, Dialogue vii, § 8.

in every step each letter should suggest to your thoughts that particular quantity it was appointed to stand for." By this means the facility and range of thought are vastly increased.

There is peculiar danger, however, in this use of words as temporary substitutes for thoughts. Campbell shows that by it many judicious and well-informed persons are sometimes led to talk and even to write nonsense without knowing it.[10] Thus, one might trippingly, or, as we say, thoughtlessly, speak of "a bilinear figure," or "an involuntary donation," or say "the weather is cold as blazes." The Psalmist corrected himself: "I said, in my haste, all men are liars;" which was well, for this saying included him, and therefore, if true, was very likely a lie. And this reminds us of the saying of Hobbes, that "words are the counters of wise men, but the money of fools." It is consequently needful, says Mansel, at the end of a process of thought, and occasionally at intermediate stages, to submit the result to the test of an example, and ascertain the possible coexistence of the attributes in a corresponding object of intuition. The existence of a class is possible only if the existence of the individual members is possible; hence symbolical cognition supposes intuitive cognition actual or possible as its condition, and derives its validity from it. The test of thought, then, as a possibility, is an image of an example, which is possible only in the absence of self-contradiction. We must, then, envisage our notions, look them in the face, and, thus realizing them, insure that they do not involve contradictory attributes. This is done by the intuition of a case, or an example, called, by the Germans, *Anschauung*, which may well be translated an envisaging.[11]

Symbolic conception, then, is that in which an arbitrary sign present to sense or imaged by the mind, and associated with the attributes of a general notion, is regarded as significant of all the members of the class.

As employed in symbolic thinking, the concept may now be defined as a collection of attributes united under a sign, and representing possible objects of intuition.

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[10] *Philosophy of Rhetoric*, bk. ii, ch. vii.

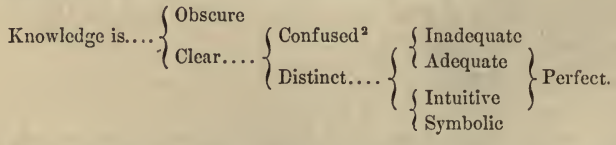
[11] *Prolegomena Logica*, p. 35 sq. and p. 106. See also Hamilton's *Logic*, Lecture X.

II. QUALITY.

§ 1. Concepts may be viewed in four ways. First, with reference to the things, the external objects which they represent, and in which, directly or indirectly, they originate, they are considered as arising from them as their source; as constituted of the marks or qualities of the things; as applicable to one thing, or common to many: this is their Origin. Secondly, with reference to the mind, or thinking subject, they are considered as having gradations towards perfection; as being more or less clear, distinct, etc.: this is their Quality. Thirdly, with reference to their contents, they are considered as comprehending marks, or as extending to things: this is their Quantity. Fourthly, with reference to each other, they are considered in reciprocal relations as the same or different, as containing or contained, as co-ordinate or subordinate: this is their Relation. Their Origin having been considered under the previous topic, we come now to examine, secondly, their Quality.

§ 2. Leibnitz first thoroughly discussed the quality of concepts. His views were expressed in a famous little tract "On Knowledge, Truth, and Ideas."[1] In it he pointed out the distinction, already examined, between intuitive and symbolic thinking, which, according to Hamilton, superseded in Germany the whole controversy between Nominalism and Conceptualism that agitated France and England for several centuries.

Concepts have quality according as they more or less perfectly represent in consciousness their objects. The following scheme marks the degrees by which knowledge approaches perfection:



[1] In *Acta Eruditorum*, 1684. See a translation of the tract appended to Baynes's edition of the Port-Royal Logic.

[2] It would be better, perhaps, if this were named Indistinct, and then Confused might be taken as a genus to include Obscure and Indistinct.

§ 3. Knowledge is first obscure, then clear. A concept is *obscure* when our apprehension of it is so faint that we cannot separate it from others. E. g., My notion of tuberose is obscure, even if not mistaking it for a kind of rose. I have seen it, perhaps, but cannot form an image of it sufficient to clear it from my notion of lily, fuchsia, and other flowers that may resemble it. My notion of final cause is obscure if I do not separate it from material cause, formal cause, and efficient cause. So the vulgar notions of value, price, utility, capital, rent, etc., are each obscure.

We think a concept *clearly* when we can distinguish it as a unity, as a whole in complete separation from other wholes. Clearness is obtained by negative judgments, denying or setting off other concepts apart from this one, or by remarking the specific difference. E. g., We have a clear knowledge of the faces of our friends, since we readily know one from another. So we have a clear notion of horse when we know that it is not mule, nor ox, nor ass, etc. So our knowledge of justice is clear when we know that it is neither truth, nor benevolence, nor wisdom, nor power. The clearness increases according as we are able to deny of a notion or set off those notions which lie nearest to it. Again, our notion of perfume is cleared by remarking its specific difference; it is something that can be smelled.

§ 4. Clear knowledge is confused, and then distinct. I have a clear knowledge of my friend, yet that knowledge is *confused* or indistinct, since I cannot tell how or by means of what I know him, I cannot describe his features. The artist, however, who painted his portrait knows distinctly his several features. Sometimes an artist can pronounce clearly that a work of art is badly done without being able to give a reason for his judgment. His notion is confused; he says there is something in it, he cannot tell what, that is wrong. My notion of gold is confused. I cannot characterize distinctly either its qualities or its varieties.

A concept is *distinct* when, viewed as a plurality, we can discriminate the marks that constitute it; being confused so long as these marks are indistinguishable. Distinctness is obtained by affirmative judgments. Analytic abstraction precedes, and is followed by, a synthesis,—the mark is affirmed of the thing. Thus the marks become severally known, and we can thereafter discriminate them. The knowledge is then distinct. It is natural and logical when one undertakes to explain any obscure matter, that he should begin by clear-

ing it, especially of those things that lie nearest to it, declaring it is not these, and then proceed to render it distinct by stating what it is. E. g., Justification is not pardon, it is righteousness, etc.[3]

Distinctness of thought has two modes. We think a concept distinctly when we distinguish the marks which it connotes from each other,—this is its distinctness in intension; and, again, when we can distinguish the things which it denotes from each other,—this is its distinctness in extension. Intensive distinctness is obtained by definition, the enumeration of the marks. Extensive distinctness is obtained by division, which discovers the things contained under the concept. Thus, a chemist's notion of gold is distinct: he can both name its marks, i. e., give its intension, and name its varieties (if varieties there be), i. e., give its extension. My notion of thought was obscure until I separated it clearly from perception, memory, and imagination, and it is now becoming distinct by studying its characters and kinds. Our notion of red is very clear, but intensively indistinct; we cannot name the particulars by which we distinguish it from blue. It is, however, extensively somewhat more distinct, as we can name the varieties scarlet, crimson, pink, etc. A primitive notion, such as identity, being ultimate, cannot be analyzed, is without marks, is therefore indefinable, and can be cognized only *per se*. Though perfectly clear, it has no distinctness, either intensive or extensive.

§ 5. Distinct knowledge is inadequate, then adequate. We think a distinct concept *adequately* when the relative number and importance of the marks which it connotes are sufficient to correctly represent the things which it denotes. "When everything," says Leibnitz, "that enters into a distinct notion is distinctly known, when the last analysis

[3] If we would understand Leibnitz, we must keep in mind that he does not distinguish kinds of knowledge, but degrees, and that these graduate insensibly into each other. When I discern in an object some one quality which another has not, this may be sufficient ground for me to declare that they are two, the one is not the other, and so far my knowledge is clear. But this would not, perhaps, be sufficient ground for me to describe the object; I cannot yet tell what it is, and my knowledge is, therefore, still indistinct or confused. But as I discern more and more marks, my knowledge of the object gradually passes from what we call merely clear but indistinct, into distinct knowledge, and I can then describe it. When the distinctness becomes more complete, I can define it inadequately, but not until all its marks are discerned can I define it adequately, i. e., enumerate all its distinctive marks. The whole process from obscurity to perfection consists in a discerning of more and more marks.

is reached, then the knowledge is adequate, of which I scarcely know whether a perfect example can be offered; the knowledge of numbers, however, approaches near to it." Perhaps we have a nearly adequate knowledge of a chess-board, its definition consisting of so few marks, and they so nearly ultimate and simple: a square composed of sixty-four equal squares of alternately opposite colors. Dr. Thomson says, "We may consider any knowledge adequate which carries the analysis sufficiently far for the purpose in view." E. g., A machinist has an adequate knowledge of the machines he has invented, constructed, and used. But this is practical, not logical, adequacy. The great bulk of our knowledge is logically inadequate.

§ 6. Distinct knowledge is also either intuitive or symbolic. We think a distinct concept *intuitively* when we image an example, an individual, containing in it all the marks connoted by the concept, and itself contained under the class of things denoted by the concept. Notions not very complex, and especially those of visual objects, are readily exemplified in an image; but when one is very complex, we are not able to image it completely. Thus we could not image a chiliagon. Even were some such figures before the eye, we could not perceive the difference between one of 1000 sides and one of 1001 sides. "When, however," says Leibnitz, "we are able wholly, or at least to a great extent, to form this image, I call the knowledge intuitive."

"But, for the most part," he continues, "especially in longer analyses, we do not behold at a glance the whole nature of a thing, which would be intuitive knowledge, but we employ signs instead of things. We commonly omit, for the sake of expedition, any explication of these signs in present thought, knowing or believing that we have such explication in our power. Thus, when I think of chiliagon, a polygon of a thousand equal sides, I do not always expressly consider the nature of 'side,' of 'equality,' of 'a thousand,' but I employ these words in the place of the ideas which I have concerning them." This is *symbolic* thinking. All large numbers, such as those which state the velocity of light (186,000 miles per second), the distance of the sun (91 millions of miles), and also all such very complex notions as religion, civilization, the English constitution, war, etc., are known to us only symbolically. Our knowledge of primitive notions, as *unit*, is readily intuitive, while our knowledge of composite ones is, for the most part, symbolical.[4]

[4] Leibnitz was not the first, as Hamilton and Thomson intimate, who re-

§ 7. If knowledge be at the same time both adequate and intuitive, it is *perfect.* We think a concept perfectly when it is clear, distinct, adequate, and individualized in an intuitive example. It is evident that knowledge logically perfect is hardly, or only in rare cases, attainable by the human mind. But we are too easily content with obscure or indistinct knowledge, and thus our thoughts are often vague, or even self-contradictory and absurd, without our becoming aware of it. Then we believe that we see, when really we are blind.

But should our concepts become logically perfect, still our knowledge would be very far from absolute perfection. "Truth," says Cudworth, "is bigger than our minds, and we are not the same with it, but have a lower participation only of the intellectual nature, and are rather *apprehenders* than *comprehenders* thereof. This is, indeed, one badge of our creaturely state, that we have not a perfectly comprehensive knowledge, nor even such as is adequate and commensurate to the essence of things." Yet it is the ability to form concepts of things, to comprehend and understand the many in one, to classify and arrange in order of relations the objects of knowledge, that is above all others the great power of intellect, the glory of the human mind, and that which constitutes its immeasurable superiority over the brute. But, on the other hand, it is the necessity of forming concepts at all, the necessity of resorting to a fiction of unity in plurality, the necessity of making a minute part stand for a vast whole, that marks the imperfection and finite character of the human mind. However perfectly this may be done, it is merely the perfection of a logical device, not the perfection of knowledge. To know things in any measure, the human mind must *think* them, and this constitutes its immeasurable inferiority to the divine mind, which does not think at all, but knows, by the immediate intuition of the things themselves, all things, at once in their real plurality and totality.

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marked this distinction between intuitive and symbolic thinking, though certainly he impressed it on modern philosophy. I find the same distinction clearly implied by Aristotle, in *De Soph. Elench.* ch. i, as follows: "Not being able to point out the things themselves that we reason about, we use names instead of the realities as their symbols. Then the consequences in the names appear to be consequences in the realities, just as the consequences in the counters appear to the calculator to be the consequences in the objects represented by the counters. As, in calculation, those who are unskilled in manipulating the counters are deceived by those who are skilled, so, in reasoning, those who are unacquainted with the power of names are deceived by paralogisms." See *supra*, pp. 28, 29.

III. QUANTITY.

§ 1. We are next to consider concepts with reference to their contents, a view lying more strictly within the province of Logic than the two preceding, which belong rather to Psychology. That a concept may be viewed as a quantity is manifest, since it consists of a variable plurality of marks, and is applicable to a variable plurality of things. And this indicates that the quantity is twofold. It is either an *intensive* or an *extensive* quantity. A concept viewed intensively is said to connote its marks, which are reduced to unity in thought by being conceived as inhering in one substance; viewed extensively, the concept is said to denote its objects or things, which are reduced to unity in thought by being conceived as constituting one class or group, each member of the class possessing all the marks. Its marks, then, constitute the connotation of a concept; its objects constitute the denotation of a concept.[1]

The intension of a concept, or its comprehension or depth, is determined by the greater or smaller number of marks contained in it, and of which it is the sum. For example, the concept *man* is composed of the marks *existing*, *living*, *sentient*, *rational*, all thought as inhering in one substance. This explication of the connotation of the concept *man* is its determination or definition; thus, Man is a being, living, sentient, and rational.

The extension of a concept, or its sphere or breadth, is determined by the greater or smaller number of specific concepts, or of objects contained under it. For example, the concept *man* contains under it the specific concepts *logician*, *chemist*, *artist*, *mechanic*, etc. Again, the

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[1] This important distinction, though taken in general terms by Aristotle, escaped the marvellous acuteness of the schoolmen, and remained totally overlooked until the publication of the Port-Royal Logic, 1662. It was therein, for the first time in modern philosophy, taken and applied by Arnauld, with whom it was doubtless again original. It passed thence into most of the subsequent works on Logic. In Germany the doctrine was developed, but in England nothing beyond Arnauld's exposition was attempted until Hamilton expounded and applied it, borrowing largely from Krug, Esser, and other German writers, as an integral part of the science. That he overestimated its consequences will be seen in the sequel.

concept *logician* contains under it the objects *Aristotle*, *Porphyry*, *Boethius*, *Arnauld*, *Hamilton*, and the rest. This explication of the denotation of a concept is its specification or division.

Observe that while both quantities are said to contain, a concept viewed intensively is said to contain *in* it; or to comprehend, marks, but viewed extensively it is said to contain *under* it other concepts, or things.

§ 2. It is evident that if the number of marks constituting the content of a concept be few, it may extend to a great number of things; and, on the other hand, if the marks are many and distinctive, the concept can include and be predicated of only a small number of things. Thus the concept *bird* has only a few marks, such as *existing*, *living*, *sentient*, *feathered*, *winged*, *biped*, etc.; but it is applicable to, or contains under it, a great variety or number of things; whereas the concept *duck* has more marks, such as *web-footed*, etc., and the variety or number of things thereby denoted is less. Hence we have the general law that the greater the intension, the smaller the extension; and the smaller the intension, the greater the extension; or, in other words, the two quantities are in inverse ratio.

Concepts are modified in thought by changing their content. Thinking in marks, we think out things, and *vice versa*. In theoretical strictness, the thinking in one mark is the thinking out one class or thing, and *vice versa*, and the ratio is exact; but in actual thought, owing to the incompleteness of our concepts, the ratio is very far from exact, and the law applies only in a loose, general sense. The theoretical statement, however, should be limited to the essential and original marks, and does not refer to the accidental and the derivative. Original marks carry their derivatives along with them by necessary implication. The latter, therefore, do not really increase the intension, but only render it more explicit.

It follows from the above that the minimum of intension is the maximum of extension. A concept in which the intension or depth is a minimum is one in which a plurality of marks can no longer be distinguished, i. e., it has but one mark. Such a concept is *being* or *thing*, which connotes only the mark *existing*. It is called a simple concept as opposed to complex or compound. Now the extension or breadth of a simple concept is at a maximum. Thus the concept *being* or *thing* contains under it, or extends to, everything that exists, everything in the universe.

On the other hand, the minimum of extension is the maximum of intension. A concept in which the extension or sphere is at a minimum is one in which a plurality of objects can no longer be distinguished, i. e. it includes in its sphere or applies to but one object. Such a concept is *Aristotle*, or *Hadrian's tomb*, or *Virginia*, or *the sky*, or *to-day's lecture.* Each of these denotes only one object, and is called an individual, because it cannot be logically divided. Now the intension or comprehension of an individual is at a maximum. Thus the concept *Aristotle* contains, or conceivably contains, in it, or comprehends, an indefinite plurality of marks, so numerous as to defy all computation; a number which, theoretically, is equal to the number of the things in the universe.

§ 3. Under a previous topic it was said that an abstract term is the name of a mark thought as a thing. This is a device of thought, bringing mere qualities into a form which enables us to make them the subjects of judgments. A quality, being thus treated as a concept, must be thought as itself possessing the two quantities intension and extension; that is to say, an abstraction is both connotative and denotative.

A compound quality thought as an abstraction connotes its components. E. g., "The wisdom (*abstract*) that is from above is first pure, then peaceable, gentle, easy to be entreated, full of mercy and good fruits; without partiality, and without hypocrisy." Here both positive and negative elements, which, taken together, compose *wise*, are attributed to *wisdom* as its intension or connotation; they are its marks. Now we may say, Charity is wisdom from above, and thus convey into it (*connotare*), or attribute to it, all these marks. Again, an abstraction denotes its several kinds. The wisdom just described is one kind. But we are told there is another, and that "This wisdom descendeth not from above, but is earthly, sensual, devilish."[2] There are, then, at least two kinds of wisdom, and these constitute its extension or denotation. It is evident that the kinds denoted by an abstraction are themselves abstract notions; whence it follows that an abstraction can be predicated only of another, as Charity is wisdom.

Evidently these marks of the abstraction may be attributed to the concrete notion. The above marks may be affirmed of "The spirit-

[2] Epistle of James, iii, 15 and 17. See also 1 Cor. from i, 17 to ii, 16, where St. Paul discusses several kinds of wisdom.

ually wise, or of the carnally wise man." But an abstract has qualities that do not belong also to the concrete. E. g., "The wisdom that cometh from above is more precious than rubies, more to be desired than gold, is a defence better than strength, better than weapons of war." This cannot be said of "The spiritually wise man." The abstraction, then, connotes a new series of marks. What is this series? It does not consist of the component, derivative marks, but of original marks, attributable to the quality merely as a quality: e. g., Wisdom is desirable, ennobling, and rare; that is, it is a desirable, ennobling, and rare quality.[3] With the first series the abstraction is not so complete, so absolute, as with the second, wherein the mark is viewed more thoroughly as a thing. Many more things, therefore, can be said of the abstract than of the concrete notion, which, perhaps, is one reason of the favor shown it by thinkers. A primitive notion, such as *single*, having no components, is, when taken abstractly, without the first series. We can say of *singleness* or *unity* only those things belonging to the second series.

§ 4. It has been already stated that we may think a predicate either as a mark or as a class; as, Facts are stubborn, or, are stubborn things. The one we may call thinking in the intensive quantity; the other, thinking in the extensive quantity. It is true that one quantity implies the other, and we do not think the one without at the same time thinking the other. But in ordinary thinking one of two is in vivid consciousness, while the other, though within consciousness, is comparatively, and it may be very, obscure. Now either phase of thinking may become habitual, one person more attentively considering the qualities of a thing, another regarding it as a member of a class. I am inclined from observation to believe that thinking in intension

[3] Mill, in his *Logic*, bk. i, ch. ii, § 5, says: "A non-connotative term is one which signifies a subject only (e. g., a proper name), or an attribute only. Whiteness, length, virtue, signify an attribute only. None of these names, therefore, are connotative." But is not prudence a virtue? He afterwards modifies this statement, saying: "Even abstract names, though the names only of attributes, may in some instances be justly considered as connotative; for attributes themselves may have attributes ascribed to them; and a word which denotes attributes may connote an attribute of those attributes." His example is the word "fault;" equivalent to "hurtful quality." "This word is a name common to many attributes, and connotes 'hurtfulness,' an attribute of those various attributes." E. g., Slowness, in a horse, is a fault. This means that the quality in a horse which receives this name is a hurtful or undesirable quality.

is more usual with cultivated, and in extension with uncultivated, persons. Compare the scholarly synonyms of mark,—*quality*, *property*, *attribute*, *characteristic*, etc., — with the vulgar synonyms of species, —*class*, *sort*, *kind* or *kin*, *group*, *variety*, *set*, *lot*, etc. Children, too, seem to prefer extension; and hence pupils in Logic usually find more difficulty in understanding the theory relative to intension, this quantity being less familiar. Also, it seems that the literature of thought, from the early days of Greek philosophy until quite modern times, shows a strong inclination to the extensive quantity, describing things by classes; and that the tendency of modern thought is to the intensive quantity, describing things by attributes. Certainly, the literature of Logic, from Aristotle to Arnauld, treats exclusively of extension. Again, this appears in rude languages as compared with the refined, as might be presumed; since a language, in its early stages, gives common names to things in groups, as sorts or kinds; but as it progresses, adjectives multiply, largely derived from the substantive nouns.

If, however, these are facts, they would seem curiously at variance with this other fact, that the quantity of intension is given at once in the very nature of things, since everything has qualities which can be directly apprehended; whereas the quantity of extension, the distribution of things into genera and species, does not exist in nature, for nature gives only individuals, but is a creation of mind itself, and created only through the quantity of intension. The intensive quantity is primary and natural; the extensive, secondary and factitious.

IV. RELATION.

§ 1. In considering the reciprocal relations of concepts, we will view them first intensively.[1] Notions thus viewed are identical or different. Of notions absolutely identical strictly there are none; for unless there be some difference, they cannot be distinguished, and are therefore one. Indeed, the phrase "two things identical," taken strictly, is a contradiction in terms. Yet in Logic we speak of identical notions, meaning those which, having reference to the same object, differ only in being conceived by different minds, or by the same mind at different times, these slight differences being considered as not belonging to the notion itself. Notions whose proper differences are not intrinsic or essential, but only extrinsic or accidental, are relatively identical. Such notions are also called similar, or cognate; and the essential attributes being all common, they are called reciprocating or convertible. Thus signs taken from different languages, as "Compassion and sympathy," "Conspectus and synopsis," "Achromatic and colorless," stand often for similar or cognate notions; and the terms of a definition, as "Grace is unmerited favor," are convertible notions, for each comprises the same essential marks.

Notions are absolutely different when there is no similarity. Strictly there are none; but the term is loosely applied in extreme cases when the similarity is very slight and unimportant, as in "Blue and heavy," or "Money and memory." Notions are relatively different when they have at least one important mark common and one diverse; thus "Saint" differs from "Sinner," "Wise" from "Unwise," "A bright day" from "A dark day."

§ 2. Again, notions viewed intensively are congruent, incongruent, and conflictive. Congruent notions are such as agree, or may be connected in thought. All identical notions are congruent; also many that are not identical; as "Learning and virtue," "Beauty and riches," "Magnanimity and stature;" for though in themselves very different,

[1] The doctrine is, in general, Hamilton's, drawn mainly from Esser, Krug, and Drobisch. See Hamilton's *Logic*, pp. 150–158.

they can easily be combined. Incongruent notions are such as cannot unite in the same object; as "A loud circle," "A bright toothache." Aristotle puts the question "Is happiness praiseworthy?" To this there is no proper answer, for it has no proper meaning. It is an incongruous jumble. Notions are conflictive when the difference is such that one involves a negation of the other; as "Virtue and vice," "Beauty and deformity," "Wealth and poverty." Such notions are said to be in opposition.

Opposition is principally of two kinds, contradictory and contrary. Contradictories are only two; and to affirm or deny either, denies or affirms the other; both cannot be, but one must be; they are reciprocal negatives; as "Blue and not-blue," "Walking and not-walking," "Jew and Gentile," "Simple and complex," "One and another," "A and non-A," etc. In case of contradictory opposition, there are, by the principle of Excluded Middle, only two conflictive notions conceivable. These are disjunct notions. Contraries also are only two; but while they cannot coexist, it may be that neither exists; both may be denied through the affirmation of something else, a *tertium quid*. Thus "White and black," "Running and lying," etc., are contraries. A color may be neither white nor black, but gray. I may be neither running nor lying, but sitting.

In order to define contraries more exactly, we must first define disparate notions. These, like disjunct notions or contradictories, cannot be associated in one notion; they exclude, they deny each other, they are conflictive. They differ from contradictories as contraries were said to do; i. e., it may be that neither of two exists. But disparate notions are more than two. They constitute a series of co-ordinate notions graduating between two extremes; as "White, gray, black;" "Running, walking, standing, sitting, lying;" "Old, middle-aged, young;" "Day, twilight, night." Now the two extremes of a disparate series are contrary notions; e. g., "Day and night," "Wise and foolish," "Tall and short," "Love and hate," "Infinitely great and infinitely small." Aristotle, in the *Categories*, vi, 14, says: Contraries are those which in the same genus are most distant from each other.

It must be observed that pure Logic knows nothing of disparates and contraries, as they necessarily involve matter. When we abstract from the matter of a notion, and consider only its form, it is impossible to know that one notion opposes another, unless one is the mere negative of the other, as A and non-A. Therefore, pure Logic knows no opposition between notions except contradiction.

§ 3. We note one other distinction between concepts viewed intensively. As comprehended, they are either involved or co-ordinate.

One concept involves another when the latter forms a part of the sum of the marks constituting the comprehension of the former.

Two concepts are co-ordinate when they are coexclusive, and both immediately comprehended in the same lower concept.

For example: Socrates involves both famous and Athenian. These are co-ordinate. But Athenian further involves Greek; and Greek, European; and European, human. It is evident that these latter notions are not equally proximate and immediate in "Socrates," that some are given only through others, and that they are to each other in the relation of part and whole. Thus thought evolves the simple out of the complex; and the perfecting of knowledge consists in this progressive unfolding into clear and distinct consciousness the intension of notions originally obscure and confused.

In speaking of concepts as involving, and of marks as parts of a whole, these words are used in a peculiar sense. The parts are not *partes extra partes*, for each mark permeates and informs the whole concept. Thus when I think of *chalk* as both white and brittle, the whiteness and the brittleness are thought to coexist throughout.

§ 4. We now pass to a consideration of the relations of concepts in the quantity of extension, which, however, be it constantly kept in mind, is but a different aspect of the same thing. These relations are of three sorts, inclusion, intersection, and exclusion.

1st. Of Inclusion. One concept is included in another when the sphere or extent of the one coincides with, or is contained under, that of the other. There are two cases of inclusion:

(*a.*) Coextension; as when the spheres coincide or are common.

(*b.*) Subordination; as when one is contained under the other, as a species under a genus, or as an individual under a species.

2d. Of Intersection. Two concepts intersect when their spheres have a common part, and each a part not common.

3d. Of Exclusion. One concept is excluded from another when their spheres have no part common. There are two cases of exclusion:

(*a.*) Co-ordination; as when, though mutually exclusive, both are immediately contained under the same concept.

(*b.*) Non-co-ordination; as when, while mutually exclusive, they are not both immediately contained under the same concept.

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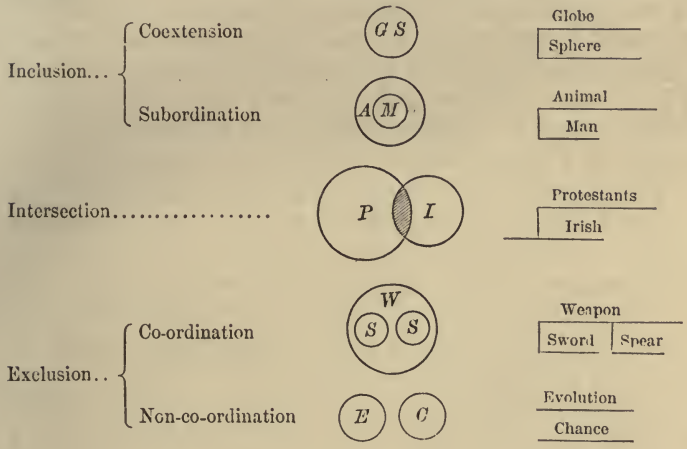
Let us now restate the above, and symbolize by Euler's circular notation,[2] in which the sphere of a concept is represented by a circle; and also by Hamilton's linear notation,[3] in which the extent of a concept is represented by a horizontal line; the relation of two or more, by such lines standing one under the other, and by their comparatively greater or less extent; affirmation being expressed by a vertical line joining two horizontal ones; negation, by the absence of such connection.



Of these relations there are only three that call for special remark, —subordination, intersection, and co-ordination. Subordination will be treated at once; intersection under the topic Definition; and co-ordination under Division.

§ 5. When one concept is subordinate to or contained under another, it differs from the higher concept by comprehending more

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[2] The invention of this method of sensualizing the logical relations of concepts by circles is usually attributed to Euler, who made use of it in his *Lettres à une Princesse d'Allemagne*, 1768. It is found, however, in a posthumous work of Christian Weise, Rector of Zittau, who died in 1708. Ploucquet employed the square, and Maass the triangle, instead of the circle.—Drobisch's *Logic*, § 84; see also Thomson's *Outline*, § 104; and Hamilton's *Logic*, pp. 133 and 180.

[3] This is a modification and an improvement of Lambert's linear notation as found in his *Neues Organon*, 1764. It is to be preferred to the circular notation. Both represent only relations in extension, not those in intension, and therefore, though convenient and helpful, are inadequate. See Hamilton's *Logic*, p. 670 sq.

marks and by extending to fewer individuals. It is called a species. Thus *sword* is a species of *weapon; man* is a species of *animal. Sword* is contained under *weapon;* it comprehends more marks, but it extends to fewer things; it is the narrower notion. The superior concept, since it contains under it more things, is the more general notion, and hence is called the genus. Thus *weapon* is the genus of *sword; animal* is the genus of *man.* The notion *animal* extends to many things besides *men;* it is the broader notion.

It is manifest that genus and species are merely relative terms; for the genus may be contained under some higher concept, and then relative to this higher genus it is a species. Thus *weapon* is a species of the genus *instrument.* Of course the species may contain under it some lower concept, and then become the genus of that lower species. Thus *sword* is a genus containing under it the lower species *sabre, rapier*, etc. A concept that is alternately a genus relative to lower concepts, and a species relative to some higher concept, is called a subaltern genus.

A genus is a universal notion, or *a universe*, because it binds a plurality of parts into the unity of a whole. This is the logical, direct from the etymological, meaning of universe, *ad unum versus.* A universe, then, means, strictly, *E pluribus unum.* It is called, by way of eminence, the Logical Whole.[3] A species, since it is but a part of this whole, is a *parti*cular notion. We should distinguish between the usual meaning of *universe*, as that unlimited highest genus which comprises all things in one, and *universe* considered as a limited genus which unites only some things.

A universe or genus is usually present to the mind of a speaker, within which his thoughts revolve, and under which, often without naming it, he is bringing in his statements. If we apprehend his assumed universe, we may follow and understand his thoughts; if not, confusion is inevitable from the ambiguities of language. Thus the word "civil" has many meanings; it is opposed to "natural," to "military," to "ecclesiastical," to "discourteous," and so on. Now if "civil service" be spoken of, and it is apprehended that the talk is under the tacitly implied universe of "the departments of government," then we understand that it is intended to exclude "military" and "ecclesiastical," and confusion is avoided. In rude parlance we

[3] "Universale totum quoddam est; quippe multa complectitur ut partes. Dicitur totum logicum, quia logicæ munus est de universis disputare."—Burgersdyck.

say, we must know what, in general, one is talking about, in order to understand his particular statements.

Both genera and species are called classes, and the arrangement of things according to genera and species is called classification. The psychological process by which we classify has been somewhat anticipated in the account given of generalization and specialization, which terms are synonymous with generification and specification. When we think the similar to be the same, we form a genus including all the similar things. Thus in contemplating *man* and *brute* we experience the shock of similarity; we abstract from each what is similar; we think it the same, or common to both; we give it a name, and thus establish the class, the genus, *animal*, containing under it *man* and *brute* as species. On the other hand, when we think the dissimilar to be diverse, we form a species, excluding a portion of the things considered. Thus in contemplating *animals* we experience the shock of dissimilarity; we abstract from man the quality *rational*, which marks the diversity; we affirm it of man and deny it of the rest. Thus we establish two species of *animals*, the *rational* and the *irrational*, or *men* and *brutes*.

Finally, the species as parts make up the genus as a whole. These are *partes extra partes*, for they do not coexist, as do marks, but are actually separable groups of things; e. g., *diamonds* and *rubies* are species of *jewels*. Consequently, it is possible to symbolize geometrically, by circles or lines, the relations of concepts viewed in extension, which is not practicable when they are viewed in intension.

§ 6. It should be observed that subordination in the quantity of extension corresponds to involution in the quantity of intension. Also while the term generalization is applicable to either quantity, the term specification relates to extension, and corresponds to the intensive term determination. For determination is a thinking in, a synthesis, a concretion of marks, and this, since it throws out things, specifies a concept. Determination, then, restricts the denotation by amplifying the connotation, and terminates only in individualization.

§ 7. Many concepts are related to each other as correlatives. According to the Law of Relativity, knowledge always includes two things. We know heat by transition from cold; light by passing out of the dark; up by contrast to down. There is no such thing as an absolute knowledge of any one property; we could not know mo-

tion if we were debarred from knowing rest; our first parents had no knowledge of good until it was "bought dear by knowing ill." We may be thinking more of one member of the couple than of the other, of the heat rather than of the cold, of the straight line rather than of the crooked; but if either exists, the other always coexists with it in consciousness. The one is the explicit, the other the implicit, subject of the thought.

This would seem to occasion double names throughout all the universe of things, and language, if complete, would contain no single names, but consist of couples. Accordingly we find a great many couples, specifically called "Correlative Terms," in each of which, if either member be expressed, the other is implied; as "Parent and child," "Ruler and subject," "Cause and effect," "Heavy and light," "Rich and poor," "Genus and species," "Positive and negative."

The last example, "Positive and negative," "To affirm and to deny," is probably the basis, or origin, and the generalization of all the rest. One of the two has usually more or less of a negative character; and in cases where names have not been adopted for both correlatives, one exists in thought as a negative. Hence for every positive concrete name a corresponding negative may be framed as correlative to it by attaching a negative particle, such as the prefixes *un-*, *in-*, and the suffix *-less;* as "Conscious and unconscious," "Temperate and intemperate," "Godly and godless," "A and non-A."[4]

§ 8. Another mode in which concepts are related is expressed by the old and almost disused logical terms First Intention and Second Intention.[5] A first notion or intention is a concept of things formed by the first or direct application of the mind to the object. It denotes things. The concepts which we have been using as illustrations are all first intentions. The object *Socrates* is regarded by the mind as *Greek*, *man*, *animal*, *body*, etc. A mental state may be thought as *a smell*, *a sensation*, *a feeling*, *a consciousness*. All these are first intentions. A second notion or intention is a concept generalized from first intentions. It denotes first intentions or concepts of things. It is the conception under which the mind regards its first intentions as related to each other. Thus the relation of *animal* to *man*, and of

[4] See Bain's *Logic*, p. 2 and p. 55.

[5] *In-tendere*. "Ego dico intentionem nil aliud esse quam attentionem ac diligentiam animæ in alicujus rei consideratione."—Zabarella, *De Reb. Nat.* p. 871.

man to *animal*, is expressed in the second intentions *genus* and *species*. These are concepts of concepts. Adopting, then, the definitions of Mansel, we have the following:

A First Intention is a concept of a thing or things, formed by the mind from materials existing without itself.

A Second Intention is a concept of other concepts, formed by the mind from materials existing within itself.

First intentions precede in order of time, for, as Boethius explains, men *first intended* to give names to things before they intended to find names for their mode of viewing them. The first is the real meaning of a word, the second is its logical value. "Of the first intention," says Hobbes, "are the names of things; of the second are the names of names and speeches." This is the true distinction, but marred in expression by the ultranominalism of the writer.[6]

The distinction between first and second intentions is nearly identical with that between matter and form. Logic is not occupied with things as they exist in nature, but with the way the mind conceives them; not with matter, but with form; not with first notions, but with second. Nearly all logical terms are names of forms, or second intentions; as Universe, Concept, Mark, Property, Accident, Definition, Judgment, Syllogism, Subject, Predicate, etc. Hence Logic is said to treat of second intentions applied to first; and may be well defined as the Science of Second Intentions. Avicenna, the Arabian philosopher, in *Meta.* ch. ii, says, "Subjectum Logicæ sunt intentiones intellectæ secundo, quæ apponuntur intentionibus primo intellectis, secundum hoc quod per eas pervenitur de cognito ad incognitum."[7]

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[6] "Prima notio est conceptus rei quatenus est, ut animalis, hominis; secunda notio est conceptus rei quatenus intelligitur, ut subjectum et attributum."—Pacius.

[7] The distinction is very important, and seems clear enough, but has been remarkably mistaken. Aldrich misstates it; Whately blunders sadly in a guess at it, but with admirable candor adds in a note (*Logic*, p. 202), "I must confess that, after the most patient attention to the explanations given of it, I have never been able to comprehend what is meant by it." We are indebted chiefly to Mansel (see notes in *Aldrich*, pp. 20, 21) for clearing away the mist. See also Thomson's *Outline*, § 16. It seems that of old the same trouble existed, and the profane used to make fun of the venerable scholastics and defame their darling Second Intentions with such burlesque questions as this: "Utrum chimæra bombinans in vacuo posset comedere secundas intentiones?"

V. DEFINITION.

§ 1. In order to give to our thoughts scientific precision, and to systematize them into a scientific whole, we must perform a double operation. First, we must consider what we think, i. e., what is comprehended in thought. Secondly, we must consider of what and how many things we think, i. e., to what and how many objects the thought extends. The comprehension of thought is developed by Definition; its extension, by Division. Our thoughts by this means are rendered distinct, the internal or intensive distinctness being secured by definition; the external or extensive distinctness, by division. Thus we approximate perfection of thought (ii, § 4).

It has already been stated that definition is the explication of the essential and original marks of a thought or concept (iii, § 1). Thus, to repeat the example, *Man* is defined as *rational, sentient, living, existing.* It is manifest, however, that this mode of statement is awkward, and in most cases impracticable. Observing, then, that the notion *animal* involves *sentient, organized, existing,* all the marks that are common to *man* with other concepts, we substitute for them this notion, and define summarily, "Man is rational and animal." The mark *rational,* not included in this summation, is distinctive, as belonging to *man* alone of all the notions that connote *animal.* A logical definition, then, consists of two, and only two, essential and original marks, one of which is common and the other distinctive.

Since the notion defined contains implicitly the marks which the definition contains explicitly, it follows that they are reciprocating or convertible concepts (iv, § 1). Either may be substituted for the other. Thus, "A triangle is a polygon of three sides," and "A polygon of three sides is a triangle." Or, as "Every rectilineal figure may be divided into triangles having a common point," so "Every rectilineal figure may be divided into polygons of three sides having a common point."

Simple notions, as containing no plurality of marks, are incapable of definition. The notion *being,* for example, having only one mark, *existing,* and no differential or distinctive element, is an indefinable, an indefinite notion. It is distinguishable only from *nothing,* a mere

empty negation having no content. Indeed, a simple notion, having but one mark, cannot in strictness be called a concept. On the other hand, an individual cannot be logically defined, since practically we cannot form a notion comprising all the essential and original marks which it has in common with any other notion or thing. An individual can only be described.

§ 2. It is obvious that definition, according to the above account, relates primarily to the intension of a concept. The scholastic logicians, however, viewed it in the extensive quantity, and their view and nomenclature are most usual with us. According to them, a definition consists of the proximate genus and the specific difference. The proximate genus is that class under which the notion defined is immediately contained. Thus *animal* is the proximate genus to the concept *man*. The specific difference is that which distinguishes the notion defined from all other species of that genus. Thus *rational* is the specific difference that distinguishes *man* from all other species contained under *animal*, as *beasts*, *birds*, *fishes*, etc. Let it be remarked that *rational* is also the generic difference, since it distinguishes the notion *man* from the genus *animal*. Such is the scholastic definition *per genus et differentiam*. Other examples are: "Snow is frozen (=specific difference) mist" (=proximate genus); "Logic is the science (=p. g.) of the necessary forms of thought" (=s. d.); "Eloquence is the power of influencing men's conduct (=p. g.) by means of speech" (=s. d.).

These two elements, the proximate genus and the specific difference, make up the whole intension of every notion, for the proximate genus connotes all the marks common to the several species. But to make the explication complete, it is further necessary to define the genus. This done, the same necessity again appears, and is met. We proceed in this manner until we reach a simple notion as the highest and final genus, which cannot be defined. For example:

A carnivore is a flesh-eating (=d.) mammal (=g.).
A mammal is a vertebrate (=g.) suckling its young (=d.).
A vertebrate is an animal (=g.) having an internal skeleton (=d.).
An animal is a sentient (=d.) organism (=g.).
An organism is a living (=d.) being (=g.).

Here we have the whole connotation, "A carnivore is flesh-eating, suck-giving, internal-skeletoned, sentient, living, existing."

§ 3. Concepts often intersect; that is, two concepts often have a common part, and each a part not common. There are Irish Protestants; also there are Irish not Protestants, and there are Protestants not Irish. Some black things are heavy, some not; some heavy things are black, some not. The common part is a species which is contained under each or either of the total concepts as a genus. In other words, whenever a certain group of things may be referred to either of two genera, these genera intersect, the group being a common part.

Now the two portions of a definition, the genus and the difference, may be each viewed as a concept in extension. If so, they will be seen to intersect, and the notion defined to be the common part. Thus the notion *rational* intersects the notion *animal; man*, being both, is the common part; while there are animals that are not rational, as the beasts of the field; and there are rational beings that are not animals, as angels. Ordinarily, we think of man as an animal, bringing him under this notion as a proximate genus; and we use the mark rational as a specific difference to characterize him, to mark him off from other animals. But it is perfectly competent to refer him to rational being as the genus, and to use animal as the differential mark; thus, "Man is a rational being (= p. g.) having animal nature" (= s. d.). Therefore the two portions of a definition are convertible in thought, and it depends wholly upon the use made of them in thought as to which should be called the genus, and which the difference. So, if a watch is a portable timepiece, it may be thought either as a sort of portable thing or as a kind of timepiece; if a concept is a bundle of marks, it may be thought either as a kind of bundle or as meaning that kind of marks which are bundled together. Aristotle observes that specific difference is of the nature of genus.

§ 4. Since a definition is the explication of all the connotation of a thought, the perfection of its definitions is the perfection of a science. In studying a prepared science, we begin with the definitions; but in constructing a science, we end with the definitions. True, in its early stages, we necessarily make constant use of provisional, imperfect substitutes; and so it was that Socrates, presiding at the birth of science,[1] spent his whole life in searching for and analyzing definitions.

[1] The mother of Socrates, Phænarete, was *μαῖα*, a midwife; and, in allusion to this, his method of eliciting truth by questioning was called the *maieutic* method.

But as a science progresses, its definitions are modified, gradually improved, and made real; and when they are finally perfected, the science is perfected.

This gives occasion to distinguish three kinds of logical definition *per genus et differentiam*, the *nominal*, the *real*, and the *genetic*. This distinction is grounded on the matter; pure Logic, as it treats of the form only, does not know kinds of definition. Consequently, if we consider the form only, each of these three kinds of definition exhibits the proximate genus and specific difference. When we look into the matter, we discover such variations and imperfections as justify the above distribution.

Nominal definitions express the meaning of a word as it is popularly understood and used, not explicating all the marks (since common usage requires much less than exact science), and freely employing those that are accidental, derivative, or peculiar. Thus, "A pension is an allowance for past services;" "A violin is a musical instrument having four strings and played with a bow;" "The east is where the sun rises." The definitions given by the dictionaries are mostly nominal. A mere heaping-together of synonyms, as "Law is a rule, decree, or statute," or merely giving the etymology, as "Centaur" means "bull-goader," though often called nominal definitions, are obviously no definitions at all. The imperfect, provisional definitions, spoken of above as preliminary, in order to prepare the way for real ones, are nominal definitions.[2]

Real definition is concerned with the real nature of things; it unfolds all the essential marks in their original form, and these only, and adds none that are not implied in the subject defined. It is therefore strictly analytic. Such are the perfected definitions of a science. An unexceptionable example can hardly be found. The exactness of mathematical thought gives approximations. Thus, "A circle is a plane figure whose periphery is everywhere equidistant from the centre." In practice the distinction between the nominal and the real definition cannot always be clearly descried. They graduate into

[2] The nominal definition, according to Aristotle, is one in which there is no evidence of the existence of the object to which the definition is applicable; as a centaur. Subsequent logicians, especially the recent ones, differ widely from Aristotle and from each other in stating its meaning and distinguishing it from the real. The statement in the text agrees with some of the best authorities, and seems to accord best with popular usage. It is a point of little importance. On the whole subject, see Mansel's *Appendix to Aldrich*, note C.

each other. The requisite that the latter shall consist of the essential and original marks, which constitutes the distinction, evidently relates exclusively to matter, not at all to form. Hence, as said, pure Logic knows nothing of this distinction.

A genetic or causal definition concerns itself with the rise or production of a thing; considers it, not as being, but as becoming. Thus, "A cone is a solid generated by the revolution of an angle about one of its sides." The notion defined not being given, but made, this definition is synthetic.

Logical definitions are sometimes, though improperly, called definitions *a priori*, to distinguish them from definitions *a posteriori*. A definition *a posteriori* generalizes the conditions, or the consequences of a concept, or explicates, not the marks connoted but the things denoted. E. g., "Malaria is that which induces fever;" "Mind is that which knows and feels, desires and wills." Obviously these are not definitions at all, and hence are also called pseudo-definitions. The second example, which merely unfolds the denotation of mental activity, is, of course, strictly a logical division.

An Explication, unqualified, evolves only some of the marks. An Exposition is a series of explications. A Description gives marks or characteristics as concrete in the thing. It deals, therefore, only with the individual, giving any number of its marks, the selection being governed merely by the purpose.

§ 5. A few practical Rules, some of them deduced from the above principles, and useful in forming good definitions, are admissible here. A good definition must be—

1st. **Adequate.** If the genus is not proximate, the definition is too wide. If the difference is not common to all members of the class, it is too narrow. E. g., "Man is a rational being" (too wide); or, "is a praying animal" (too narrow). A convenient test of adequacy is convertibility (§ 1).

2d. **Not negative.** A definition ought to tell what a thing is, but some tell merely what it is not. E. g., "Parallels are lines that do not meet;" "Pleasure is the feeling opposed to pain." Negative statements serve to render a notion clear, and are valuable as precursory to definition, but they do not render a notion distinct (ii, § 3). If, however, the notion defined is essentially negative, as *shadow*, *freedom*, *gentile*, *want*, etc., then its definition is properly negative. E. g., Cuvier, defined an invertebrate as "An animal destitute of an internal skeleton."

3d. **Not tautological.** A definition must not contain the name of the thing defined, nor a derivative nor a synonymous nor a correlative term, for this is to define a thing by itself. This vice is called defining in a circle or reciprocally, or, by the ancients, "diallelon" (διά, ἀλλήλων). It is a sort of logical seesaw. E. g., "Life is the sum of the vital functions;" "A cause is the concurrence that produces an effect." Here the fault is immediate. It may be mediate. E. g., "A board is a thin plank," and "A plank is a thin board;" "Law is the expressed will of a ruler," and "A ruler is one who gives laws." There is a similar vice in reasoning, called by the same names.

4th. **Precise.** It must contain nothing unessential or superfluous. E. g., "Oats is a grain which in England is generally given to horses, but in Scotland supports the people" (Dr. Johnson). This specific difference is unessential. So, "Man is a risible animal." This definition does not fail, nor violate strictly logical purity, but it offends against scientific system or arrangement of thoughts. Again, "A triangle is a figure having three sides and three angles." Here is superfluity. Derivatives should be excluded as superfluous, for they are already contained in their originals. E. g., "The circumference of a circle is a curved line returning upon itself," etc. Every line returning upon itself is a curved line; hence "curved" is superfluous.

5th. **Perspicuous.** It should be intelligible, literal, and brief. We define only to make a thought more distinct; hence terms more confused than the one defined violate perspicuity. E. g., "Net-work is anything reticulated or decussated at equal distances, with interstices between the intersections" (Dr. Johnson). "The soul is the first entelechy or energy of a natural organized body possessing life potentially" (Aristotle). This is obscure, says Leibnitz. Again, all figurative language should be excluded. Tropes, for instance, do not indicate what a thing is, but only something similar. E. g., "The Divine Nature is a circle whose centre is everywhere and the circumference nowhere." Many terms, however, originally metaphorical have ceased to be so. These may be used, and sometimes must be, especially in mental science. Finally, brevity is certainly a merit, but extreme brevity may leave a matter more obscure than needless prolixity.[3]

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[3] See Hamilton's *Logic*, pp. 341–349. His treatment is borrowed almost entirely from Krug, *Logic*, §§ 121–123. See also Mansel's notes in *Aldrich*, pp. 38–43, and Appendix, note C. Aristotle discusses Definition in *Anal. Post.* bk. ii. See especially ch. x.

§ 6. **Praxis.** Analyze, classify, and criticise the following:

1. A line is length without breadth.—*Euclid.*
2. Science is classified knowledge.
3. A pump is a machine for raising water.
4. A beggar is a person who asks alms.
5. Motion is the translation of matter through space.
6. Words are signs of thoughts.
7. A spheroid is formed by the revolution of an ellipse about its diameter.
8. Philosophy is the science of principles.
9. The sun is the orb giving the light of day.
10. An angle is the inclination of two lines to each other.
11. Philosophy is the recognition of mathematical ideas as constituting the world.—*Oken.*
12. The soul is the principle by which we live, feel, move, perceive, and understand.—*Aristotle.*
13. Mind is spiritual substance; or, is the conscious subject.
14. Mind is the unextended.—*Bain.*
15. Attention is consciousness concentrated.
16. Perception is the faculty by which we immediately cognize external objects.
17. A dragon is a serpent breathing flame.
18. A synopsis is a conspectus of the chief points.
19. Logic is the art of reasoning.
20. Logic is the light-house of the understanding (*pharus intellectus*).
21. A pension is an allowance made to any one without an equivalent. In England it is generally understood to mean pay given to a state-hireling for treason to his country.—*Dr. Johnson.*
22. Green is a color compounded of blue and yellow.
23. Dirt is matter in the wrong place.—*Lord Palmerston.*
24. Truth is the agreement of a cognition with its object.
25. A spaniel is a species of dog.
26. A whale is a fish inhabiting the polar seas, and furnishing oil as an article of commerce.
27. Animal is the genus denoting men, beasts, birds, fish, reptiles, etc.
28. Wealth is things useful, necessary, and agreeable.
29. Pain is a disagreeable affection of mind or body.
30. A feeling is a mental affection involving either pleasure or pain.
31. Beauty is the feeling we experience in recognizing unity amidst variety.

32. A Sphinx is an imaginary monster having the head and bust of a woman, and the body of a lion with wings.
33. A circle is a line returning upon itself, all the points of which are equidistant from a given point.[4]
34. A triangle is a figure having three sides.
35. A point is that which hath no parts nor magnitude.—*Euclid.*
36. A fable is a place where animals talk to each other, which also they do not do so.—*From a little girl's composition.*
37. Man is the star-gazing, laughing, food-cooking, trading, provident, instrument-using, two-handed biped.
38. Man is the measure of the universe.—*Protagoras.*
39. Man is the featherless biped.—*Plato.*
40. Common salt is sodium chloride; or, is chloride of sodium.
41. An elephant is an animal that drinks through its nostrils.
42. A dog is a digitigrade quadruped, having fixed claws, four toes, and a recurved tail.
43. Excise: a hateful tax levied upon commodities, and adjudged not by the common judges of property, but by wretches hired by those to whom the excise is paid.—*Dr. Johnson.*
44. Honesty is integrity, is probity, is fair-dealing; or, is the best policy; or, is uprightness in respect to transactions relating to property.
45. Time is a measured portion of indefinite duration.
46. Motion is the act of potential being up to the measure of its potentiality.—*Aristotle.*
47. A plane triangle is a figure produced by a plane cutting a limited cone through its axis.
48. Virtue is a voluntary act done in obedience to the law of God.
49. Monarchy is a form of political government in which one man is sovereign.
50. Capital is wealth destined to consumption.
51. A proposition is a sentence indicative.—*Whately.*
52. Silence is the entire absence of sound or noise.
53. Health is the condition of a living body free from disease or pain.

Define the following terms, both really and genetically, and then consult a geometry:

54. A line,—A straight line,—A curved line,—Parallel lines,—An angle,—A right angle,—A plane,—A figure.

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[4] Hamilton's example of Real Definition (*Logic*, p. 343).

VI. DIVISION.

§ 1. The correlative of Definition is Division. As definition relates primarily to the intension, or depth, of a concept, so division relates primarily to its extension, or breadth. A definition explicates or evolves marks; a division explicates or evolves subordinate concepts or things. The one develops the comprehension; the other, the sphere. By definition the connotation is analyzed; by division, the denotation. By definition the notion is rendered internally or intensively distinct; by division the notion is rendered externally or extensively distinct. Thus the notion *man* is defined by unfolding its connoted parts, *rational* and *animal;* it is divided by unfolding its denoted parts, as *logician* and *non-logician.* Only by division, says Aristotle, can we be assured that nothing has been omitted from the definition of a thing.

§ 2. As preliminary, it is needful to distinguish two kinds of wholes in or under which the mind thinks the objects presented to it. They are as follows:

1st. The Logical or Qualitative Whole. This is of two sorts:
- (*a.*) The comprehensive, characteristic, or intensive whole, whose parts are marks evolved by Definition.
- (*b.*) The universal, generic, or extensive whole, whose parts are species or things evolved by Division.

2d. The Mathematical or Quantitative Whole. Of two sorts:
- (*a.*) The integral whole.
- (*b.*) The collective whole.[1]

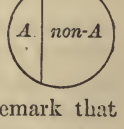
The logical whole, with which we are at present more particularly concerned, is purely subjective, a creation of thought. It is qualitative; i. e., it is the concept consisting of a bundle of qualities or marks, and containing other concepts. These its parts are separable only by mental abstraction. There are two species. The first, the intensive whole (called in the old Logic a metaphysical whole), whose parts are

[1] Logic commonly distinguishes also the Physical Whole, and some others; but we shall find need only for the above. See Hamilton's *Logic*, pp. 142–144.

marks, has been considered under the previous topic. The second, the extensive whole, whose parts are kinds or things unfolded by logical division, is more especially before us.

A mathematical whole is an individual, either objective or subjective, viewed as a quantity, and consisting of parts actually separable. These can be evolved only by the whole being cut asunder, i. e., by partition, which must be clearly distinguished from logical division. Such parts are neither marks nor kinds. This whole is of two species. First, the integral whole is one in which its parts originate. They may be homogeneous, as *a polygon* severed into *similar triangles;* or heterogeneous, as *the human body*, consisting of *head*, *trunk*, and *limbs.* Anatomy is a science of partition, of dissection. *A sword*, which divides into *sabre*, *rapier*, etc., is parted into *hilt* and *blade*, etc. Secondly, the collective whole is an aggregation of similar parts, one originated by the parts. Such are the notions of *an army*, *a forest*, *a town.* These are formed by the repetition of the notions of *a soldier*, *a tree*, *a house.* We must not confound the notion *army*, which is a general or class notion, a logical whole, with the notion *an army* taken as a collective notion, an individual thing formed by a collection of other individual things.

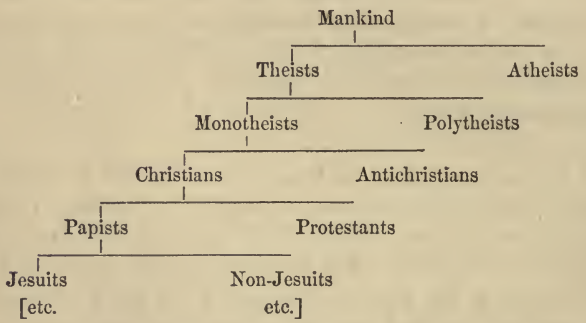
§ 3. It has been already seen how by specialization we form subordinate groups, or species. Since pure Logic considers only the form, each genus or universal whole can contain under it only two species, marked with A and non-A. For A being a generic difference, i. e., a mark not found in the genus or divisum, but found in some of its members, we know *a priori*, without any research into the matter of thought, that the members are exclusive of each other and exhaustive of the divisum. This is division by dichotomy, and the members are contradictories. For illustration: *animals* are *rational* and *irrational*, or *vertebrate* and *invertebrate; angles* are *right* and *oblique;* the *oblique* are *acute* and *obtuse;* the *ancients* were *Greeks* and *barbarians*, or *Jews* and *Gentiles*, or *bond* and *free.* The process viewed intensively, as thinking in marks, is called determination; viewed extensively, as establishing species, is called specification. In relation to each other, the two species are co-ordinate, as being of equal rank in respect of the divisum; but we remark that either may be of indefinitely greater breadth than the other.



The negative member of the dichotomy is characterized by the absence of the mark A, or, in other words, by the negative mark non-A.

Hence we have a peculiar class of concepts called negative, privative, or infinitated concepts. In some cases their sphere is very wide, denoting almost everything, and connoting very little, almost nothing positive. E. g., Ungodly, unhappy, apathy, blindness, senseless, dark, cold, infinite, freedom, shadow, atheist, idle, sober, dead, etc.

§ 4. When the process is pursued beyond a single division,—that is, when a species is regarded as a subaltern genus and subdivided into lower species,—then it is requisite at the outset to select some one mark of the original divisum as a ground or principle on or in reference to which the several divisions shall be made. This generic mark so chosen is called the ground of division, *fundamentum divisionis*. For example, in dividing *Mankind* we select his *religious character* or *creed* as the ground of division, and, subdividing upon the same principle, we obtain a logical series, thus:



The number of distinct forms in which this mark, the principle of division, appears in the things to be divided will determine the extent of the series. This procedure obviously has respect to the matter of thought, and is not strictly pure Logic. We add that, if it is proposed to establish a real division, i. e., one unfolding the true nature of the things contained under the divisum, or, in short, one rigidly scientific, it is requisite to select as a principle of division an essential and original mark of the divisum, and to adhere to it throughout. So logical perfection requires, but it is, in fact, rarely practicable in an extended series.

And this suggests that the distinction made between nominal and real definition may well be carried out relative to division. A nominal or artificial division would be one made for some transient purpose

or to attain a practical end; or one tentative and precursory to a real division; or one popularly accepted and useful, such as the hundreds that may be observed on every page, and in every few minutes of conversation. A real or scientific division would be one proposing to divide notions and things according to their true and essential nature, in order to attain correct objective knowledge of things as they are. Such division develops natural kinds, and is to be looked for in the more refined sciences. The Linnæan artificial divisions of flora were precursory and tentative; those of Jussieu's natural system are real and more rigidly scientific.

§ 5. In divisions not purely logical, but having respect to the matter, it often happens that we have those more than dichotomous; we may have a trichotomy (*τρίχα*, threefold; *τέμνειν*, to cut), or a polytomy. E. g., "Doctrines are helpful, harmless, hurtful." This arises from two causes. Either it is an abbreviation by which a series of subordinate species is condensed into one co-ordinate statement, as, "Angles are acute, right, and obtuse;" or, "Mankind are Christians, Jews, Mohammedans, polytheists, and atheists;" or, "Plants are endogens, exogens, acrogens." Or it arises from the lack of a sharp definition of our concepts. There is between very many of our thoughts a wide border-land which it is impossible to assign clearly to either, constituting a *tertium quid*, a third species which it is necessary to insert in order to exhaust the divisum. Thus we have our *twenty-four hours* divided, with reference to their light, into *day*, *twilight*, and *night*. So we have "White, gray, black;" "Riches, competence, want;" "Young, middle-aged, old," etc. For many of these mediate species we have no names, as between *sick* and *well*; *strong* and *weak*; *long* and *short*; *wise* and *foolish*, etc.

We have remarked that in a strictly logical division the two members, A and non-A, are contradictories; no member of that universe can be both, nor can be neither. In a trichotomy or a polytomy the members are disparate notions. Thus, *brook*, *creek*, *river*, are disparate notions contained under the genus *streams of water*. The two extremes of such a division, as *brook* and *river*, are logical contraries. A thing of this genus cannot be both, but may be neither; it may be the *tertium quid*.

Let it be also noticed that in many cases a notion which seems to have been originally a mere negative of its co-ordinate notion has had thought into it a positive character, so that either may be now thought

as positive and the other as negative; or perhaps both are really positive, and no mere negative exists. Thus, *white* and *black*,—the mere negative is *dark*. So *true* and *untrue* or *false*; *happy* and *unhappy*; *honor* and *dishonor*; *temperate* and *intemperate*, which last has become inverted. So *protestant*. So also *pleasure* and *pain*. Plato taught that pleasure is merely the absence or negation of pain; the Hedonists taught the reverse; but unquestionably both are positive. Also, it was taught anciently that evil is the mere negation of good; and to-day there are those who hold that good is the absence of evil; but both good and evil are positive, and in this case there is no intermediate ground. Actions are either good or bad; there are no indifferent actions.

Finally, a polytomous division admits of one, and only one, strictly privative or negative notion. Thus, "Some men lend, some borrow, some do both, others do neither;" "Plants are monocotyledonous, dicotyledonous, and acotyledonous or flowerless." The intermediate ground, well named the undefined or indifferent part, often takes this negative character; as "Men are very industrious, positively lazy, and neither the one nor the other."

§ 6. The importance of the correlative processes of definition and division cannot well be overrated. They are the reflex respectively of analysis and synthesis, in the balance of which lies the perfection of knowledge.[2] "Such is the excellency of definition and distribution," says an old logician, "that almost they alone do suffice for the absolute putting-down of any art; therefore, the wise Socrates, *in Phædro Platonis*, saith that if he find any man who can cunningly divide, he will follow his steps and admire him for a god."

We shall do well, then, to observe the following practical directions. From the account given, we first present for forming divisions this Canon: Assemble representative instances of the objects denoted by the divisum, and, having fixed upon a generic mark as a principle of division, select a mark immediately involving this principle for a specific difference; then divide the denotation by affirming the specific difference of the species which it determines, denying it of all other contained objects. In subsequent divisions pursue a similar course,

[2] When a notion is adequately defined, and thoroughly divided, we have attained a complete knowledge of its characters and kinds, and this process exhausts its content. See Kant's *Logik*, § 98.

involving in each new specific difference the one immediately preceding, and, of course, the original principle of division.

To this canon we now append the following RULES, useful as a further guide to correct division:

1st. Each division throughout a series should be governed by the same principle, which should be an essential and important mark of the first divisum.

The intervention of a different ground of division in the series gives rise to the logical fault called "Cross division." Thus: "Men are Europeans, Americans, negroes, and pagans." This is an abbreviated series in which the ground of the first division is *geographical;* the second, *color;* the third, *religion.* The members evidently cross or overlap each other; a man may be all of the last three. This very common vice, when more concealed, is detected and the division tested by dichotomy. That is to say, any trichotomy or polytomy, if correct, may be reduced to a dichotomy by taking any one member as positive and including all the others under its negative. If this can be done with each member, without cutting any one, the division is sound. Thus, "Physical substance is animal, vegetable, mineral." Tested: "P. S. is A and non-A (=V+M);" or is "V and non-V (=A+M);" or is "M and non-M (=A+V)." This test applied to the following will clearly demonstrate that it is logically vicious: "The religious sects of Great Britain are Catholic, Calvinist, Episcopal, and Dissenting."

The principle selected must be essential, if we would attain to real, scientific knowledge. It must be important, determining many other attributes, if we would evolve an extended and valuable series. The purpose with which an artificial division is made determines its ground. In civil affairs it would be useless and absurd to divide *men* into *horsemen* and *footmen;* but in military affairs it is important. Words in a grammar are divided according to syntactical relations; in a dictionary, alphabetically. Medical botany and the florist's manual will divide plants differently, and both deviate from Jussieu. We sort our books by *size*, to fit our shelves; by *subjects*, for handy reference; by *binding*, for show.

2d. Dividing members must, as parts, equal the whole divisum.

No one must exhaust the divisum; as, "Mankind are rational beings and politicians." Together they must exhaust it; as, "Governments are monarchies, oligarchies, and democracies." This is insufficient; there are other forms of government. Together they must not more than exhaust it; as, "Vertebrates are quadrumana, bimana, quad-

rupeds, and bipeds." Bipeds and bimana overlap in man. Leibnitz calls this last fault "communicant species." So, "Imaginative writers are poets, dramatists, and writers of tales." Again, "Sciences are deductive and inductive." These species are communicant, since the latter makes large use of deduction. There is no science non-deductive.

3d. *Divisio ne faciat saltum.*

Each species must emerge directly from its own proximate genus. Thought must not overlook and overleap its immediate parts and spring from the divisum to remote species. This the theory requires; but practically, for the sake of brevity, such a *saltus* is allowed, thought passing through intermediate steps to guard against error. Thus we may say that "Mathematics treats of infinitesimals, as well as of magnitudes of assignable quantity." This last member equals "non-infinitesimals." The genus "mathematical subjects" is far from being proximate to these species.[2]

§ 7. **Praxis.** Are the following sixteen examples Partitions or Divisions, or neither? If Divisions, are they correct? If not, point out the defects. If correct, reduce to dichotomous statement.

1. Propositions are affirmative, hypothetical, and negative.
2. Thought is by conception, or by judgment, or by reasoning.
3. The mental faculties are sensation, perception, imagination, memory, and judgment.
4. Is the year or are the seasons divided into spring, summer, autumn, and winter?
5. A flower consists of calyx, corolla, stamens, and pistil; and the pistil consists of ovary, style, and stigma.
6. Literature consists of writings historical, religious, poetical, classical, and current.
7. Matter is solid, liquid, and aeriform. What is the principle?
8. Languages are Aryan, Semitic, and Turanian.
9. Rectilineal figures are triangles, rectangles, parallelograms, and figures of more than four sides.

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[2] See Hamilton's *Logic*, Lect. xxv. His doctrine is drawn mostly from Esser's *Logic*, §§ 134–137. See also Thomson's *Outline*, § 55; and Drobisch's *Logic*, § 119. Division was a favorite method with Plato for the demonstration of Definitions, which Aristotle censures (*Anal. Post.* bk. ii, ch. v), and teaches that its chief use is to test definitions when obtained. Among the later Peripatetics the method was more esteemed. Modern logicians have drawn chiefly from Bœthius's work *De Divisione.* Cf. Cic. *Top.* ch. vi, and Quintil. v, 10. See also Kant's *Logic*, § 113; and Trend, *Elem.* § 58.

10. The Federal domain consists of states and territories; the states, of Northern, Southern, etc.; and each state is divided into counties.
11. The elements of a true civilization are, a wise and just polity, a general intelligence, and an æsthetic culture.
12. Job's family contained sons and daughters. Job's children were sons and daughters. The sons of Zebedee were James and John.
13. The fine arts are drawing, painting, sculpture, architecture, poetry, and photography.
14. Wealth naturally divides itself into three portions — 1st. That which is reserved for immediate consumption, and of which the characteristic is that it affords no revenue or profit; 2d. The fixed capital, which affords profit without circulating or changing masters; 3d. The circulating capital, which affords a profit only by circulating.—*A. Smith.*
15. Profits are divided into interest, insurance, and wages of superintendence.—*Mill.*
16. The origin of colonies is to be traced either to the necessity for frontier garrisons, as among the Romans, or to the poverty or discontent of the inhabitants of the mother-country, as among the Greeks.
17. Divide and subdivide *Triangle* so as to include the scalene, the right-angled, the equiangular, the obtuse-angled, and the isosceles.
18. Make several divisions of *Citizens*, stating the principle in each, into these given species: Laity, aliens, naturalized, peers, clergy, baronets, native, commons.
19. Divide *Man* on the principle of age, sex, family relations, color, stature, riches, rank, education, occupation, and disposition.
20. Are *Books* or is *A Library* divided into folios, quartos, octavos, and duodecimos?
21. Is the distinction of *The Ten Virgins* into five wise and five foolish a logical division or a partition?
22. Divide and subdivide the *Officers of the United States Government* with reference to their official functions.
23. Divide and subdivide *War* on any designated principle.
24. Divide and subdivide *Pleasures* on the ground of their effect on the mind and character.
25. Give the divisions and subdivisions of Topic iv, on *Relation.*
26. Reduce the definitions in v, § 2, to dichotomous divisions.
27. Reduce the divisions in vi, § 4, to a series of definitions.
28. Reduce the definitions in vii, § 6, to dichotomous divisions.

VII. COMPLETE SYSTEM.

§ 1. In concluding this general division of Logic treating of the Concept, it is needful to gather up into one some of the results obtained, and this will give occasion to remark a few additional points. The notion of a series of related concepts has been anticipated, especially under the last topic. We proceed to examine the form of such a series when it is evolved into a complete system.

As preliminary, and at the risk of some repetition, we will present and remark upon the following scheme of the two quantities:

Intension.		Extension.
Existing	Minerals, Plants, Brutes, Men.	
Existing, living	Plants, Brutes, Men.	
Existing, living, sentient	Brutes, Men.	
Existing, living, sentient, rational	Men.	

The most obvious point here illustrated is the law of thought that as intension increases, extension diminishes, and *vice versa;* that the maximum of either one is the minimum of the other; that these two quantities of thought are in inverse ratio.

In ascending the series we think out marks and think in things in the same act. For each mark thrown out, a concept is brought in. Now this act, on the intensive side, this thinking out marks, is abstraction; for in it we draw away a complement of marks, and thus abstract these from at least one other which passes out of consciousness. Thus, we first abstract *existing, living, sentient,* from *rational;* then *existing, living,* from *sentient;* then *existing* from *living.* On the side of extension there is, for each abstraction, a generalization. In thinking out *rational,* we think in *brutes,* i. e., the marks *existing, living, sentient,* are generalized as belonging to *brutes* in common with *men;* and these two classes of things are united in the more general or generic class which we term *animal.* Hence, generalization is also generification. It follows that abstraction and generalization are what might be called directly parallel correlatives; directly parallel, as moving in the same direction in the different quantities.

In descending the series, we think in marks and think out things in the same act. This act, on the intensive side, is determination,

because the bringing in a mark, while it narrows down and fixes specifically or definitely the limit of a smaller class of things, also attains a fuller, deeper knowledge of them. Determination, which in the scheme descends, is the inverse correlative of abstraction, which ascends. On the side of extension, there is for each determination a specialization. In thinking in *sentient*, into *existing*, *living*, we think out *plants*, i. e., the notion organism is specialized into animal by excluding vegetation, and we have established a subordinate, special, or specific class, *animal*. Hence specialization is also specification. Specialization, which descends, is the inverse correlative of generalization, which ascends. Finally, determination and specialization may also be called directly parallel, or, simply, parallel correlatives.

It should also be observed that on the one side abstraction is analysis, and determination synthesis; while, on the other side, the order is reversed, specialization is analysis, and generalization is synthesis. Hence the movement that is analysis in one quantity of thought, is synthesis in the other. The neglect of this distinction by logical authors has led to much confusion in the use of these terms.

§ 2. The isagogue of Porphyry to the Categories of Aristotle, written in the third century, was designed as a detailed explanation of the relations of genera and species. From its doctrine subsequent logicians constructed a scheme which, because of the form it presented, was called by the Latins the tree of Porphyry (*arbor Porpyriana*), and by the Greeks the ladder (κλίμαξ).[1] It exhibits a hierarchy of concepts representing a complete system. The following scheme presents the device in a modified form, with the same matter already used:

Second Intentions. Concepts of Concepts.	First Intentions. Concepts of Things.	Intension or Depth. Marks connoted.	Extension or Breadth. Things denoted.
Summum Genus	Being or Thing	Existing	All Things
Species or Sub-genus	Organism	Ex., Living	All Organisms
Species or Sub-genus	Animal	Ex., Lv., Sentient	All Animals
Infima Species	Man	Ex., Lv., Sn., Rational	All Men
Individual	Aristotle	Ditto and Father of Logic	One Being

[1] For a delineation of it, as given by Thomas Aquinas, see Mansel's *Aldrich*, p. 32. The isagogue will be found appended to Owen's translation of the Organon of Aristotle (Bohn's ed.); and also prefixed to St. Hilaire's *Logique D'Aristote, traduite* (Paris, 1844). The doctrine of the isagogue is drawn largely from the writings, and sometimes is expressed in almost the very words, of Plato.

§ 3. It is evident that the mind, rising from individuals to classes, and by successive generalizations forming wider and wider classes or genera, at each step diminishing the marks connoted, at last must reach a notion of widest generality, connoting but one mark, above which, of course, it cannot rise; and the process necessarily ceases. This highest, widest notion is called the "*summum genus*," and is defined as the genus that cannot become a species. It is represented in the above scheme by *Being* or *Thing*, containing in it only the notion *existing*, and containing under it *all things*. This is a simple notion and cannot be defined, not being referable to a genus.

The Aristotelian logicians consider the *summa genera* as fixed by nature, and ten in number, corresponding to the ten Categories or Predicaments of Aristotle.[2] By the Categories, Aristotle means, metaphysically, a classification *a posteriori* of the modes of objective or real existence; logically, a classification of the most general terms that can be predicated of any subject whatever. They are as follows, illustrated by his own examples:

1. Substance;—it is a man, a horse, etc.
2. Quantity;—it is two cubits long, three cubits, etc.
3. Quality;—it is white, grammatical, etc.
4. Relation;—it is double, half as large, greater, etc.
5. Action;—it cuts, burns, etc.
6. Passion;—it is cut, is burned, etc.
7. Place;—it is in the Agora, in the Lyceum, etc.
8. Time;—it is to-day, was yesterday, last year, etc.
9. Posture;—it is reclining, seated, etc.
10. Possession;—it is having shoes, armor, etc.

Everything that can be spoken of or thought of comes under one or the other of these Categories; in other words, whatever can be a subject of predication is in one or the other of these Predicaments. Each is, therefore, the highest generalization of a series of notions, each a summum genus. Aristotle, in his logical writings, whatever place they may hold in his metaphysics, evidently intends the Categories to be an enumeration of the widest notions signified by single terms. They have excited a world of discussion, been sharply criticised, banished repeatedly to metaphysics, and as often recalled to Logic. Kant objects to them: 1st. That the analysis is not made on any one principle; 2d. That the enumeration is incomplete; 3d.

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[2] *Categoriæ*, ch. iv. See also *Topica*, i, 9; and *Metaphysica*, iv, 7.

That empirical notions are intruded among the pure, and derivative among the original. Hamilton objects that the summum genus of each series is not absolute, but included under one higher.[3]

He redistributes the series thus:

- Being, *ens.*
  - *Per se*, i. e., Substance, *substantia*, (1).
  - *Per accidens*, i. e., mode of substance.
    - Absolute
      - Matter, *quantitas*, (2).
      - Form, *qualitas*, (3).
    - Relative, *relatio*, (4).
      - Action and Passion, *actio et passio*, (5 and 6).
      - Place, *ubi*, (7).
      - Time, *quando*, (8).
      - Posture, *situs*, (9).
      - Possession, *habitus*, (10).

Practically, particular summa genera are assumed in different departments of thought. Usually, that notion is accounted the summum genus which is characterized by the mark selected as the principle or ground of the division. This summum genus is the subject of the science. Thus, in bòtanical science, "Plant" will be the actual summum genus; in zoology, "Animal;" in chemistry, "Compound substance," in political economy, "Wealth;" and so in more common-

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[3] See *Logic*, pp. 139–141; and his note in Reid's *Works*, p. 687. See also Kant's *Kritik der r. V.* p. 65; Mill's *Logic*, p. 45 sq.; and Mansel's *Aldrich*, Appendix, Note B. For historical matter, see Trendelenburg's *Geschichte der Kategorienlehre.*

A popular, concrete illustration of the Categories was given by Cornelius to his pupil in *Martinus Scriblerus*, which, as a mnemonic, we quote as follows:

"Cornelius was forced to give Martin sensible images. So, calling up the coachman, he asked him what he had seen at the bear-garden. The man answered: I saw two men fight for a prize; one was a fair man, a sergeant in the guards, the other black, a butcher; the sergeant had red breeches, the butcher blue; they fought on a stage about four o'clock; and the sergeant wounded the butcher in the leg. Mark, quoth Cornelius, how the fellow runs through the Predicaments: Men (*substantia*), two (*quantitas*), fair and black (*qualitas*), sergeant and butcher (*relatio*), wounded the other (*actio et passio*), on a stage (*ubi*), four o'clock (*quando*), fighting (*situs*), blue and red breeches (*habitus*)."

Another mnemonic is as follows:

1 2 4 3 5 6
Arbor sex servos fervore refrigerat ustos,
7 8 9 10
Ruri cras stabo, nec tunicatus ero.

These two mnemonics will also serve to illustrate the statement that Logic is an analysis not merely of scientific, but of the most common-place thinking.

place matters. See example in vi, § 4, where "Mankind" is the summum genus. But the frequent use of the words "thing," "being," etc., shows what constant mental reference is had to the true summum genus. Indeed, whenever we do not know the proximate or an approximate genus of an object, or do not care to be exact, we mount up on eagles' wings, and call it "a thing." Thus: "A comet is a curious thing." Also, whenever we wish to consider an object relative to some one mark especially, or exclusively, we call it a thing, thus omitting all others by a direct reference of it to the summum genus; as, "Wine is a hurtful thing, because," etc. So, also, when we wish to emphasize some one mark; as, "Cruelty is a hateful thing."

§ 4. On the other hand, when the mind descends in thought, adding marks and rejecting things, it must finally reach a class of things that contains under it only individuals, a class that connotes a maximum plurality of common marks, and denotes a minimum plurality of things. Here the process of logical division into kinds must cease. This deepest, narrowest class is called the "*infima species*;" and is defined as the species that cannot become a genus. It is represented in the scheme by *Man*, containing in it many common marks, and containing under it only individual human beings.

The Aristotelic logicians consider the infima species also as fixed by nature, and expressed in terms like *man*, *horse*, etc. Classes, such as *negroes*, *mustangs*, etc., would not, by them, be admitted to be species at all, but only accidental varieties. But the whole question of natural kinds belongs entirely to the naturalist, and with it Logic has nothing to do. Pure Logic cannot discriminate between essential and accidental marks. The logician gets nothing from objective nature but individuals, and elaborates from them his system without any other restriction than the primary laws of thought. Hence the division into logical kinds continues until no mark, common to even two individuals, remains. The species that comprehends all the common marks is theoretically the infima species, for that only cannot be made a genus by further division.

The individual then, not being a kind, is not a logical part, i. e., cannot be obtained by division. The constituents of the infima species may, however, be estimated numerically, may be counted, and hence it is spoken of as containing under it individuals. But the individual, as the word indicates, is also described as that which cannot be divided. What, then, is the difference by which to distinguish the individual

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from the infima species? It is that, while the infima species consists only of common marks, the individual possesses, besides these, at least one particular mark, represented in the scheme by *Father of Logic.* This particular mark determines only a numerical, and not a specific difference; therefore, the individual cannot be defined, but only described. Such is the logical individual. The actual, or real, individual possesses also a distinct existence in space or time. It can be severed only by partition, and can be discriminated only in perception, external or internal. Its numerical differences are endless.

§ 5. The scheme before us is obviously very meagre and brief, presenting no more than is requisite to exemplify the principles of classification. The extent of any series is, theoretically, incalculable, but practically, and in view of the matter of thought, the upper and lower limits are soon reached. If the characters which afford the principle of such a division are only external and contingent, there is a division in the wider sense; if they are internal and constant, there is a division in a stricter sense; if they are not only internal, but also essential and original, there is a division in the strictest sense. Starting with any assumed summum genus, even the wider divisions must soon practically terminate in an infima species, though the strictest divisions, as in the botanical natural system, may, treated by dichotomy, extend through some hundreds of steps. But pure Logic takes no account of characters as accidental or essential, as congruent or repugnant. As far as the laws of thought are concerned, it is permitted to unite, in an act of conception, any attributes which are not contradictory of each other. The number of attributes in the universe not thus logically incompatible with each other, is infinite, and the mind, therefore, finds no limit to its downward progress in the formation of subordinate notions.

Hence, theoretically, the summum genus and the infima species are both unattainable except *per saltum.* We may approximate, but never reach them. This impossibility is expressed in two laws, as follows:

1st. The law of homogeneity:—Any two notions the most dissimilar must, in some respect, be similar. Consequently they can always be subordinated to some higher concept.

2d. The law of heterogeneity:—Any two notions the most similar must, in some respect, be dissimilar. This dissimilarity furnishes the ground for a new division, which process, therefore, may be continued *ad infinitum.*

§ 6. Before dismissing the tree of Porphyry, attention must be recalled to the relations of definition and division. Definition looks up the scale; division, down. When a subject is to be fully treated, we first define it. We give the specific difference, which sets it apart from co-ordinate notions, and then the proximate genus, the one next above, which involves all the marks of the preceding genera, including the highest. Thus the definition comprises all the scale lying above its subject. Next we proceed to divide and subdivide until we reach and include the lowest species. Thus division, moving downward, exhausts the scale. The system then is complete, the work is thoroughly done, the treatment is scientifically expansive and exhaustive.

It is not necessary that this order should be rigidly observed. In the progress of a treatise the form of definition may often replace division, and one or the other will preponderate according to the point in the scale at which a beginning is made, or according to the inclination of the writer or the nature of the subject. In Plato's Republic, one of the noblest examples of logical method, successive definitions of justice are brought to the test and rejected until a satisfactory one is obtained. Then division preponderates, in the enumeration of the powers of the human soul, and of the classes in a State that answers to them; as well as of the declinations through which the perfect polity, if it could be constructed, would have to pass. The whole is fused together and adorned by a dramatic element, in such a manner as to render this dialogue the finest work of heathen philosophy. In the Nicomachean Ethics of Aristotle, definition predominates, but with considerable aid from division. Thus he enumerates the opinions of men about "the good," and rejects all but the right one. Defining that under the name of "happiness," he is led on to define the parts of his first definition; and, in the case of the moral and intellectual virtues, he does not consider his explanation complete without a division of both classes.[4]

Since definition and division are convertible correlatives, a scientific system may be expressed entirely either in tabulated divisions, or in a series of definitions. These are, *mutatis mutandis*, the same thing. We may begin with the summum genus, and, descending, exhaust the scale by a series of divisions. Or, we may begin with the infima species, and, ascending, exhaust the scale with a series of definitions. Any specific concept being defined, it is requisite to define the proxi-

[4] See Thomson's *Outline*, § 128.

mate genus to which it is referred, and again the proximate genus to which this is referred, and so on, until the summum genus is reached; whence a series, a complete system. As a crude illustration, we give from Political Economy the following:

Wages is circulating capital paid in remuneration of labor.
Circulating capital is capital consumed in a single use.
Capital is wealth destined to reproductive consumption.
Wealth is things useful or agreeable, which cannot be obtained without labor or sacrifice.

This series is readily convertible into divisions; and, to speak generally, definitions and divisions are mutually convertible.

Certain sciences, as Botany and Zoology, are sometimes called the classificatory sciences, because they exhibit their matter mostly in the form of divisions. But all sciences are classificatory, and those referred to should rather be called the dividing sciences, in opposition to defining sciences, such as exhibit their matter mostly in the form of definitions. Chemistry, for example, is eminently a defining science. It exhibits very few divisions. Having named the elements, it employs hardly any other technical names, a compound substance being known generally only by its definition, which takes the place of a name, as "Potassium iodide," "Nitrate of cupric oxide," etc. It would be quite possible to state the relations of chemical substances as genera and species.

§ 7. It is thus, in the manner and with the formal results which have now been described, that we do think, and, governed by the necessary laws of pure thought, it is thus that we must think. Our thoughts are elaborated and rendered distinct by being co-ordinated and subordinated, by being divided and defined, until they are gradually built up into systems more or less imperfect, more or less incomplete. And, be it observed again, this is the case not merely in refined science, but is equally true of our every-day thinking, and that about the most trivial matters. The difference is not in kind, but in degree, the common-place thinking being only more multifarious and imperfect. Every common noun in language occupies a place in some one of the countless hierarchies of concepts which the human mind, for various purposes, has been led to form. Nay, far more than that, every common noun is the point of intersection of a multitude of linear systems crossing each other at all possible angles, and interweaving with each other, so that each occupies a place, not merely

in one, but in many series. It is true that in most minds there is much confusion and disorder in this fabric of thought, an entangling evinced by the indefinite and very ambiguous character of common words. Still, the greater part of the humblest mental life is occupied in generalizing and specializing, in systematically arranging and correcting the arrangement of thoughts.

When Captain Cook landed a cow in the South Sea Islands, the savage natives exclaimed in astonishment: It is a kind of goat! The goat being the only horned animal known to them, they generalized this mark. They specialized by thinking in the difference *large;* so their definition was: A cow is a large goat. It may be hoped that they have now corrected the matter of this classification, but in form their logic was at once perfect. If I should speak of a button, a child might ask: What do you mean by button? It being by no means easy to define this familiar thing, I may escape, and satisfy the querist by naming and describing the different kinds of buttons; or, perhaps more easily still, show it a specimen, saying: This is a button, —which will do pretty well, since, according to the scholastic aphorism, *omnis intuitiva notitia est definitio.* This is more easy, for in it I decline to think the matter, and throw the burden of thinking it on the child. Every book, whose author has well digested his subject, illustrates the point. In turning the leaves, we find the whole divided into parts, the general divisions being so called by way of eminence; the parts are subdivided into chapters; these into sections; these into paragraphs; these into sentences; this external, formal partition corresponding to the internal logical division of the subject-matter. So it is, in matters small and great, we are governed, though for the most part unconsciously, by logical law; and whoever adjusts his notions of things according to their true relations, in systematic order, each clear of others and distinct in itself, his is the cultivated, well-stored intellect, he is eminently the thinker.

§ 8. The inaccuracy in the *usus loquendi* of familiar words requires that they should be largely set aside in building up a science. Hence nearly every science has many unusual, technical terms, sharply defined, and located in its system: such words as are not likely to be drawn into vulgar use, and have their edges worn off by the attrition of every-day handling. In these technicalities a science arranges its classifications in obedience to the logical principles we have discussed, and when its system is complete, it then has attained that logical per-

fection which is specially characteristic of science according to its ideal definition.

Owing to the multiplied divisions in sciences, many have adopted peculiar names also for the several subaltern genera, in order to mark the relative place of each step in the ascending and descending series of classes, and thus mark out clearly and conveniently the various degrees of generalization. Thus the system of Zoology, as given by Agassiz, slightly modified from Cuvier, is as follows:

Second Intentions.	First Intentions.
Kingdom.....	*Animal*, Vegetable, Mineral.
Branch.....	*Vertebrates*, Articulates, Mollusks, Radiates.
Class.....	*Mammals*, Birds, Reptiles, Fishes, etc.
Order.....	Bimana, Quadrumana, *Carnivora*, Herbivora, etc.
Family.....	*Cats*, Dogs, Civets, Weasels, Bears, Seals, etc.
Genus.....	*Felis* (the true Cat), Lynxes, etc.
Species.....	*Lions*, Tigers, Panthers, Leopards, etc.
Variety.....	Nubian, Arabian, Persian, Indian Lions.

The student of Logic would do well to make a thoughtful visit to any well-arranged Museum of Natural History. It presents a logical universe. The summum genus is material product of nature. On entering he finds this universe logically divided, on the principle of succession in time, perhaps into two floors; the lower presenting ancient products, Geology; the upper, recent products, subdivided into Zoology and Botany—extant life, in opposition to the extinct life in the lower division. Between these two floors is, perhaps, a gallery, a sort of *tertium quid*, devoted to Lithology and Mineralogy. We enter the lower apartment, the Hall of Geology. It is subdivided into two halls, one for Paleontology, the other for Structural Geology. The first of these has many co-ordinate subdivisions, the *fundamentum divisionis* being again historical. At first glance the ground seems to be *size*, large specimens being grouped centrally on the open floor, the small being in side cases. But these large specimens mostly belong to the same geologic age, and hence the fault is not serious. Whatever logical offence, however, is involved, it must be pardoned as practically unavoidable. The side cases, we observe, are labelled, each representing a geologic age; one is the Silurian age; another, the Devonian; another, the Carboniferous; and so on in the order of time. If we approach the last named, we find it subdivided into fossiliferous fauna and flora. On the side of the flora we find one set of shelves devoted to the tribe of Phœnogams; another, to that of Calamites;

another, to that of Cryptogams. Looking on the shelves of the latter, we find the Lepidodendrons, the Ferns, and the Equisetæ. In some cases a single shelf is subdivided, giving infima species, and then at last we come to the individual specimens. And so with each of the other departments. In this distribution, it will be observed that the principle of succession in time is abandoned when we come to the interior of the case, and a new ground of division adopted. This is a logical fault, and gives rise to cross divisions. It is, however, justified by the practical results.

If these collected objects were arranged merely to please the eye, they might furnish amusement, but not scientific instruction. It is this logical arrangement according to important natural affinities, evolving a complete system, that distinguishes this museum by the specific difference, scientific. As a product of thought, it offers this peculiar advantage to the student of Logic, that it presents a logical system displayed, not in words, but in the things themselves.

§ 9. We now close this general division of Logic. In it we have considered how thought does and must elaborate its highest and most complete results. We are about to enter upon the second part, which, however, is only another aspect of the details. Before proceeding to the new view, attention is recalled to the three fundamental laws which govern pure thought in every aspect. Their application at each step has been so obvious, that we have felt it needless to point it out. A general example may be here given. If any genus, X, is divided by dichotomy into its species, A and non-A, then the genus X must be affirmed of both these species in turn by the Law of Identity; e. g., Every A is X, and Every non-A is X. The species must be denied of each other by the Law of Contradiction; e. g., No A is non-A. One species being denied of a thing, the other must be affirmed by the Law of Excluded Middle, there being no middle ground; e. g., Whatever is not non-A is A. Such applications should be constantly made in the progress of the subject. The Laws should never be forgotten, as they are the very corner-stone, the root of the whole Theory of Thought.

PART THIRD.—OF JUDGMENTS.

I. THE PROPOSITION.

§ 1. To judge is to bring one thing in or under another. A judgment, as a product of thought, is the issue or result of comparison. Two things or notions compared are apprehended as similar or as dissimilar, and the judgment pronounces that they agree or that they disagree. By virtue of this declared relation, the duality of the notions is reduced to a unity; the two terms being thought in relation are unified. Necessarily one is thought as determining the other. For both cannot be thought as merely determining, since there is then nothing determined. On the other hand, both cannot be thought as merely determined, since there is then nothing determining. Hence, one must be determining, the other determined, the one of the other. Therefore, one is thought as an attribute or mark contained in the other, which is thereby determined; or else it is thought as a class which the other is contained under, and thereby determined.

Before proceeding, it will be well to reiterate that the considerations upon which we are entering are not an advance beyond those just concluded. We are not to advance, since the arrangement of thoughts into a complete system is logical perfection. We are to pass again over a portion of the same ground, but to consider it from a different point of view. The almost complete identity of concept and judgment has already been remarked. A concept is an implicit judgment; a judgment is an explicit concept. E. g., "Man" is a concept that implicitly involves the marks "rational" and "animal;" the judgment "Man is rational and animal" differs from the concept only in that it unfolds, or explicitly states, its content. We are not, then, upon new ground. It is sufficiently apparent that in forming a hierarchy of concepts, every time we subordinate or co-ordinate notions, at every step of division or definition, we pronounce a judgment. What is now proposed is to consider the parts and kinds of these

judgments, and the limiting laws which regulate their formation or determine their validity, to investigate the grounds upon which we do and must judge in determining the relations of our concepts. This is true not only of immediate judgment, but also of reasoning; for oftentimes we cannot determine directly the relation between two concepts, but must do it by comparing each with a third. Let us, then, keep in mind that in what follows we are only improving our knowledge of the modes by which the mind progresses towards completely systematizing its thoughts. And let us also remember that every step is governed by the three primary laws, and, in pure Logic, by no others.

§ 2. A judgment expressed in language is called a proposition. What is subjectively a judgment is objectively a proposition. The first treatment is to sever it by partition into three portions. These are, according to what was said above: 1st. The notion of something determined, called the Subject; 2d. The notion of something determining, called the Predicate; 3d. That which expresses this recognized relation between the two, called the Copula. These terms are correlatives, each implies the existence of, neither can exist without, the other. In every express judgment something is spoken of —that is the Subject; something is said of it—that is the Predicate; that which says this—that is the Copula. Thus, "Snow is Pure;" "Sin is Pardoned;" "Sighs are Prayers;" "some Sentences are Propositions;" "some Stars are Planets;" or, to indicate merely the form, "S is P." The subject and predicate, being the extreme parts in this partition, are called the Terms of the proposition. It is not at all requisite that these terms should consist of single words; they may be composed of many words in intricate grammatical relations. E. g., "The very many difficulties we encounter in the study of an abstruse science (=*subject*) are (=*copula*) to be overcome by persistent effort stimulated by a desire to acquire knowledge" (=*predicate*).

> "With taper light
> To seek the beauteous eye of heaven to garnish (=*subject*)
> Is (=*copula*) wasteful and ridiculous excess" (=*predicate*).—*Shaks.*

The metaphysical meaning of subject and substance is supposed to be understood.[1] We observe that the logical subject must always be a substantive noun (which may consist of many words), i. e., some-

[1] See Hamilton's *Metaphysics*, pp. 104 and 110.

thing thought of as having substance. *Non-entis nulla sunt predicata.* The predicate may be either substantive or adjective, i. e., attributive. We may, however, take the view that, in accordance with its etymology, the subject means that which is thrown under or contained under the predicate.

§ 3. In Aristotle the predicate includes the copula, and this is still the usage of grammarians. But logicians now reckon the copula as a distinct co-ordinate part. Since a judgment always expresses the present relation of two notions now in mind, the copula must always appear as the present tense of the verb *to be.* E. g., "For the mind *is* its own kingdom in which an eternal now does always last."

The copula admits of only one qualification, negation. Hence in a negative sentence the negative particle, wherever it may occur, is considered as a part of the copula. E. g., "The quality of mercy *is not* strained;" "*No* chastisement *is* joyous;" "Britannia needs *no* bulwark," i. e., Britannia *is not* needing a bulwark.

The old logicians held that the copula may be otherwise modified in order to express the degree of certainty that attends the judgment. This is the "Doctrine of Modality." Thus,—

Modal judgments are
- Problematic; as, A may be B.
- Possible; as, A can be B.
- Impossible; as, A cannot be B.
- Necessary; as, A must be B.

The latter two are called "Apodeictic" or "Demonstrative." Recent logicians reject the doctrine of modality, and account the modifiers as a part of the predicate; thus, "A *is* something that may be B." They hold, as above stated, that the copula can be modified in no way whatever except by the negative particle.[2]

The meaning of the copula is ambiguous, or, rather, it has quite a number of different significations. In a following section it will be seen that it may be interpreted either as "comprehends," or as "is contained under." Thereafter we shall find that it sometimes means "is equal to;" and other meanings will appear as we progress. We need to remark here only that it requires interpretation. Always, however, it implies existence, modified or limited by the predicate. Aristotle says:[3] "The copula affirms merely a relative, not an absolute, exist-

[2] See Hamilton's *Logic*, p. 181 sq.

[3] *De Sophistici Elenchi*, v, 3.

ence." From "Ptolemy *is* dead," we cannot infer that "Ptolemy *is*," i. e., actually exists; but only that he exists to us as a dead man can, by remembrance or tradition. So, "Ptolemy *is not* alive" denies his existence relative to life, but implies it in the other sense.

In merely existential propositions the verb *to be* declares absolute existence, it is both copula and predicate. Thus, "I am," "*sum*," means "I *am* existing," or "I *am* a being." The predicate in such case is the summum genus, or its single, simple mark. So, "Enoch *was not*;" "It is fine weather to-day;" "There was a sound of revelry by night" (Byron); "There is none that doeth good, no not one;" "There are men that practise self-denial," i. e., some exist, a very few. Some propositions may be construed as existential or otherwise; as, "It is impossible to love and be wise" may be construed either "To love and be wise cannot *be*," i. e., cannot coexist, or "To love and be wise *is* impossible." So, "There are six Richmonds in the field." So also,—

> "That I have ta'en away this old man's daughter,
> It is most true; true, I have married her."—*Shaks.*

That is, these facts exist; or, these accusations are true.

Very often in common speech the copula is absorbed in verb forms or elided, and the whole proposition may consist of a single word. E. g., "Stars twinkle,"=Stars *are* things that twinkle; "*Cogito*,"=I *am* thinking; "*Pluit*,"=It *is* raining (existential); "*Ilium fuit*,"=Troy *is* something which formerly existed (existential); "Did he come yesterday?" Ans.—"Yes,"=He *is* one who came yesterday; "He loved,"=He *is* one who was loving, or did love. All verbs are perhaps fundamentally one, the verb "*to be*," their variety arising from the incorporation of various attributive notions with this simple verbal element, and its own past and future forms being adverbial notions incorporated with the present tense.

§ 4. In accordance with its postulate,[4] Logic requires that all propositions shall be transformed, as in the above examples, so that, without addition, retrenchment, or distortion of the thought, the three parts, Subject, Copula, Predicate, shall severally appear. The process is sometimes quite troublesome and awkward, but nevertheless must be performed. E. g., "So he said" becomes "What has just been said

[4] See Part 1st, ii, § 8.

is the thing which he said;" "If he should come to-morrow, he will probably stay till Monday" becomes "The happening of his arrival to-morrow *is* an event from which it may be inferred as probable that he will stay till Monday."

It may be observed in this connection that the proposition often exhibits rhetorical inversions, and a displacement of minor parts. E. g., "Great is Diana of the Ephesians;" "Few and short were the prayers we said;" "Flashed all their sabres bare" (Tennyson); "Gold and silver have I none, but such as I have give I to thee;"

> "From peak to peak the rattling crags among
> Leaps the live thunder."—*Byron.*

> "These things to hear
> Would Desdemona seriously incline."—*Shaks.*

> "There is a tide in the affairs of men
> Which, taken at the flood, leads on to fortune."—*Shaks.*

As the subject naturally comes first, Logic further requires that order be restored, the order of the parts as stated above. All such inversions corrected, all elisions supplied, and the three parts stated distinctly in their natural order, constitute the reduction of a proposition to its strict logical form. Hence every proposition must, for logical purposes, be reduced to one or the other of the two invariable forms: S is P, or, S is not P.

§ 5. Aristotle, having before him a notion or thing, asked himself, what and how many kinds of things may be predicated of it? The result was his ten Categories of first intentions.[5] He next asked, what and how many are the *kinds of predicates;* or, in other words, what are the second intentions of all possible predicable things? The result of this inquiry was his equally famous doctrine of the four Predicables. It is that every judgment affirms or denies of its subject one or the other of these four relatives,—

1st. Definition; as, Man is a rational animal	=All of the essence	Convertible.
2d. Property; as, Man is risible	=None of the essence	Convertible.
3d. Genus; as, Man is an animal	=Part of the essence	Inconvertible.
4th. Accident; as, Man is a biped	=None of the essence	Inconvertible.

Aristotle affirmed that every judgment is in the form of one or another

[5] See Part 2d, vii, § 3.

of these four Predicables, and is contained under one or another of the ten Categories. Porphyry and the Schoolmen enlarged the number of the Predicables to five, by substituting Species (as predicable of individuals) and Specific Difference for Definition. This was the reverse of improvement; for, as Aristotle himself had remarked, each of these is of the nature of Genus, and interchangeable with it.[6] The doctrine of the Predicables, however, like that of the Categories, has ceased to play a prominent part in Logic.[7]

§ 6. Various divisions are made of judgments or propositions for logical purposes. As the genus divided is the same in each case, and as a different principle is used in each, it is evident that there will be cross divisions. Thus, an intensive judgment may be either affirmative or negative; and an affirmative judgment may be either intensive or extensive. This is not, however, a logical fault here, since the several divisions are not proposed as steps in a series, but are independent of each other.

The first division to be considered is that judgments are intensive and extensive. This distinction is grounded on the relation of subject and predicate, as containing and contained, as reciprocally whole and part. In the intensive judgment the subject is the whole, or major term; the predicate is the part, or minor term. Thus, "The earth is spherical." Here let us view the notion "earth" as an intensive whole, consisting of a complement of marks. We then attribute to it the mark "spherical," which thereby enters into, or is recognized as a part of, this whole; for it is only one mark out of many that characterize our notion "earth." This is an attributive judgment. It is conventionally expressed thus: "The earth *comprehends* spherical." On the other hand, in the extensive judgment the predicate is the whole, or major term; the subject is the part, or minor term. Thus, "The earth is a sphere." Here let us view the notion "sphere"

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[6] Part 2d, v, § 3.

[7] See Mansel's *Aldrich*, *Appendix*, Note A, for a discussion of the Predicables. The doctrine will be found in *Topica*, i, 8, where Aristotle says: Every predicate either reciprocates with its subject, or does not. If it reciprocates, it expresses either the whole essence (*τὸ τί ἦν εἶναι*) of the subject, or none; in the former case it is called Definition; in the latter, Property. If it does not, it expresses either a part of the essence, or none. In the former case it must be a part of the definition, either Genus or Difference. In the latter case it is evidently Accident, for accident is that which is neither definition nor property nor genus, and yet is present with a thing.

as an extensive whole, constituted of a great many things, such as the other planets, their satellites, the sun, all globular fruits, the geometrical sphere, rain-drops, etc., all which things are included under our notion "sphere." Now in the given judgment we declare that the earth is one of these things, a part of the great complement of things denoted by "sphere." It is conventionally expressed thus: "The earth *is contained under* sphere." This means: "My notion of the earth is contained under my notion of sphere." For another example: "Men are mortal;" this is intensive, attributing the mark mortality to men, the major term. Again, "Men are mortals;" this is extensive, "mortals" is the major term, a genus embracing also "brutes" and "plants," and "men" is a species contained under this genus which is predicated of it. Let it be remarked how the copula is here interpreted and replaced by "comprehends" and "is contained under." [8]

Not only is the copula ambiguous, but most frequently there is nothing in the entire proposition to show which of the two quantities is thought. And, indeed, mind readily passes from one view to the other, and any proposition whatever is easily capable of being interpreted either intensively or extensively. While this is of logical moment, important in a theory of thought, it is not of the smallest practical consequence. One person might peruse a volume viewing every proposition intensively, another read the same volume viewing every proposition extensively, and the knowledge acquired by each would not, for that reason, differ appreciably. It is a fault of the old Logic, however, that all of its nomenclature and treatment has exclusive reference to the quantity of extension.

This seems the proper place to observe that, while a logical proposition may have an individual subject, it cannot have an individual predicate. For the predicate of a logical proposition in extension is a genus; in intension, is a mark. An individual can neither be the one nor the other. We may say "Great is Diana;" but this is a mere rhetorical inversion; "Diana" is the subject, and the predicate is "great." Again, we may say "The favorite pupil of Plato was Aristotle;" but this is not a logical, but an equivalent proposition; one, not in the qualitative, but in the quantitative whole.

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[8] Says Arnauld: "J'appelle *comprehension* de l'idée, les attributs qu'elle enferme en soi. J'appelle *étendue* de l'idée, les sujets à qui cette idée convient."—*Port-Royal Logic*, pt. i, ch. vi.

§ 7. The second division of propositions is into categorical and conditional. The grammatical forms of sentences or clauses, since they are expressions of mental states, correspond with the generic faculties of mind, thus,—

Sentences are	Interrogative,	expressing	Cognition.
	Conditional,		
	Categorical,		
	Exclamatory,	"	Feeling.
	Optative,	"	Desire.
	Imperative,	"	Will.

An interrogative sentence, if the question is real and not merely rhetorical, shows that a comparison is being made which has not yet reached an issue in judgment. It is the search after ground for judgment. Conditional sentences or propositions express a comparison so nearly complete that only certain grounds or premises remain in question. Such doubtful or contingent matter is stated as a condition. They are of two sorts, conjunctive and disjunctive. E. g., "If you understand this, you can explain it;" and, "This view is either accurate or inaccurate." Conditional propositions are indicative, subjunctive, or potential. A categorical proposition expresses a comparison completed; not making reference to any condition, it is absolute. It also appears in the indicative, subjunctive, and potential moods. Logic is concerned only with the conditional and categorical forms; for these only are propositions, none of the other four forms expressing a declaration. The consideration of the conditional judgment is postponed until we shall have finished our examination of the categorical judgment. For the present, then, the words judgment, proposition, etc., unqualified, will be understood to mean the categorical.

The term "categorical" is originally legal, it means "accusing." In Logic it means a downright statement, a predication or attribution unqualified by condition, and hence simple or absolute. A categorical proposition, then, is one in which the predicate is unconditionally affirmed or denied of the subject.[9]

§ 8. The third division of judgments is into total and partial. It

[9] As used originally by Aristotle, the term "categorical" meant merely "affirmative," as opposed to "negative." By Theophrastus, his successor in the Lyceum, it was employed in the sense "absolute," "simple," "direct," as opposed to "conditional." In this signification it has continued to be employed by all subsequent logicians. See Hamilton's *Logic*, p. 207.

is called their Quantity.[10] The quantity of a judgment or proposition is determined solely by the quantity of the subject, according as this is total or partial. The following scheme exhibits the important divisions,—

Propositions are { Total or definite.... { Individual, e. g., All the world's a stage. / Universal, e. g., All men are players. } Partial or particular { Indefinite, e. g., Some men love. / Semi-definite, e. g., Some men seek reputation. }

The quantity of the subject, and hence of the proposition, is indicated by the predesignations *all*, *some*, etc. It is often the case that no sign of quantity is prefixed. A judgment always has quantity either total or partial in the mind of the thinker and speaker, but the hearer is frequently left to surmise the quantity intended from the context, or from the matter. Thus, "Birds breathe," i. e., all do, the predicate being of the essence; "Birds sing," i. e., some do, the matter being accidental or contingent. Some logicians class these as "indefinite propositions," very like as some grammarians specify a "doubtful gender." But, as seen in the scheme, we have another and a better use for the word "indefinite," and these undetermined, or, as Hamilton calls them, "preindesignate" propositions, do not properly constitute a class. When we undertake to reduce such a proposition to strict logical form, it is needful, generally, to designate the quantity of the subject by its sign.

Individual propositions are those in which a whole, the subject, is judged of or viewed as a single, indivisible unity. It may be a proper noun, as, "Cæsar is ambitious;" or an object designated as an individual by the definite article, or by any demonstrative word or phrase; as, "The world is round;" "This man is crazy;" "The whole head is sick, and the whole heart faint;" "All Jerusalem went out to meet him." It may be a collective whole, as, "The senate has adjourned;" "The college of Apostles was typified in the twelve tribes."

Universal propositions have subjects which are logical wholes. The total number of objects within a divisible but undivided class are judged of; as, "All men are players," i. e., all taken together; "Every

[10] This term is unfortunately ambiguous, being used to express two quite different relations; the quantity of thought or of concepts being intensive and extensive, the quantity of judgments being total and partial. If not heeded, this various application of the term is liable to confuse. The quantity of a judgment has no reference whatever to intension or extension.

man is a player," i. e., all taken severally. Such terms are said to be distributed; because what is said is said distributively of each object in the class. It seems, then, that "*All*" is ambiguous, meaning either all as a unity, as in individual propositions; or all as a plurality, as in universal propositions. The former is called the cumular meaning; the latter is called the exemplar, and is its most usual meaning. The signs of universality or distribution are *all*, *every*, *each*, *both*, *any*, *none*, *neither*, *always*, *never*, *whoever*, *wherever*, etc. Names of material substances, as gold, stone, salt, water, flame, etc., are singular and universal without predesignation. They each denote any and every portion of one kind of substance.

Partial or particular propositions are those in which we judge of a number of objects, less than the whole denoted by the naked subject. That is, we judge not of all, but only of some. The old logical meaning of *some* is *some at least, perhaps all;* hence it is only "may be particular." This is the indefinite *some.* De Morgan proposes to call it "vague" instead of "particular" or "indefinite;" and instead of "universal," he proposes "full." The word *some* also is ambiguous; either it is some at least, perhaps all, as, "Some men love," perhaps all do; or it is some at most, not all, as, "Some men seek reputation," not all,—which is clearly true if we mean only that reputation which is found "in the cannon's mouth." The first is the wholly indefinite judgment; the second is the semi-definite, it excludes "all." Whether the predesignation *some* is indefinite or semi-definite, is generally to be determined by the context or matter, but Hamilton, who introduced into our Logic this distinction, which he considers of importance in reasoning, insists that *some* is always thought as semi-definite when the other term of the judgment is universal; a rule that is certainly objectionable.[11] A subject qualified by the article *a* or *an* (except when it means *any*) is particular and semi-definite; as, "A German invented printing," i. e., some one German did. If we substitute for "a German" the name "Faust," the proposition becomes total and individual. The signs of partial or particular subjects are *some*, *not all*, *not every*, *a few*, *there are—that*, *a* or *an*, *one*, *two*, *three*, etc., *sometimes*, *somewhere*, etc. There are also signs that approximate a whole, but, being less than the whole, are still, if taken strictly, particular, though in common speech often tantamount to *all;* as, *many*, *most*, *almost every one*, *the large majority of*, etc. The following

[11] See Appendix to his *Logic*, p. 531.

are nearly total negatives: *few*, *very few*, *hardly* or *scarcely any*, *little*, *small*, *slight*, *rare*, *seldom*, etc.; e. g., "Few are saved," i. e., Nearly all are not, perhaps none; hence indefinite. But *a few* is affirmative; e. g., "A few are saved," i. e., a small number are, perhaps all; hence indefinite. Terms qualified by such signs, or merely thought as particular, are said to be undistributed.

§ 9. The fourth division of judgments is into positive and negative. The positive proposition affirms, by the Law of Identity, that the subject and predicate are in the relation of equivalence, or in that of part and whole, contained and containing. The negative proposition denies, by the Law of Contradiction, such relation, excluding subject and predicate, each from the sphere or comprehension of the other. By the principle of Excluded Middle, no third form of declaration is possible; the relation in question between subject and predicate either does or does not exist, it is yea or nay. Hence, as has been said, every proposition is of the form "S is P," or "S is not P." The ground of this division of judgments is called their Quality.[12]

But let us examine the meaning of negation a little more particularly. Oftentimes a negative judgment simply denies one thing of another, no more. If we say, "Smoke is not vapor," the meaning probably is that these two notions, though liable to be confounded, are essentially so unlike that they should be set entirely apart in thought. There is no thought of a genus. It is simply a holding back from error. So also, if it is said, "Smoke is not fluid," the mark is simply denied as comprehended in the subject, no more. Again, in other negative judgments there is a thought of a genus, which is denied to the subject; as, "Smoke is not a gas;" i. e., the genus gas does not contain under it smoke as one of its kinds. Smoke is excluded from it, simply rejected from the sphere of gases, but no more. Or, thirdly, there may be a mental reference of both notions to a containing genus, under which they, as co-ordinate species, are denied of each other; as, "Men are not brutes." Here the thought is, most likely, limited to the universe *animal*, while *man* and *brute*, as co-exclusive and exhaustive of the genus, are thought as contradictories. Lastly, the notions may be thought merely as disparate, or

[12] Another unfortunate confusion of terms, for the quality of judgments as positive or negative has no reference whatever to the quality of concepts heretofore discussed.

perhaps contrary, and as such denied of each other; as, "Man is not a beast for burdens, nor a reptile for bruising;" or, "See Folly waltzing far from Wisdom's way," i. e., Folly's way is not Wisdom's way. In this case, also, there is a thought of a containing genus or universe limiting the notions, which have much in common, to a narrow sphere.

In a negative judgment the negative particle qualifies the copula, though it may not stand in connection with it. E. g., "Not a drum was heard" (Wolfe); "Not every mistake is culpable;" "No man is wiser for his learning" (Selden); "No drunkard shall inherit eternal life;" "There is none that doeth good, no, not one;" "That goodness is no name, and happiness no dream" (Byron). A negative judgment is said to have "a negative copula," which phrase, taken strictly, is a contradiction in terms, but is used to designate the qualified copula. It is needful to observe that affirmative propositions often contain negatives as a part either of the subject or of the predicate, and should not be mistaken for negative propositions. E. g., "Not to know me argues yourself unknown" (Milton); "He that does not heed, stumbles;" "To wonder not is a rare art," "*Nil admirari prope res est una*" (Horace). In those the negative is a part of the subject. In the following it is in the predicate: "Even in that extremity the general cry was, No surrender" (Macaulay); "On my bended knees I supplicate you, reject not this bill" (Brougham); "We cannot do without it." It should also be remarked that propositions are often essentially negative, wherein no negative particle appears. E. g., "The brute perishes;" "He is blind;" "Darkness and silence fall on land and sea." These also are, in form, logical affirmatives. Negative thought may also be conveyed in affirmative forms by means of such phrases as *beyond, far from, the reverse of, on the contrary, wanting* or *deficient in, devoid of*, and the like.

When the negative particle qualifies the predicate, the judgment is affirmative; it is not a mere denial, but something is affirmed of the subject, though the predicate is a negative notion. We have already remarked that many notions, originally pure negatives, have in usage had thought into them a positive character.[13] These are no longer pure, and are generally accompanied by the thought of a narrow genus or universe, which is not the case in a pure negation; e. g., *helpless, unpleasant, unwell, uneven, indirect, immortal*, etc. Thus, if I

[13] See *supra*, Part 2d, vi, § 5.

say "The soul is immortal," there is affirmed of it, besides the negative notion of infinity, the positive notion of continuous existence. This is a thought very different from that of the pure negative "non-mortal." But it is impracticable to analyze exhaustively the various shades of meaning thus acquired. So, setting them aside, we shall speak only of purely negative predicates.

Affirmative judgments, having a predicate purely negative, combine an act of affirmation with an act of negation. These have been classed by Kant as a third species under quality, the negativo-affirmative, called by him "Infinite or Limitative Judgments." It will be best to give Kant's own explanation, as follows:

"In transcendental Logic, infinite must be distinguished from affirmative judgments, although in general Logic they are rightly enough classed under affirmative. General Logic abstracts all content of the predicate (though it be negative), and only considers whether the said predicate be affirmed or denied of the subject. But transcendental Logic considers also the worth or content of this logical affirmation, an affirmation by means of a merely negative predicate, and inquires how much the sum total of our cognition gains by the affirmation. For example, if I say of the soul, "It is not mortal," by this negative judgment I should at least ward off error. Now by the proposition "The soul is non-mortal," I have, in respect of the logical form, really affirmed, inasmuch as I thereby place the soul in the unlimited sphere of non-mortal beings. Now, because, of the whole sphere of possible existences, the mortal occupies one part, and the non-mortal the other, neither more nor less is affirmed by the proposition than that the soul is one among the infinite multitude of things which remain over when I take away the whole mortal part. But by this proceeding we accomplish only this much, that the infinite sphere of all possible existences is in so far limited that the mortal is excluded from it, and the soul is placed in the remaining part of the extent of this sphere. But this part remains, nothwithstanding this exception, infinite, and more and more parts may be taken away from the whole sphere without in the slightest degree thereby augmenting or affirmatively determining our conception of the soul. These judgments, therefore, infinite in respect of their logical extent, are, in respect of the content of their cognition, merely limitative."[14]

It remains to state here the Aristotelic rule for the distribution of

[14] *Critique of Pure Reason*, p. 59.—Meiklejohn's Tr.

the predicate. We have shown in the previous section that the distribution of the subject is according to the quantity of the judgment; that universals distribute, and particulars do not distribute, the subject. Now the distribution of the predicate, which takes place in thought without any verbal sign, depends on the quality of the judgment. The RULE is: *Negatives distribute the predicate, affirmatives do not.* Some simple examples will suffice to illustrate this rule. Thus, "All houses are buildings," i. e., *some* buildings only, for there are some buildings that are not houses, as forts, bridges, ships, etc.; hence this predicate is undistributed or particular. Again, "No houses are pyramids;" i. e., not *any* pyramids, since no pyramid can be called a house; hence this predicate is distributed or universal. Again, "Some houses are dwellings," i. e., *some* dwellings only, for tents, caves, and ships also are dwellings; hence the predicate is particular. Again, "Some houses are not dwellings," i. e., some houses, such as shops, factories, churches, are not *any* dwellings; hence the predicate is here universal.

It is evident that this rule, which comes from the old Logic, and which Hamilton, as we shall see, impugns as altogether defective, has exclusive reference to the extension of the terms. Its view is that when we affirm, we thereby include the subject in the class denoted by the predicate as merely a part of it; and that when we deny, we thereby exclude the subject from that class wholly.

§ 10. In order to facilitate the statement and analysis of the syllogism, logicians combine the quantity and quality of judgments. There result four forms, which they symbolize by vowel letters,[15] as exhibited in the following

TABLE OF THE PROPOSITIONAL FORMS.

Quantity.	Quality.	Symbols.	Formulæ.	Examples.
Universal	Affirmative,	—A—	All S is (some) P.....	All oaks are (some) trees.
Universal	Negative,	—E—	No S is (any) P.....	No oaks are (any) vines.
Particular	Affirmative,	—I—	Some S is (some) P.....	Some are (some) evergreens.
Particular	Negative,	—O—	Some S is not (any) P...	Some are not (any) shrubs.

[15] It is curious to note that these symbolic letters were first adopted by an old logician, Petrus Hispanus; they being the first two vowels in the words *affirmo* and *nego.* We may add that the old logicians abounded in mnemonic devices, and, accordingly, the said Petrus supplied the following stanza,—

Asserit A, negat E, sed universaliter ambæ;
Asserit I, negat O, sed particulariter ambo.

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Individual propositions (§ 8), since the subject is a total, are usually considered as universals, and symbolized by A and E.

§ 11. The fifth division is of propositions rather than of judgments. Propositions are Simple, Complex, and Compound.

A Simple proposition consists of only one judgment; i. e., it contains not more than one subject and one predicate. It may, however, consist of many grammatical elements; as, "Well-organized and skilfully administered governments are productive of happiness in their subjects."

A Complex proposition involves with the principal judgment one or more subordinate or incidental judgments. This subordinate element appears as a clause, incidental to the principal subject or predicate. E. g., "A man *who is learned* is respected;" "*Whoever is right* is safe;" "*Who steals my purse*, steals trash" (Shaks.); "A little fire is quickly trodden out, *which, being suffered, rivers cannot quench*" (Shaks.); "Ill blows the wind *that profits nobody*" (Shaks.). In these the clause is in the subject, though the latter two are, the first partly, the second wholly, inverted. In the following the clause is in the predicate: "I am monarch of all *I survey*" (Cowper); "The cry is still '*They come*'" (Shaks.); "*When I was a boy*, I used always to choose the wrong side" (Johnson); "*When the age is in*, the wit is out" (Shaks.); "*What I have written*, I have written." In the following there are incidental clauses in both subject and predicate: "They *that are wise* shall shine *as the stars* (shine);" "Shylock, *who was a hard-hearted man*, exacted the payment of the money *he lent* with such severity that *he was much disliked by all good men*" (Lamb).

A subdivision of incidental clauses may be made into Explicative and Limitative, or Restrictive. The Explicative clause merely unfolds the marks connoted by the notion it qualifies; as, "Man, *who is born of woman*, is of few days and full of trouble;" "Jonah sought to evade the God *who is omnipresent*." Explicative clauses express judgments not now made, but formerly made, and now renewed subordinately. Limitative or restrictive clauses, which may also be allowed to include the concessive clause removing restriction, are those which, as the terms indicate, limit or restrict the notion they qualify; as, "Men *who are avaricious* are discontented." This is not said of all men, but is said of all in a limited class. So, "He is well paid *that is well satisfied*" (Shaks.); "Honesty, *when it is mere policy*, is not a

virtue." The concession in "I will trust him *though he slay me*" removes a conceivable restriction. So in "*Live we how we can*, yet die we must" (Shaks.). In "They strive *that they may enter in*," and "They take heed *lest they fall*," the predicates are limited by purpose; one positively, the other negatively. When the restrictive is a condition, the categorical proposition may easily be converted into a conditional. Thus the example above may become "If men are avaricious, they are discontented."

We now observe that, these incidental clauses of all kinds being regarded merely as substantive, adjective, or adverbial elements, the complex proposition is in Logic treated as simple. It was needful to discuss it only that we may be forewarned not to mistake clauses for principal propositions; and, in reducing a proposition to strict logical form, that we may be careful to subordinate them in place to the principal subject or predicate. Thus, "He, *who*, though *he is rich, is saving*, is one *that can share* with him *who is needy* without lessening *what is enjoyed*;" here the form is, S is P. Indeed, the complex sentence is often directly reducible to one that is strictly simple. Thus, the first example given above, "A man *who is learned* is respected," reduces to "A man of learning," or "A learned man, is respected."

The Compound proposition is one that comprises two or more judgments, co-ordinate, or nearly so; and these, for logical purposes, require to be separated and stated independently. It is of two kinds, according as the compounding elements are more or less obvious. The first kind, wherein these elements are quite evident, has received no specific name, and needs only the illustration of a few examples; as, "Art is long, and life is fleeting" (Longfellow); "Every man desireth to live long, but no man would be old" (Swift).

> "We are such stuff
> As dreams are made on; and our little life
> Is rounded with a sleep."—*Shaks.*

> "Men may come, and men may go,
> But I go on forever."—*Tennyson's Brook.*

"*Veni, vidi, vici*," contains three distinct propositions in three words. "Pompey, Crassus, and Cæsar were triumvirs;" here are three propositions: 1st. "Pompey was a triumvir;" 2d. "Crassus was a triumvir;" 3d. "Cæsar was a triumvir." If, however, we say "Pompey, Crassus, and Cæsar were the triumvirs," then the proposition is single and simple, for the three are taken collectively as one whole. So, "Roses and lilies contend for a home in her cheek," is single and

simple; but in "Darkness and silence settle on land and on sea," there are four propositions.

> "Ho! hearts, tongues, figures, scribes, bards, poets cannot
> Think, speak, cast, write, sing, number,—hoo!—
> His love to Antony."—*Shaks.*

In this curious sentence there are six distinct propositions, and were it not that each predicate answers to its own subject we might count thirty-six.

Compound propositions of the second class, having elements less obvious, and requiring analysis, are for this reason called Exponibles. These more than the others require special attention, since they are more intricate, and in syllogizing with them it is often requisite that they be distinctly resolved. We name three species: 1st. Exclusives and Exceptives; 2d. Comparatives; and 3d. Inceptives and Desitives.

1st. Exclusives. Compounds of this species may be formulated thus:

$$\text{Only A is B} = \begin{cases} \text{A is B} \ldots\ldots\ldots\ldots\ldots\ldots = \text{A} \\ \text{No non-A is B} \ldots\ldots\ldots\ldots = \text{E} \end{cases}$$

$$\text{E. g., "Faith alone justifies"} = \begin{cases} \text{Faith justifies.} \\ \text{What is not faith does not justify.} \end{cases}$$

It is obvious that this proposition may be inverted and the exclusive particle made to appear in the predicate; thus, "Justification is by faith alone,"=B is only A.

Exceptives are exemplified in "All but one were saved," which means "Nearly all were saved" and "One was not saved;" I and O.

No useful rule can be given for the resolution of these two forms of exponibles. Generally, if not always, the elementary judgments differ in quality, and one is to be noted as direct and the other as indirect or implied. The distinction between the exclusive and exceptive forms is of no practical moment, as they are readily convertible, the only difference being that what is the direct judgment in the one becomes the indirect in the other. The following are some of the exclusive and exceptive particles: *only*, *alone*, *exclusively*, *merely*, *sole*, *solely*, *but*, etc. These particles annexed to the subject quantify the predicate universally; as, "God alone is wise," i. e., He is all the wise. Annexed to the predicate they merely limit the subject to that predicate; as, "The sacraments are but two," i. e., there are no more.

We give some examples illustrating their various modes of expression to facilitate the recognition of them hereafter. "None but the brave deserve the fair" (Dryden); "A fool thinks none except

himself wise;" "Brutus, in killing Cæsar, was merely patriotic;" "Christ is the only Saviour;" "The moon is only our satellite;" or, "is our only satellite;" "Mercy but murders, pardoning those that kill" (Shaks.); "The paths of glory lead but to the grave" (Gray); "God alone is worthy of being loved for his own sake," i. e., we ought to love God for his own sake, and all other things for God's sake; "Only those riches which you shall have given away will always abide with you," "*Quas dederis solas semper habebis opes*" (Martial, *Ep.* v, 43). Sometimes the exclusive or exceptive particle is in the sense, but not expressed; as, "(There is only) one Lord, one faith, one baptism" (*Eph.* iv, 5).

2d. Comparatives. Propositions in which we compare contain two judgments; for it is one to say that a thing is such, and one other to say that it is more or less so than another thing. Thus, the maxim of Epicurus, that "Pain is the greatest of evils," affirms that pain is an evil, and that it is the extreme one. This is more evident when we consider that the maxim may be contradicted in two ways. The Stoics denied the first component, saying that no pain is an evil. The Peripatetics, however, allowed the first component, but denied the second, saying that vice is the extreme evil. But why may not the same be said of any proposition having a qualified predicate, as "Pain is a great evil?" Because what is said here merely excludes other evils; but in the above comparative other evils are expressly included by what is said.

3d. Inceptives and Desitives. When we say that a thing has commenced, or ceased to be, such, we make two statements, one about the thing as before, the other as after, the time indicated. Thus, "I begin to believe" affirms that I now believe, and that heretofore I did not believe; and "I have ceased to believe" affirms the two contraries. Observe that to say simply "I believe" says nothing of the past. Again, "With Augustus Rome began to be marble," and, "With Augustus Rome ceased to be of brick." These may fairly be interpreted as saying, "Augustus found Rome of brick, and left it marble." That inceptives and desitives are compounds becomes a little more evident when we consider that a question such as "Have you quit drinking?" affirms the component that you have been drinking, and questions only the second, whether you are now drinking.

It should be observed that many judgments which are not classed as compound, whose outward form is simply "S is P," nevertheless imply in thought an indirect judgment. This is true of every semi-

definite judgment (§ 8). Sometimes, on the other hand, we convey our thoughts by indirections expressed; we merely "insinuate," leaving the direct judgment, our real meaning, to be understood. Logic always deals primarily with the latter, and, according to its postulate, gives it complete expression.[16]

§ 12. A sixth division is of judgments rather than of propositions. It is exhibited in the following scheme:

Judgments are { Analytic .... *a priori*. / Synthetic { *a posteriori* or empirical. / *a priori* or pure. }

When the predicate P belongs to the subject S as something which is contained, though covertly, in the concept S, the judgment is called Analytic. Since the predicate adds nothing to the conception of the subject, but only unfolds its constituent marks, which are thought already, though confusedly, in the subject, it is also called Explicative. E. g., All bodies are extended. I need not go beyond the concept body in order to find extension connected with it, but need merely to analyze the conception in order to discover this predicate in it. The analytic judgment is *a priori*. It is not grounded on experience, because I need not go out of the sphere of my conceptions to form it, and hence resort to the testimony of experience is quite unnecessary. That bodies are extended is not an empirical judgment, but itself stands firm *a priori*. It is a necessary judgment, its necessity arising from the ground of identity. Analytic judgments are highly important, but are so only because by them we attain the distinctness of conception which is requisite for a sure and extended synthesis in the progress of knowledge.

When, however, the predicate P lies completely out of the concept S, though connected with it, the judgment is called Synthetic. Since the predicate increases the conception of the subject by something which was not contained in it, which no analysis could have discovered in it, this judgment is also called Augmentative or Ampliative. E. g., All bodies are heavy. This predicate is something totally

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[16] The above analysis of compound propositions, derived mostly from Arnauld, is intended to serve logical purposes, and is not even for these supposed to be exhaustive. To the student of Logic it will be sufficient in most cases, and generally illustrative and helpful.

different from what I necessarily think as contained in the concept body, and adds to the content of that notion.

Synthetic judgments are subdivided into those *a posteriori* and those *a priori*. The former are judgments from experience, which as such are always synthetical. I cognize by analysis the concept body through the marks extension and impenetrability. But now I augment my knowledge. Looking back on my experience of body, I find weight always connected with the above marks; so I amplify my conception by predicating of it this additional mark, saying, Body is heavy. Experience is the ground of this synthesis, because the notions body and weight, though one is not contained in the other, still belong to one another contingently as parts of a whole of experience.

But synthetic judgments *a priori* are not grounded on experience, nor does experience help us at all in forming them. E. g., Every event has a cause. The concept event implies antecedent time, from which I could form an analytic judgment. But the conception of a cause lies quite out of the concept event, and indicates a thing entirely different. This judgment, therefore is not analytic. Moreover, the experience from which I derive the conception of event does not include an experience of cause, and hence experience is not the ground of the judgment. Again, the judgment has a universality which experience can never give, expressing a necessity that cannot come of experience, which is essentially contingent. Such a judgment is, therefore, altogether *a priori*. What, then, is its ground? How is a synthetic judgment *a priori* possible? This is the question which Kant undertook to answer in his *Critique of Pure Reason*. Its importance is inestimable, for upon this class of synthetic or ampliative judgments depends the whole of speculative knowledge.[17]

§ 13. Under a previous topic[18] we considered two kinds of wholes in or under which mind contemplates its objects—the logical, or qualitative, and the mathematical, or quantitative, whole. Under the present topic we have thus far considered judgment as in the former only,

[17] See Introduction to *Kritik der reinen Vernunft*, § 4. The distinction of propositions into Verbal and Real made by Mill (*Logic*, bk. i, ch. vi), followed by Bain (*Logic*, bk. i, ch. ii, § 7), seems substantially the same as the above famous distinction by Kant of judgments into Analytic and Synthetic. Those logicians reject, however, the class of synthetic judgments *a priori*, and consider all synthetic judgments or real propositions to be *a posteriori* or empirical.

[18] Part 2d, vi, § 2. See also Hamilton's *Logic*, pp. 379, 380.

and now something must be said of it when in the latter whole.[19] For what we think about is conceived of either as general or as individual. We attain generality only by virtue of the qualities of things; and to think things in respect of their qualities is to think them in the qualitative or logical whole. On the other hand, we may contemplate the object of thought as a quantity, possessing no generality, not divisible into kinds, individual, severable only by dissection into adjacent parts, and measurable by some ideal standard. This we call thinking in the quantitative or mathematical whole.

Things of the same kind often differ in degree; and since in judgments concerning them the comparison is not respecting qualities or kinds, but respecting the quantity in its different degrees, we will venture to call these Judgments of Degree.

Two mathematical quantities can be related to each other in two degrees only; they must be either equal to each other, or else one greater than the other, either indefinitely or by so much. Hence the copula in these judgments either means "*is equal to*," "*is the same as*," or it is "*is greater than*," or, in the reverse view, "*is less than.*" It may be replaced by the sign of equality (=), or of inequality (>); for such a proposition is an equation. E. g., "The earth's diameter *is* (=) 8000 miles;" "The earth *is greater than* (>) the moon."

According to this statement, every judgment in the comparative degree, or judgment of comparison, has for its copula "*is greater than*," or its converse or obverse, "*is less than.*" This simple relation is often compounded with other notions; as in "longer" and in "shorter," in "included by," "better," "worse," "stronger," "more repulsive," "most attractive," "highest," etc. But, in brief, any terms whatever expressive of degree of comparison involve this copula, and characterize the judgment as essentially mathematical.

Here the question concerning the meaning of the copula recurs. We have seen that in the logical proposition it is to be interpreted "*comprehends*," or "*is contained under;*" and no one perhaps will question that in the strictly equivalent proposition it means "*is equal to*," or "*is the same as.*" These three relations, therefore, are ambiguously conveyed by the simple "*is.*" Now a judgment or proposition is

[19] A double sense of the word "quantity" has already been pointed out. We are now obliged to use it in still a third sense, one that has no reference whatever to intensive and extensive thought, nor to the logical distribution of terms, but in a sense more strictly mathematical, as relating to individual totals.

a declared relation between two notions or terms. Can all relations be reduced to these three? Are there not others? De Morgan insists that relations essentially distinct are very numerous, and proposes to include them all in one generalized "copula of relation," thus: "Every X has a relation to some Y," embracing the above, and also such connectives as in "X controls Y," "X causes Y," and many others. We shall subsequently see there is no need for this great extension of the copular meaning; but there appears to be a necessity for adding "*is greater than*" and its obverse to the meaning commonly recognized in Logic. It is true that a comparative judgment can be construed as compound. Thus, "The mass of the earth *is* a mass greater than that of the moon" means, as we have seen in the preceding section, "The mass of the earth is as much as the mass of the moon, and has something in addition." But if the comparison be accepted as an expressed interpretation of the copula, then comparative propositions would seem to be in thought quite simple. When I mentally compare the masses of two planets, I judge simply and directly that one *is greater than* the other, without at all thinking that one *is as much as* the other, and has something to spare. The copula thus understood, and the proposition construed as mathematical, many difficulties arising from syllogistic law disappear.[20]

Both terms of judgments of degree are always individuals viewed as mathematical wholes. There are various modes of designating individuals, such as by the definite article, by demonstrative and possessive pronouns, etc.; e. g., "Thou art the man;" "This is our home." These are integral wholes. Collective wholes often occur; e. g., "A legion is ten cohorts." Another mode is by a proper name, or by some particular mark; as, "Aristotle is the Father of Logic." Every proposition whose predicate is quantified, as "all" or "some," is

[20] The two copulas above described express the relation of degree between two individual wholes. The relation between the whole and its parts, merely as such, is expressed by the copula "*is part of*;" e. g., "The thumb *is part of* the hand;" "An arc *is a part of* the circle." This is a quantitative judgment of a different kind from that of degree, but does not seem to require especial exposition.

It is worthy of note that "*comprehends*" in the qualitative whole is similar to "*is greater than*" in the quantitative; and "*is contained under*" is strikingly like "*is less than.*" But there is another correspondence more real. We quantitatively as well as qualitatively think the relation of whole and part, and "*is contained under*" corresponds to "*is a part of.*" For example,—

The preachers are contained under (or, are a class of) teachers.
The preachers are a part of (or, are a section of) the teachers.

thereby brought into the mathematical whole, and the "all" is not distributive, but cumular; e. g., "All men are all reasoners;" "Ducks are some birds." Here both terms are individual totals. The algebraic equation, as, "$6=2\times3$," and "$x^2-y^2=(x+y)\ (x-y)$," is a judgment of the same character, its two members are individuals. All such judgments are properly called quantitative, because primarily, fundamentally, and essentially they always relate to space or time, the bases of mathematics, the science of quantity.

An individual may be known by the test that its parts are not kinds. We have seen in § 6 that an individual cannot become a predicate in the logical whole. In the mathematical whole the predicate, as well as the subject, being always an individual, the individual predicate is, therefore, the characteristic mark of a judgment of degree.

A consequent peculiarity of these propositions, and a test of their equivalency, is that they are all simply convertible. No special symbol is needed. Since the subjects are total, they are treated like individual propositions (§ 8), and symbolized by A and E (§ 10), with this marked difference, that whereas individual propositions are inconvertible (ii, § 7), the proposition of degree is always and only simply convertible. When the terms are not equivalent, the copula "*is greater than*" must in conversion be substituted by "*is less than*," and *vice versa*. When they are equivalent, either term may be substituted wherever the other occurs.

Singular terms must be discriminated from individual, with which they are apt to be confounded. "A man" is a logical qualitative whole, meaning "one single member of the class man," and is a thought very different from "That man," which is a mathematical, quantitative whole. The first is singular; the second, individual. Singular propositions are liable to be confused with equivalent propositions, because of the oneness of the terms in both; but surely it is evident enough that in "A horse is an animal" there is generality and no equivalence; whereas in "This horse is my animal" there is equivalence and no generality.

Likewise let us distinguish between coextensive and equivalent notions. Two coextensive logical wholes are aptly symbolized by two concentric circles whose radii are equal. But it should be kept in mind that these circles are mathematical quantities, and hence are equivalent, or rather equal. But coextension belongs to the logical whole, and is essentially qualitative. The following are coextensive notions: "Honesty and probity;" "Triangle and trilateral;" "En-

dogens and monocotyledons;" "Acotyledons and flowerless plants;" "Double-refracting and polarizing crystals;" "To conquer one's passions and to become master of one's self." But when the fact of coextension is neither expressed nor thought of, i. e., whenever the judgment containing such terms in extension is simple, the subject is construed in thought as contained under the predicate. And when the coextension is thought, still the copula cannot be replaced by the sign of equality and read "*is equal to*," but it should be read "*is coextensive with.*"

Also we must not be embarrassed by the factitious generality of many quantitative propositions, and doubt that the terms are individual totals. "A=B;" this means, "The quantity A *is equal to* the quantity B;" or, since equal quantities, purely as such, are indistinguishable, "The quantity of A *is the same as* (is identical with) the quantity of B." "Men are stronger than boys" means "The strength of men *is greater than* the strength of boys." "Every diameter is a double radius" means "The length of every diameter *is equal to* the length of two radii." "The superior planets move more slowly than the inferior" means "The speed of the superior *is less than* that of the inferior." "Iron is not as heavy as lead" means "The specific gravity of iron *is less than* that of lead." "Circus jokes are old as the hills" means "The age of the one *is equal to* that of the other." "Women love best" means "Woman's love *is greater than* any other." "The color of her eyes is the color of the skies." It will be noticed that mere abstract qualities are thought quantitatively, i. e., as individual totals, and when abstract, if indistinguishable as greater and less, are identified by "*is the same as.*"

It remains to observe that the logical and the mathematical wholes are often readily convertible in thought, such transference requiring few verbal changes or none to adapt the expression to the mode of thought. Thus "Mankind," which in the very form of the word expresses a general notion containing under it species, may be replaced by "The human race," which is individual, having no species, and can only be partitioned into sections. So there are kinds of army; and there are wings of an army. Being or thing is general, including all kinds of existing things; but the Universe is not a general notion, but a mathematical whole, a collection of all things into a unit, the only one not a part of any other, and is capable only of dissection. Again, the term animal is general; but animals may be thought as a collective whole comprising many individuals similar in certain es-

sential respects, and this whole may be severed by thought into parts, such as the part saved in the ark, and the part destroyed by the deluge. The indefinite article qualifying a predicate may be interpreted in either of two ways; thus "Gold is a metal" means logically and strictly that gold is a kind of metal, but we may think it mathematically, that gold is a part of metals taken as a collective whole. In short, perhaps any general notion may be thus transmuted or reduced in thought to a mathematical quantity, a collective whole consisting of many similar individuals, its species becoming dissevered members.

This weighty fact, and the essential difference between the two modes of thought, not being recognized, is the reason, I apprehend, why Hamilton and a number of subsequent logicians have attempted the reduction of all propositions to equations, and proposed thereby to supersede the old logical system.[21] But such reduction is artificial. It exhibits the processes of thinking, not as they really occur, but in forms into which they may be construed by more or less violence. Such a presentation of Logic is possible only because of the power which the mind has of transmuting its notions from logical wholes and parts into mathematical wholes and parts.

On the other hand, the old Logic was limited to the logical whole and part. A Latin logician would probably deny that what we have called a judgment of degree is a case of predication at all—predication belonging only to the logical relation—and would insist on all such forms being construed in the logical whole. Hence, perhaps, no symbol was assigned to such propositions, nor were they otherwise recognized. But these propositions abound, they are in constant use, they frequently stand as premises in all kinds of reasonings, and mathematics consists of them. We may, it is true, transmute them into propositions strictly logical, but we then incur the most serious embarrassments in the attempt to bring them under syllogistic law. Moreover, this again is artificial, not natural, not thought as it is, for we reason with mathematical propositions without any such transference. It is needful, then, to admit them to a prominent and important position in Logic if we would truly represent human thinking.

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[21] Notably George Boole in his *Mathematical Analysis of Logic* (1847), and his *Investigation of the Laws of Thought*. A very good résumé of his principles will be found in Bain's *Logic*, pp. 190–207. Jevons would make Logic mechanical! See his "Logical Machine," facing the title of his *Principles of Science*.

§ 14. **Praxis.** In each of the following propositions, is the form categorical, or conditional, or what? (§ 7). If categorical, is it simple, or complex, or compound? (§ 11). If simple or complex, reduce to strict logical form (§ 4), and interpret the copula (§ 6). Affix the symbol of quantity and quality (§ 10). If compound, resolve it into its elements, and affix the symbol to each. If mathematical, express it as an equation or inequality.

1. It is the duty of every man to fear God and honor the king.
2. Very few patriots are disinterested. There is no place like home.
3. Nothing is harmless that is mistaken for virtue.
4. Men are all sinners. No news is good news.
5. All these claims upon my time overpower me.
6. One truth is clear, whatever is, is right.—*Pope.*
7. Not many if any metals are without lustre.
8. Not being rich is not always an evil. Diogenes was no fool.
9. Except the self-existent, there is nothing beautiful, but that which is not.—*Rousseau.*
10. Hardly any virtue is safe from passing into vice.
11. Virtue is teachable, if it is knowledge.
12. All is not gold that glitters. The rich are not therefore happy.
13. None but Aryans are capable of the highest civilization.
14. Jefferson was the father of the University of Virginia.
15. Ah! few shall part where many meet,
    The snow shall be their winding sheet.—*Campbell.*
16. Charity affords relief as far as possible.
17. He who truly loves most is not he who flatters.
18. The quarrel toucheth none, but us alone.—*Shaks.*
19. After his death, resistance and order were no more.—*Gibbon.*
20. I propose my thoughts only as conjectures.—*Burnet.*
21. Whereto serves mercy, but to confront the visage of offence?
22. That thou art happy, owe to God.—*Milton.*
23. George Eliot is Mrs. Lewes. Arrows are swifter than eagles.
24. Though this be madness, yet there's method in it.—*Shaks.*
25. Those here present constitute the class in Logic.
26. There is no fireside, howsoe'er defended,
    But has one vacant chair.—*Longfellow.*
27. Saltpetre is nitrate of potassa. That horse won the race.
28. There are who ask not if thine eye be on them.—*Wordsworth.*
29. There's a divinity that shapes our ends,
    Rough hew them how we will.—*Shaks.*

30. The time has been my senses would have cooled to hear a night shriek.—*Shaks.*
31. Nothing is so easy as to object. He is as wise as Solomon.
32. Some books are to be tasted, others to be swallowed, and some few to be chewed and digested.—*Bacon, Essay L.*
33. Our revels now are ended. There's few or none do know me.
34. Who lived king, but I could dig his grave?—*Shaks.*
35. The longer the day, the shorter the night.
36. That he is mad, 'tis true; 'tis true, 'tis pity;
And pity 'tis 'tis true.—*Shaks.*
37. There's not a joy the world can give,
Like that it takes away.—*Byron.*
38. His alms are far beyond his means.
39. I will not let thee go, unless thou bless me.
40. The author of *Novum Organum* was not the inventor of Falstaff.
41. Even a fool, when he holdeth his peace, is counted wise.
42. It will hardly be sufficient to resolve only a few of these examples.
43. The most skilful of generals was Napoleon.
44. Every sly act is nothing less (or else) than dishonest.
45. Logic is the science of the necessary forms of thought.
46. Not every one that saith unto me, Lord, Lord, shall enter in, but he that doeth the will of my Father.
47. A circle is the figure of greatest area.
48. Your duties are not another's. My tasks are all but impossible.
49. He was too impulsive a man not to have committed many errors.
50. Yonder forest is the refuge of outlaws.
51. He first and last will reign sole king.—*Milton.*
52. Congress legislates for the Union.
53. Mankind are all men and women. All testimony is merely probable.
54. God's word, exclusively, is to be received without question.
55. The most sublime act is to put another before thee.
56. Le salut des vaincus est de n'en point attendre.—*Tr. from Virgil.*
57. Nobilitas sola est atque unica virtus.—*Juv. Sat.* viii, 20.
58. Nullas habet spes Troja, si tales habet.—*Seneca.*
59. Nemo læditur, nisi a scipso.—*Id.*
60. Melior est sapientia quam vires, et vir prudens quam fortes.
61. Latin has been a dead language for five hundred years.
62. That which survives is the fittest.

II. INFERENCES.

§ 1. Under the previous topic we have examined seven modes of dividing judgments or propositions. An eighth remains, so important that each part calls for separate and extended consideration. This division is grounded on the various processes by which judgments are formed, and may be stated as follows:

Judgments are { Intuitions. / Inferences. { Inductive. / Deductive. { Immediate. / Mediate.

Intuitions are the synthetic judgments of Kant already described, one kind being empirical, the other pure. These are the ground of all knowledge, the ultimate premises from which arise all other judgments. They lie on the threshold of Logic, but their discussion belongs to Philosophy, the science of principles.

Inferences are defined by Aristotle to be "enunciations in which, from something laid down and admitted, something distinct from what we have laid down follows of necessity." Locke says, "To infer is nothing but, by virtue of one proposition laid down as true, to draw in (*inferre*) another as true." Says Mill: "It is the act of drawing a conclusion from premises." More generally, to infer is to derive a judgment from one or more premised judgments.

Inductive inferences are synthetic. They are universal judgments derived from particular cases of empirical intuition, and furnishing premises for subsequent deduction. Their importance is so great that an adequate discussion of them will require a distinct treatise.

Deductive inferences are analytic. They are inferred judgments of equal or less generality than that of the premises. They are the subject of Deductive Logic, and are of two kinds, immediate and mediate.

When two notions known as related are, in a modified form, concluded of each other without the intervention of a third notion as a medium of comparison, the inference is immediate. In this case one judgment is derived directly from another. There is but one premise, the given judgment; and the derived judgment merely represents the given matter in a modified form.

A mediate inference or a reasoning is accomplished through a third notion used as a medium of comparison. It has two premises.

Immediate inference will be treated under the present general topic. Mediate inference, or Reasoning, is the subject of the subsequent part.

§ 2. Let us at the outset, for the sake of clearness, distinguish between implied and inferred judgments, which McCosh would identify.[1] An implied judgment is one that actually exists together with the given judgment, either merely in thought or involved covertly in the expression. An inferred judgment is one that only virtually or potentially exists in the given judgment, and is derived from it. The statement of the one is nothing new; there is no advance, no progress of thought, but only its full expression; that of the other contains something new, there is a step forward, a progress of thought. In the inferred judgment there is always either a different subject, or a different predicate, from that of the premise, and perhaps both.

The different quantities of thought, the intensive and extensive, are hardly, in strictness, to be considered as implying each other, much less can we consider that one is inferred from the other; they are merely different aspects of the same thing, which necessarily coexist, one having merely accidental preponderance in thought.

In indirect speech there is always an implied judgment. So also the semi-definite proposition involves an implied judgment. Thus, if I say "Some men are rich," it is accompanied by the thought that "Some men are not rich;" but this, being an actually coexistent thought, is not inferred. It would be evidently an entirely unwarranted use of the term to say that one of these judgments is inferred from the other. We cannot say that since some men are rich, then it follows that some men are not rich. An exponible contains an implied, indirect judgment which is expressed, though covertly. Thus, the example given might be stated, "Only some men are rich." Here, "Only some" expresses covertly that some are not.

Again, what Thomson, followed by McCosh, calls "Immediate Inferences of Interpretation" are not inferences, but mere implications. Thus, in "John loves Mary," it is implied, but not inferred, that "John lives," that "Mary lives," and that "There is such a thing as love."[2]

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[1] *Logic*, p. 108. Cf. Mill's *Logic*, bk. ii, ch. i, § 2; and Thomson's *Outline*, § 83.

[2] Thomson's *Outline*, § 89; and McCosh's *Logic*, p. 115. Refer also to what was said of the force of the copula, i, § 3.

Finally, the "Immediate Inference by the Sum of several Predicates" of Thomson and McCosh, is not an inference at all, but merely a compound judgment of the obvious sort. Thus, "Copper is red, malleable, ductile, and tenacious" is merely compounded of "Copper is red," "Copper is malleable," etc. It is strange phrasing to call it an inference from these components. It is also quite remarkable that McCosh includes under this head the bringing together the components of a definition.

§ 3. As preparatory to an account of those several kinds of immediate inference for which we shall have subsequent use, we state a prohibition applicable to all deductions in the form of the following RULE: *The quantification must not be increased.* We may infer from *all* to *all*, from *some* to *some*, from *all* to *some*, but not from *some* to *all*. It is sufficiently evident that what is said only of *some* furnishes no ground for a statement concerning *all*.

§ 4. **Active and Passive.** The change from active to passive, and *vice versa*, is the first form of immediate inference to be noticed. The two forms are usually regarded as merely equipollent, but they seem to be rather an inference, the one from the other. In "God made the world," something is said of "God;" he is the subject of thought. In "The world was made by God," the subject is "The world," and something is said of it. The inversion, too, is only partial, since the notion "made" is in the predicate in both cases. Hence I would prefer to consider this change as an immediate inference; but it is a question of little importance.

§ 5. There are two kinds of immediate inference introduced into Logic by Leibnitz, which, being very similar, may be stated together,—

Added Determinants. The same mark may be added to both terms of a judgment. The new judgment thus formed is inferred from the other. Thus, since "Coal is fuel," then "Cheap coal is cheap fuel;" since "Science is system," then "A false science is a false system." The extent of both subject and predicate is narrowed, is more closely determined. This is thinking in a mark, going from genus to species. We add that the subtraction of the same determinant from both subject and predicate is also legitimate, but not an inference.

Complex Conceptions. This inference is parallel to the other. The two terms of a judgment may be added as marks to the same concept.

Thus, since "Science is system," then "A scientific arrangement is a systematic arrangement;" and since "Coal is fuel," then "The consumption of coal is the consumption of fuel." Two judgments may be amalgamated on this principle, the terms of one being added as marks to the terms of the other. Thus, since "A museum is a collection of interesting objects," then "A scientific museum is a systematized collection of interesting objects."

§ 6. **Infinitation.**[3] This mode of immediate inference passes from the merely negative judgment to the infinite judgment of Kant (i, § 9). It places the subject in the outer, infinite sphere of things, and limits it only by the subtraction of the predicate from that sphere. Thus, from "The soul is not mortal," I immediately infer that "The soul is non-mortal." These propositions express different thoughts. They are not equal, not identical, but merely similar. The inverse inference is included under the same name; i. e., the reduction of an infinite proposition to a mere negative, is also, for convenience, called infinitation. Thus, from "Quakers are non-combatants," we immediately infer that "Quakers are not combatants." Also purely affirmative and doubly negative judgments are said to be infinitated thus, since "Man is mortal," then "No man is non-mortal;" and *vice versa*. Hence, for immediate inference by infinitation, the Rule: *Change the quality of the judgment and of the predicate.* This is done, if the premise has either a negative copula or predicate, by simply transferring the negative particle from one to the other; if both are negative, by subtracting it from both; if neither, by adding it to both. Observe that, though the quality of the judgment is always changed, the quantity remains unchanged. This process Bain calls "Obversion," but he denies that it is properly an inference, insisting that the two notions are mutually implied under the law of Relativity.

To avoid awkward compounds with *non*, we make use of a privative prefix or suffix, as *in-*, *un-*, *dis-*, *-less*, etc., although, as has been repeatedly remarked, words so formed are often not pure negatives. For example, they often mean, not the privation of the quality, but the existence of it in a low degree; as, *unwise*, *careless*. So uncompounded negative terms are generally impure; as, *night*, *crooked*. We

[3] Commonly called by the old logicians Æquipollence. We use this word, however, in a sense more accordant with its etymology, to mean the same thought only in a different phraseology. See Part 1st, ii, § 8.

are, then, to be on our guard in using such terms to express infinitation, lest we derive too much. Under this precaution we add some illustrations as follows,—

Since	All metals are fusible;	then	No metal is infusible.....	A yields	E
"	No miser is happy;	"	Every miser is unhappy.....	E "	A
"	Some sins are pardonable;	"	Some sins are not unpardonable...	I "	O
"	Some men are not gentle;	"	Some men are ungentle.....	O "	I

We may pursue a thought through a series of immediate inferences, as in the following example,—

Since Some invisible things are not intangible;..... = O
Then Some invisible things are tangible;..... = I
(Convert simply.)
Then Some tangible things are invisible;..... = I
Then Some tangible things are not visible..... = O

De Morgan, followed by Thomson, Bowen, McCosh, and other logicians, derives this last directly from the first by a complex rule, and classes it as a second method of infinitation; but, as it obviously involves conversion, to do so needlessly confuses two modes of inference. One other example,—

Since Every unjust act is inexpedient;..... = A
Then No unjust act is expedient;..... = E
(Convert simply.)
Then No expedient act is unjust; ..... = E
Then Every expedient act is just..... = A

Some moralists who would contend for the first proposition of this series, would hesitate to admit the last. But the inference is necessary.

§ 7. **Conversion.** In immediate inference by conversion, the subject and predicate change places with each other; i. e., the terms are transposed. Besides observing the general rule given above (§ 3), we must take heed to make a total transfer; i. e., the whole naked subject must be made predicate, and the whole naked predicate made subject. By a naked term is meant a term without its sign of quantity, *all*, *some*, etc. Thus, from "Every old man has been a boy," we cannot infer that "Every boy has been an old man;" but only "Some one who has been a boy is an old man." Hence, to avoid error, it is generally needful before converting to reduce the proposition to its strict logical form, that in which subject, copula, and predicate distinctly appear. We will consider only three kinds of illative

conversion, and these only so far as our subsequent need in syllogizing requires, which is, that we be able to convert each of the four judgments A, E, I, O.

1st. *Simple conversion* transposes the terms without changing the quantity or the quality of the proposition. It may be applied to E, and to I. Thus,—

Since No one without warm sympathies is a true poet;..... = E
Then No true poet is without warm sympathies;..... = E

Since Some good mathematicians are poor financiers;..... = I
Then Some poor financiers are good mathematicians..... = I

The judgment of degree (i, § 13), symbolized by A or E, is always and only simply convertible.

2d. *Conversion per accidens* reduces the quantity of a proposition (hence also called C. by limitation), but leaves its quality unchanged. It is applied to A, and the converse is I. Thus,—

Since All plane triangles are rectilinear figures;..... = A
Then Some rectilinear figures are plane triangles..... = I

The name was given by Boethius, because it is not a conversion of the universal *per se*, but only of a particular which the universal includes. If we hold to the rule that affirmatives do not distribute the predicate, it is evident that the predicate of the convertend, "rectilinear figures," does not change its quantity in becoming the subject of the converse. But, for the same reason, the subject of the convertend, "plane triangles," in becoming the predicate of the affirmative converse, has its quantification reduced. Also observe that our general rule (§ 3) forbids us to retrace this step—to reconvert the I into A. E also may be converted *per accidens*.

3d. *Conversion by contraposition* changes the quality but not the quantity of the proposition. It is applied to the remaining judgment O, and the converse is I. In order to contrapone we have the following Rule: *Infinitate and then convert simply.* Thus,—

Since Some pure air is not wholesome;..... = O
Then Some unwholesome air is pure..... = I

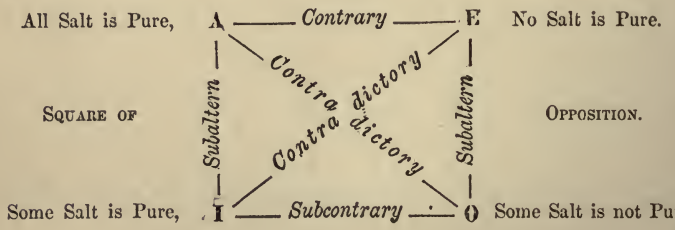
This is of course a compound process, and was devised to convert O, which cannot be converted simply, or *per accidens*. It has been also called "conversion by negation."

Upon a slight inspection it is sufficiently obvious that the doctrine of conversion has respect to judgments in extension. An intensive

judgment cannot be converted without at the same time changing its subject into a mark, and its predicate into a concept; as, "All men are mortal" converts to "Some mortals are human." Otherwise the view in converting must be changed to extension.

Again, since an individual cannot become a predicate (i, § 6), it follows that no individual judgment (i, § 8) can be converted. The symbol A or E (i, § 10), when used to represent it, must be held inconvertible. We say "Venus is pretty," and may say, "Something pretty is Venus;" but this apparent conversion *per accidens* is only a rhetorical inversion; the subject of thought is still Venus. This gives occasion to remark that no mere inversion is a logical conversion.

§ 8. **Opposition.** A subject and predicate given in either one of the four forms A, E, I, O, is in opposition to the same matter in each of the other three forms. The opposition is such that if the given proposition be taken as true, or as false, we can immediately infer the truth or falsity of at least some of the others. It is of four kinds, usually exhibited upon a diagram, thus,—



1st. *Contradictory opposition* exists between propositions having the same naked or unquantified subject and predicate, but which differ in both quantity and quality. Both cannot be true, and both cannot be false. This is merely a specific statement of the laws of Contradiction and Excluded Middle. E. g., If A, "All Salt is Pure," be sublated (denied), then by an immediate inference we can posit (affirm) O, "Some Salt is not Pure." If I, "Some Salt is Pure," be posited, then we can immediately sublate E, "No Salt is Pure." If it is true that "Every man has a conscience," then it cannot be said that "Some men have no conscience." Again, if you prove that "A doctrine, such as the connection between mind and body, is to be believed, though it is not comprehensible," you have thereby shown that "No doctrine is to be disbelieved because it is incomprehensible."

Such propositions are said, in common phrase, to be diametrically opposed. Aristotle used the diagonal for the contrary opposition of A and E, and for this reason, perhaps, the phrase "diametrically opposed" is ambiguous, it being applied both to contraries and to contradictories.[4] Contradictory opposition is the only perfect form of opposition, all others being more or less imperfect.

Proof is direct and indirect. If we wish to refute an adversary, we may show that his arguments are false, do not sustain his assertion, which, being unsupported, fails. The result is merely negative, and is often sufficient. But we may wish to go further, and prove his assertion positively false. If this is done by an attack upon his own assertion, the method is direct. But if we affirm the contradictory proposition, and, having established it, immediately infer his assertion false, the method is indirect. Thus, if one affirms with Hobbes that "All human motives are always ultimately selfish," we may undertake to prove that "Some one motive in some single case was unselfish." If this be established, then the immediate, necessary inference from this O is, that his A is false. The proof called *reductio ad absurdum* is indirect and quite similar. Euclid makes much use of it. Instead of demonstrating a proposition directly, he demonstrates that its contradictory is absurd and thence infers its truth.

2d. *Contrary opposition* exists between A and E, universal propositions differing in quality only. Both cannot be true, but both may be false. Between these propositions there is a *tertium quid*, namely I and O. If A, "All S is P," be posited, E, "No S is P," is sublated, and *vice versa*. But if either is sublated, this does not posit the other, for it may be that "Only some S is P"=I and O. To deny that "All Stars are Planets" does not afford the inference that "No Stars are Planets;" for it may be, and in this case is, true that some are, and some are not. To sublate "No wars are evil" does not give position to "All wars are evil;" for if some are, and some are not, then both the others are false.

When, however, the judgment or proposition is individual, all dis-

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[4] The Aristotelic doctrine of Opposition differs considerably from the one here given, which is the approved Scholastic form. Saint-Hilaire represents the former thus: "L'opposition (*τὰ ἀντικείμενα*) peut être de quatre espèces. Il y a: 1° celle des relatifs; 2° celle des contraires; 3° celle de la privation et de la possession (*τέρησις καὶ ἕξις*); 4° enfin celle de l'affirmation et de la negation. Cette théorie des oppositions joue un grand rôle dans le système d'Aristote."—*De la Logique D'Aristote*, Tome i, p. 172 sq. (Paris, 1838). See Aristotle's *Categoriæ*, ch. x.

tinctions in opposition disappear, or rather become merged into the simple negative, which, in such case, is the true contradictory. E. g., "Caliban is a man," and "Caliban is not a man."

In controversy opponents often take contrary positions, and either failing to establish his own gives to the other an apparent victory. E. g., One asserts that "All men are to be trusted." Another opposes this with "No men are to be trusted," but being unable to prove it in face of cited cases of some who are to be trusted, leaves the question in confusion, and his opponent in possession of the field. Indeed, they have not squarely faced each other. The opposer, in adopting the indirect method, should have undertaken, not the contrary, which is too much, but the diametrical contradictory, that "Some men are not to be trusted," which in this case would insure an easy victory.

3d. *Subcontrary opposition* exists between I and O, particular propositions differing in quality only. Both may be true, but both cannot be false. Hamilton calls these subaltern contraries, "compossible." If I, "Some S is P," be taken as true, it may be that O, "Some S is not P," is also true. But if I is false, then O must be true. If "Some Sighs are Prayers," it may also be true that "Some Sighs are not Prayers." But if it is false to say that "Some Sighs are Prayers," then it must be true that "Some are not."

Let it be noticed, however, that if, in "Some S is P," and "Some S is not P," the same "Some" is intended, then the propositions are "incompossible." In strictness they become contraries, and hence pure Logic, which takes it thus, knows no subcontrary opposition. But usually the sphere of the "Some" in the one is different from that in the other. Thus, if I observe that "Some metals (some at least, perhaps all) are fusible," it may be that "Some others, for aught I know, are infusible." Here the "Some" is wholly indefinite, and our rule holds good. But, further, if the "Some" be thought as semi-definite (i, § 8), then our rule changes from "Both may be true" to "Both must be true." Thus, I know that "Some flowers (some at most, not all) are fragrant;" then it must be true that "Some flowers are not fragrant." This Hamilton calls "integration," since the two "Somes," taken together, constitute the whole.

4th. *Subalternate opposition* exists between propositions differing in quantity only. If the universal is true, the particular is true; if the particular is false, the universal is false. If I have $100 at my credit in bank, it is evident I may draw for $5 or $10. If I have not $10 at my credit, I cannot draw $100. This is a specific application

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of the law of Identity. If it is true that "All Sin must be Punished," then we can infer that "Some, or any one, Sin must be Punished." If "Some Sin, even one, will not be Punished" be proved false, then we cannot say that "No Sin will be Punished." The reverse of the rule, however, does not hold. From "Some S is P," it does not follow that "All S is P." If "No S is P" is a false statement, we cannot infer that "Some S is P" is also false. Though to say that "No Subjects can become Predicates" is untrue, still it is true that "Some Subjects, as individuals, cannot become Predicates."

An exception is to be taken also here. If a particular proposition is thought as semi-definite, it follows that the universal is false. If "Only some flowers are fragrant," I and O, then it is false to say either that "All are," or that "None are." Also, if a universal is true, then its subalternate particular is false. If "All Scripture is Profitable," then we cannot think that "Some (some at most, not all) Scripture is Profitable." If we accept that "No Scripture should be Profaned," then we cannot consistently think that "Some (some only) Scripture should not be Profaned." In semi-definite thought the rule for subalternate opposition becomes "If either is true, the other is false." This modified form of the opposition Hamilton calls "inconsistency."[5]

Let us repeat here an exceptive remark made above, that individual propositions have only one opposite. The subject being an individual total, its quantity cannot be reduced. Hence there is no subaltern, nor diagonal contradictory. The simple contrary or negative is a complete contradiction. E. g., "Diogenes was a fool," and "Diogenes was not a fool."

The relation between subcontraries, as well as that between subalterns, is not strictly opposition. Between subcontraries there is no real contrariety, but rather a presumption of agreement, a presumption that both are true. Between subalterns the relation is that of a partial agreement, or subordination, which Hamilton calls "restriction." But for convenience and brevity, logicians treat them as species under opposition.[6]

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[5] *Logic*, pp. 530–535. Aristotle never recognizes the semi-definite judgment. With him a particular proposition is always construed as wholly indefinite.

[6] Aristotle does not mention subalternate opposition. He names subcontrary opposition, but declares it to be merely verbal, not real. He speaks of contradictories as opposites (*ἀντικείμεναι*), apparently considering these alone as really opposed. See Waitz, *Comment. on Organ.* 11b 16. Cf. Cic. *Top.* xi, § 47.

The chief results, not including the semi-definite meaning, may be summed as follows,—

All S is P, Some S is not P. / No S is P, Some S is P. } Contradictories:—One must be true, and the other false.

All S is P, No S is P. } Contraries:—Both cannot be true, but both may be false.

Some S is P, Some S is not P. } Subcontraries:—Both may be true, but both cannot be false.

All S is P, Some S is P. / No S is P, Some S is not P. } Subalternates:—{ If the universal is true, the particular is true; / If the particular is false, the universal is false.

The same matter may be tabulated also thus,—

		Contradictories.	*Contraries.*	*Subalterns.*
Universals	If A is true,	O is false,	E false,	I true.
	If E is true,	I is false,	A false,	O true.
	If A is false,	O is true.	} The others undetermined.	
	If E is false,	I is true.	}	
Particulars	If I is true,	E is false.	} The others undetermined.	
	If O is true,	A is false.	}	
	If I is false,	E is true,	O true,	A false.
	If O is false,	A is true,	I true,	E false.

Hence by the truth of universals, and by the falsity of particulars, all others are determined; otherwise only the contradictory.[6]

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[6] The old Latin logicians rather needlessly warn us that opposition cannot be correctly said to exist unless the predicate [and the subject] of both propositions is truly the same. We violate this precaution, say they, when we do not predicate in the same—

1. Manner; as, Hector is and is not a man; i. e., he is a dead man, but not a living one.
2. Respect; as, Zoilus is and is not black; i. e., he is black-haired, but red-faced.
3. Degree; as, Socrates is and is not long-haired; i. e., he is, compared with Scipio, but not, compared with Xenophon.
4. Time; as, Nestor is and is not an old man; i. e., he is not when a boy, but is at the siege of Troy.

§ 9. **Praxis.** Draw an immediate inference from each of the following propositions by added determinants (§ 5):—

1. The wages of sin is death.

Use as determinants,—inevitable, and just.

2. Novelty is pleasure.

Use as a determinant,—the greater the.

3. War is an evil.

Use,—unprovoked, welcomed with ardor, which reaches to our hearth-stones.

Infer from the following by complex conceptions (§ 5):—

4. The ignorant are ceremonious.

Use the concept,—an age.

5. Heaven from all creatures hides the book of fate.—*Pope.*

Use,—wisdom and love.

Combine each of the following pairs into one proposition (§ 5):—

6. Honesty deserves reward.
   Every man whom we meet is a neighbor.
7. The year is dying in the night.—*Tennyson.*
   The swift runner is speedily exhausted.

Infinitate each of the following propositions (§ 6):—

8. All knowledge is useful.
9. The Chinese are industrious.
10. No reptiles have feathers.
11. It is wrong to put an innocent man to death.
12. There are studies much vaunted, yet of little utility.
13. Some men's hearts are not in the right place.
14. In jewels and gold, men cannot grow old.
15. No brutes are responsible.

Convert each of the following, affixing the symbols (§ 7):—

16. Life every man holds dear.
17. Two straight lines cannot enclose a space.
18. None are free who do not govern themselves.
19. With man many things are impossible.
20. Few know themselves.
21. 'Tis cruelty to load a falling man.
22. Fame is no plant that grows on mortal soil.

23. Whoso loveth instruction, loveth knowledge.
24. Each mistake is no proof of ignorance.
25. Fair promises are often not to be trusted.
26. There falls no shadow on his tomb.
27. Full many a gem of purest ray serene,
The dark unfathomed caves of ocean bear.—*Gray.*

From each of the following premises obtain, by immediate inferences, the annexed conclusion (§§ 6 and 7):—

28. All the righteous are happy;
∴ Whoever is unhappy is wicked.
29. No human virtues are perfect;
∴ All perfect virtues are superhuman.
30. Some possible cases are improbable;
∴ Some improbable cases are not impossible.
31. Some true patriots are not popular;
∴ The unpopular are not always unpatriotic.
32. Certainty is a kind of light;
∴ Darkness is doubt.

If the following propositions are true, what opposites are also true, and what false? (§ 8):—

33. By night an atheist half believes a God.—*Young.*
34. No one is always happy.
35. Some democracies are unstable.
36. Some great orators are not statesmen.

If the following are false, what opposite propositions are also false, and what true? (§ 8):—

37. All self-confident persons have strong will.
38. No honest men become bankrupt.
39. Some private vices are public benefits.
40. Some plants do not produce seed.

III. INNOVATIONS.

§ 1. Since the revival in England of the study of Logic, which was brought about by the publication of Whately's treatise, there has been manifested much dissatisfaction with the Aristotelic doctrines as inherited from the scholastic or Latin logicians of the Middle Ages. This body of doctrine we have spoken of as the old, or Latin Logic, not meaning to intimate thereby that it is obsolete, or even likely to vanish away, but simply to distinguish it from recent doctrines. The dissatisfaction has arisen not so much from a supposed inaccuracy of the old doctrines as from their supposed inadequacy. Many important modifications and additions have been proposed by high authorities, such as Hamilton, De Morgan, Mansel, Boole, Thomson, Mill, Bain, Jevons, and others, but as yet few have been generally accepted, and the old Logic holds its ground. Hamilton has been the chief innovator, his views have been most widely discussed, and made the deepest impression; and, therefore, we will give our attention especially to them.

§ 2. Hamilton's doctrine of the semi-definite "Some" has already been stated.[1] But it is very questionable whether it should be received into Logic at all, even as a mere exception. "Some," if not wholly and simply indefinite, probably always designates either a wholly definite judgment imperfectly expressed, or else a compound judgment whose two elements are each wholly indefinite. If we say "Some members of this University are now studying Logic," this judgment in our minds would be wholly definite, a certain "Some," i. e., "All the members of the Philosophy Class are now studying Logic," without any thought whatever of other members of the University.

[1] In i, § 8. It may be remarked that, if fully adopted, its consequence to the old doctrine of Opposition (ii, § 8), enlarged by the addition of four judgments, is something fearful. The student is referred to the tabulated statement in the Appendix to Hamilton's *Logic*, p. 535, where the whole scheme is elaborately worked out. Instead of thus replacing entirely the old doctrine of Opposition with the new one of "Incompossibility," it would seem simpler and sufficient, and hence better, to treat the cases of the semi-definite meaning as exceptions to the old rules.

The judgment then is A, and the proposition should be reduced to that form, in conformity with the thought. Again, if we say "Some flowers are fragrant," meaning "some at most, not all," then this implies the counter-thought that "Some flowers are not fragrant." If this double thought be expressed in a grammatically simple sentence, for the logician postulates that it be expressed, then we have "Only some flowers are fragrant." This is an exponible compound proposition which analyzes into "Some flowers (I know not how many) are fragrant" (I), and "Some flowers (I know not how many) are not fragrant" (O). Each of these elements considered in itself, entirely apart from the other, is wholly indefinite; for the meaning of "I know not how many" must in that case be "perhaps all." The semi-definite character does not at all appear unless one judgment is recognized as limiting the other; and when this is the case the judgment is not simple, but compound. Now Logic, professing to be a thorough analysis of thought, must not stop short of its simple elements, must not recognize the compound as co-ordinate with the simple, and does not, cannot, undertake to formulate the compound modes of thought, which are legion, but evolving their elements formulates only these. Therefore the semi-definite judgment, being compound, must be denied a position among the elementary forms of thought, and if recognized at all must take its place among the abbreviated, imperfect modes of statement, subject at any moment to analysis and full discrete expression.

§ 3. The most important addition to the old Logic proposed by Hamilton is his doctrine of "The Thorough-going Quantification of the Predicate."[2] The old Logic teaches that negatives distribute the predicate, affirmatives do not (i, § 9). Hamilton teaches that in both affirmative and negative judgments the predicate may be either distributed or undistributed. Hence, to the four Aristotelic judgments of the old Logic he has superadded four others, commonly

[2] See Hamilton's *Logic*, Appendix, p. 509 sq. As Bacon called his great work the *Novum Organum*, in allusion to the Aristotelic *Organon*, so Hamilton calls his treatment of these forms the "New Analytic," in allusion to Aristotle's "Analytics," and proposes thereby "to place the keystone in the Aristotelic arch." For an excellent statement of Hamilton's views, warmly approved by himself, see *An Essay on the New Analytic of Logical Forms*, by Thomas Spencer Baynes, an admiring pupil of Hamilton's. The Essay is the more interesting from having been a prize examination paper.

called the Hamiltonian judgments. These are included in the following table.

TABLE OF THE EIGHT PROPOSITIONAL FORMS.

Best	1, υ, afa,	Toto-total,	*All* men *are all* reasoners.....	men / reasoners	C: — :Γ	Identity, or Co-inclusion.
Aff.	ii, A, afi,	Toto-partial,	*All* men *are some* lovers.....	men / lovers	C: — ,Γ	
	3, ʏ, ifa,	Parti-total,	*Some* men *are all* poets.....	men / poets	C, — :Γ	
	iv, I, ifi,	Parti-partial,	*Some* men *are some* singers....	men / singers	C, — ,Γ	
Neg.	5, ω, ini,	Parti-partial,	*Some* men *are not some* singers.	men / singers	C, ⫞ ,Γ	Non-Identity, or Co-exclusion.
	vi, O, ina,	Parti-total,	*Some* men *are not any* poets...	men / poets	C, ⫞ :Γ	
	7, η, ani,	Toto-partial,	*Not any* men *are some* lovers..	men / lovers	C: ⫞ ,Γ	
Worst	viii, E, ana,	Toto-total,	*Not any* men *are any* brutes...	men / brutes	C: ⫞ :Γ	

Some explanation, preparatory to discussion, is needed. The table consists of six columns of symbols, and one of examples. All the symbols in any one horizontal line mean the same thing. In the first column, the Roman numerals designate the Aristotelic or Latin judgments; the Arabic numerals, the Hamiltonian judgments. In the second column, the Hamiltonian judgments also are designated by vowel letters: υ for universal; ʏ, as cognate to I; ω and η, as the Greek correlatives of O and E. In the third column, **a** stands for a universal or distributed term; **i** for a particular or undistributed term; **f** (*affirmo*) stands for the affirmative copula; **n** (*nego*) for the negative copula—**f** and **n** being respectively the first consonant in each of those words. The fourth column needs no explanation; but we observe that its symbols are defective in not distinguishing the affirmative from the negative forms, and must therefore be supplemented by the words "affirmation" or "negation." The fifth column is of examples, in which it is understood that "*men*" includes both males and females; and further that "*birds*," for instance, are "*lovers*" of each other, and also are "*singers*." The sixth column is the linear notation, already described under a previous topic.

The seventh column calls for more remark. It is an ingenious device of Hamilton's, to which, however, he gave no specific name. As it is not properly symbolic, we will call it the "*Graphic Notation.*" The subject or predicate is expressed by C or Γ (gamma), the third letters respectively of the Roman and Greek alphabets. These are taken that they may be indifferent, no unconscious preference being given to either, which perhaps might not be if they were successive letters from the same alphabet. The distribution of either term is expressed by a colon standing next to it; thus C: is read "All C." The non-distribution of either term is expressed by a comma next to it; thus ,Γ is read "Some Γ." The positive copula is expressed by a pointed dash (—); the negative by the same crossed (+). The peculiar advantage of the device is that it discriminately expresses either extension or intension. Pointing to the predicate, this copula indicates an extensive judgment, and should therefore be read "contained under;" thus C:—,Γ is read "All C is contained under some Γ." Pointing to the subject, it indicates an intensive judgment, being read "comprehends;" thus C:—,Γ is read "All C comprehends some Γ." Other examples are: Γ,—,C=Some Γ is contained under some C; C:+:Γ=Not any C is contained under any Γ; Γ,+:C=Some Γ does not comprehend any C.

The meaning of "*Best*" and "*Worst*" in the table is this: We declare "*best*" when we affirm *all* of *all;* we declare "*worst*" when we deny *any* of *any.* Each of the judgments in the table declares "a worse relation between two terms than any that stands above it." The remaining points require no explanation.

§ 4. The first question before us is, Whether the four judgments, υ, ι, ω, and η, are not such as the mind forms and uses, even though it may rarely or never express some of them? True the predesignations *all*, *some*, *any*, occurring in the above table as quantifying the predicate, are not usually so expressed. Still, in the old forms we are said to think in a quantification for the predicate. Thus in A, we think "All are some;" in O, "Some are not any," etc. Now, do we not also sometimes think "All are all," "Some are all," "Some are not some," and "Not any are some?"

The evidence in favor of the natural and common use of afa, "*All are all,*" seems to be overwhelming. If we inquire into the quantity of the predicate, we shall find that this is the form whenever a property is predicated; thus, "Man is risible" means "Man is all that is

risible." So, "Animals are sentient." Again, definition seems to have this quantity; thus, "Copperas is sulphate of iron" means "Copperas is all sulphate of iron," or "All sulphate of iron is all copperas." Again, every exhaustive division yields a judgment in this form; thus, "Angles are right and oblique" means "Angles are all right and oblique;" "Length, breadth, and thickness are all the dimensions of extension;" "Mankind are all men and women;" "Pompey, Crassus, and Cæsar were all of the first triumvirate."

The form of a judgment becomes **ifa** whenever the "Some" of the subject is thought as exhausting the predicate; as, "Some of the Class are (all of the) absent;" "Some inspired men were (all of the) apostles;" "Some stars are all the planets." This appears to be predicating species of genus, which Aristotle does not provide for in his doctrine of the predicables. Perhaps he overlooked that.

Surely, then, no one will deny that judgments in which the predicate is thought as "all" are natural and in the commonest use. "The only wonder is, how they could have been almost universally rejected by logicians for over two thousand years, down to the time of Sir William Hamilton."[3] Since his day they have been accepted and incorporated into Logic by Mansel, Baynes, Thomson, Jevons, Bowen, and many others.

The form **ani**, it is said, is implied whenever genus is predicated of species; for when we say "All are some" (i. e., one part of the genus), the law of Excluded Middle compels us to think "All are not some" (i. e., the other part). If "All men are some animals," then "Not any men are some (other) animals." "No spaniel is some dog (cur)."

The law of division, that the members must exclude each other, compels us, it is said, to think the form **ini**; i. e., "Some (=one species) are not some (=any co-ordinate species)." E. g., "Some trees (pines) are not some trees (oaks)."

In general, it has been said that any limiting adjunct qualifying the predicate is equivalent to particular quantification. E. g., "A rose is a fragrant (=some) flower" (**afi**). Likewise, "A rose is not a poisonous (=some) flower" (**ani**). And to say "Some roses are not red" is to say "Some roses are not red (=some) roses" (**ini**).

The consequence to Logic of this doctrine of the thorough-going quantification of the predicate appears, at the outset, to be a simplification, and therefore advantageous. The distinction between sub-

[3] Bowen's *Logic*, p. 133.

ject and predicate ceases, it is claimed, to be of any moment. Each term quantified, it becomes indifferent which stands first, every judgment being reduced to an equation or non-equation of two terms. Consequently the old doctrine of Conversion is swept away, and we simply transpose the quantified terms at will.[4] Upon this we may remark at once, that to claim that the distinction between the subject, that which we are speaking of, and the predicate, that which we say of it, has been reduced to naught, is absurd; for to nullify this distinction would require not a mere remodelling of technical forms, but a remodelling of the forms of human thought.

But we will concede that the two affirmative Hamiltonian judgments (whatever may be said of the negative) occur in thought, and appear in our reasonings. This alone, however, does not entitle them to a position in Logic co-ordinate with the Aristotelic forms. Before deciding upon this claim it is needful to re-examine them all somewhat more closely.

§ 5. The second question, to be decided affirmatively before the Hamiltonian can be admitted to rank with the Aristotelic forms, is, Are they simple judgments? We undertake to show that, if logical, they are not simple, but compound, and hence are to be rejected.

The two negative forms, **ani** and **ini**, have been rejected by nearly all logical writers on various grounds. They at once excite prejudice by being so awkward, and so unlike the common forms of speech. Says Thomson, "They have the semblance only, and not the power, of a denial." We add, that a denial is essentially an exclusion; and an exclusion, if the quantity of the thing excluded be thought of at all, is, *ex vi termini*, of a total. A partial exclusion is meaningless, or rather a self-contradictory phrase. The exclusion of a part of a thing has meaning, and it is, that the total portion is totally excluded. Let it be remarked that a total exclusion (a tautological phrase) is different from the exclusion of a total. Moreover, we may totally exclude, and in simple judgment do totally exclude, without any thought whatever of the quantity of the thing excluded. Therefore, no simple negative judgment can have a particular predicate.

If it be said that the exclusion of a part implies the non-exclusion of another part, and that this is expressed by **ni**, we reply that such a proposition is compound, consisting of a simple negative, totally ex-

[4] Hamilton's *Logic*, p. 525.

cluding, and of a simple affirmative, including. Such compound judgments are admitted to be conceivable, they may be sometimes useful, they may occur in reasoning, they may appear as premises in syllogistic forms. But, being compound, they cannot claim a place in Logic, much less can they take rank with the simple forms, to which they are themselves reducible, and to which they must be reduced in any complete logical analysis.

The form **ifa** has been accepted by some logicians. It is at least very questionable. When we say "Some men are poets," the simple meaning is that "Some men are contained under the class 'poets,'" or, changing to intension, that "Some men are poetical." In neither case, in this simple predication, does there seem to be any thought whatever of quantity in the predicate. It is neither "all poets" nor "some poets." The quantity is indefinite in an absolute sense, i. e., it does not exist in the thought. If the question arises, we think instantly that "All poets are men," and compounding the two propositions for the sake of brevity, we may say, "Some men are all poets" (=**ifa**). This, then, also appears to be a compound proposition.

I maintain here that the predicate of an affirmative, as well as of a negative, has strictly no quantification whatever. That assigned by the old Logic is merely in view of conversion, it has no other relevance, and is given solely in anticipation of the converse. Now a term which is thought without reference to quantity cannot in becoming the subject of an affirmation (unless another judgment intrudes) be anything other than the wholly indefinite "Some" (= may be next to none, or perhaps all). Hence the scholastic rule that the predicate of affirmatives is (in view of conversion) undistributed; and therefore also A converts in thought into I, if the judgments be simple; and the rule *per accidens* cannot be swept away by any logical device. For true Logic is not a juggling with words, objectively, to see what may be done with them, but a representation of what occurs subjectively in our thoughts.[5]

But the stronghold of Hamilton's doctrine is **afa**. If this falls, all the others go with it. Let us observe that the doctrine of a quantified predicate, either old or new, is applicable only to judgments in exten-

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[5] Let us note, by anticipation, that in the syllogistic rule requiring that the middle term be distributed at least once, we are usually warned that the predicates of affirmatives are "undistributed." It should be, are "not distributed," a pure negative, meaning less than "undistributed," which is equivalent to "particular."

sion. We can see a good reason for thinking a quantity into the predicate in anticipation of conversion, as in the old doctrine; but to hold with Hamilton that the predicate is always quantified in thought is to exclude the judgments in intension from Logic. But, by a curious inconsistency, he, more than any other logician, insists upon intension, and expands Logic to embrace it fully. One of the two, however, must be given up. But, if we give up intension, we must give up extension also, for intension is primary, and then Logic ceases to be. Says Mill, "Propositions in extension have absolutely no meaning but what they derive from comprehension. The Logic of the quantified predicate takes the comprehension out of them, and leaves them a *caput mortuum*." This consequence is certainly sufficient to cast a shade of suspicion over the well-fortified **afa**.

When we make the assertion that "All triangles are all trilaterals," is it not evident that, to cover the whole ground occupied by this statement, two judgments are required: first, that "Every triangle is trilateral," and, secondly, that "Every trilateral is triangular?" How is it possible to pronounce that to be a simple judgment which is divisible into two, and especially when one of these may be thought without the other, when one may be known and the other unknown, when one may be false and the other true? If "All triangles are all trilaterals" is only one judgment, what is "All triangles are trilateral? Is it half a judgment?

In Hamilton's support of **afa** he says: "Ordinary language quantifies the predicate as often as this determination becomes of the smallest import. This it does directly by adding *all*, *some*, or their equivalent predesignations to the predicate; or it accomplishes the same end indirectly, in an exceptive or limitative form. E. g., Directly: as, 'Peter, James, John, etc., are all the apostles.' E. g., Indirectly: as, 'God alone is good,' i. e., 'God is all that is good;' 'Virtue is the only nobility,' i. e., 'Virtue is all that is noble;' 'On earth there is nothing great but man,' i. e., 'Man is all earthly great.'"[6] Now the doctrine of logicians has always been, as stated by Scheibler: *Omnis exclusiva resolvitur in duas simplices, alteram affirmatam, alteram negatam.* This view has already been discussed (i, § 12). If it be correct, if such exceptive and exclusive propositions are compound, then it appears from Hamilton's own statement and illustrations, that **afa** is a compound proposition.

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[6] *Logic*, p. 517.

It may be conceded that this form **afa** is familiar in speech, that it is natural, if you please, that men make constant use of it in reasoning, that such reasonings are easily reducible to syllogistic forms in which one or both premises are **afa**, that brevity and perspicuity are promoted by its use, and hence that it should be included in every logical analysis of the forms of human thought. But Logic in this proposed analysis cannot stop short of simple and ultimate forms. If it were an art teaching us how to reason or even how to detect error in reasoning, then there might be occasion for an elaboration and symbolizing of compound forms, though indeed the work would be endless. But as it is on the higher ground of a science, one showing how we do and must think, it is out of character to present compound forms as the results of analysis. Now whatever can be proved from All A is all B, can be proved from one or both of its elements—All A is B, and All B is A. Whatever can be proved from Some A is all B, can be proved from its elements—Some A is B, and All B is A. It is not possible that there should be a single instance in which a conclusion, provable from premises with quantified predicates, could not be proved from the same unquantified, if we set forth all those which are really involved. If there could be such an instance, the doctrine of a quantified predicate would be a real addition to the theory of thought; otherwise not.[7]

Consequently, supported by the authority of Mill, De Morgan, Bain, and others, we object to the intrusion of the compound form **afa**, and its train, among the simple forms, and reject the doctrine of "The Thorough-going Quantification of the Predicate," taught by Hamilton. We are glad to escape from the fearful complications into which it leads, and rest in the comparative simplicity of the Aristotelic Logic; and we honor the old logicians in the belief that, during the two thousand years of their acute discussions, these forms were surely considered, and were not allowed in their system because they did not belong to the fold, and if admitted would ravage the flock.

§ 6. The foregoing argument is sufficient to refute Hamilton's doctrine, and exclude his forms from among the Aristotelic. The view taken is complete as against him; but it does not completely exhibit the ultimate character of **afa** and its cognates. Let us examine their nature yet more closely. We have pronounced them compounds. It

[7] See Mill's *Examination of Hamilton's Philosophy*, ch. xxii.

would perhaps be more accurate to say that each results from the compounding of two simple logical judgments, and becomes a simple mathematical judgment. This needs some explanation.

Hamilton speaks continually of distributed and undistributed predicates. The old Logic, too, uses the same expressions, but only, as we have said, precursory to conversion, which, indeed, is already accomplished as soon as the quantification is thought into the predicate. We have denied that the predicate of a purely simple logical judgment has, or can have, any quantification whatever, affirming that it is absolutely indefinite. We now add, that a quantitative predesignation thrust in upon a predicate by the compounding of two simple judgments removes the judgment from the logical or qualitative whole, and transfers it to the quantitative or mathematical whole. Hence, if we view the judgment in reference to its origin, we may call it compound, or compounded; but if we view it in its own sense, we must no longer call it a logical, but a simple mathematical judgment (i, § 13).

For, consider the meaning of "all" in the predicate. It is not, it cannot be, the distributive, divisive, exemplar "all," but is always the total, indivisible, cumular "all," a mathematical whole. E. g., "All men are bimana;" this is the distributive "all," meaning that all, each, and every man is in the class, or has the mark, *bimana*. But let us say "All men are all bimana;" this does not mean "Every man is all bimana," nor "All men are every bimana," nor "Every man is every bimana," which is nonsense. It means "All men (as a mathematical, total, collective whole) are all bimana" (as *ditto*). Thus "all" in the predicate is never distributive, but cumular, and enforces the "all" of the subject also to be cumular. So also the total predicate of a negative is a mathematical, not a distributed total; and "some" in the predicate is a mathematical part. More generally, whenever the quantity of the predicate is designated, both terms are individuals, and the judgment is mathematical. The effect of thus quantifying the predicate is to transmute the judgment from the qualitative to the quantitative whole, in which it is simple. This shows that Hamilton's "distributed predicate" is a complete misnomer, and the fact is fatal to his doctrine.[8]

[8] To avoid future misapprehension, we will note that, though denying to the Hamiltonian forms the rank and important position assigned to them in Logic by their author, we may have occasion to use them, for the sake of brevity, in syllogizing. Also we shall be free to use his nomenclature and notation, which we esteem a valuable contribution to the appliances of technical logic.

PART FOURTH.—OF REASONINGS.

I. THE SYLLOGISM.

§ 1. The logical and natural treatment of a subject requires that its definition be first ascertained, which fixes its relations to superior notions; then that its subdivisions be ascertained, which fixes its kinds. First its connotation is settled, then its denotation. Thus, in general, let us proceed with the subject now before us.

We have already defined thought in a general sense to be the bringing one notion in or under another. This duplex definition obviously refers to the two quantities of thought, the intension and the extension. The distinction between these is thorough-going; we met it at the outset in concepts; we found that given judgments may be construed in either quantity; and we shall find the same to be true of reasonings. As every notion may be viewed either as a complement of marks, or as a kind of a thing, so every reasoning may be viewed either as a bringing-in marks into a notion, or as a bringing the notion under a genus. But let it be remembered that intension and extension always coexist, and that thought is readily transmuted from the one into the other.

We have also said that thought is either by conceiving or by judging. Now let it be again observed that conception and judgment are not two kinds or species of thought, but one and the same thing in a different form, or viewed under different aspects or phases. Every concept is an implicit judgment, and every judgment is an explicit concept. Consequently the definition given above of thought is equally the definition of conceiving and of judging.

There are, however, two kinds of conception, the immediate or direct, and the mediate or indirect. The first has been treated in Part Second. It is the direct comparison of two notions by which they are immediately conjoined, or disjoined. The second occurs when we are through ignorance unable to make a direct comparison, and re-

sort to a medium, i. e., some third notion, which being directly compared with each of the two former enables us to see their agreement or disagreement, and consequently to conjoin or disjoin them. This is mediate conception. Immediate conception has received no specific name, and is always understood when the unqualified word is used. Mediate conception is called reasoning. This, then, is the logical definition: Reasoning is mediate conception.

Let us exemplify reasoning in this view. I have the notion *man* and the notion *free-willed.* On comparing these, I am unable to decide whether or not this mark belongs to that concept. By the principle of the Law of Excluded Middle I am constrained to believe that it either does or does not; but which, I cannot immediately determine. So I seek a medium of comparison. I take the notion *responsible*, and I see directly that the notion *man* involves this notion *responsible*, likewise that the notion *responsible* involves the notion *free;* and thus I see that the notion *man* involves as one of its marks the notion *free.* This is the intensive view. If I proceed rather in the extensive quantity, the matter would be expressed thus: I am unable directly to decide whether or not *man* is a kind of *free-agent.* But I know that the class *free-agents* contains under it the species *responsible agents*, and that this contains under it *man;* and so I am able now to think that the class *free-agents* contains under it *man* as one of its kinds.

In exact accordance with this, we now observe that there are two kinds of judgments, the immediate or direct, and the mediate or indirect. Immediate judgment has been considered in Part Third. It also is the direct comparison of two notions, but issues in the explicit declaration that they are conjoined or disjoined. Mediate judgment occurs when, not being able directly to judge this agreement or disagreement, we seek a third notion as a medium of comparison, and explicitly state that each of the other notions does or does not agree with this third; and thus we are enabled to conclude explicitly whether they do or do not agree with each other. This is mediate judgment. Immediate judgment has received no specific name, and is always understood when the unqualified word is used. Mediate judgment is called reasoning. The logical definition, then, is: Reasoning is mediate judgment.

It is quite evident that there is no essential difference between mediate conception and mediate judgment; the difference is merely formal, and is usually neglected. Also it is evident that a mediate judgment when expressed in words will exhibit three propositions. Let

us not be misled by this appearance to suppose that a reasoning is three judgments. Aristotle insists, and all logicians agree, that the reasoning, which is the act of mediate comparison, and which from two given judgments having a common part concludes a third, is but a single act of mind, a single thought, only one judgment.

We will now exemplify reasoning viewed as a mediate judgment. I do not know whether to affirm or deny that man is free. So having found a medium of comparison, I express myself thus,—

Man is responsible;
One responsible is free;
therefore, Man is free.

This is evidently thinking in the quantity of intension. Treating the same matter extensively, I would say,—

Every responsible-agent is a free-agent;
Every man is a responsible-agent;
therefore, Every man is a free-agent.

In order expressly to distinguish the intensive from the extensive quantity, we interpret the copula of the former as "*comprehends*," and that of the latter as "*is contained under*." The above more explicitly stated would then be as follows,—

Intensively {
The notion man *comprehends* the notion responsible;
The notion responsible *comprehends* the notion free;
∴ The notion man *comprehends* the notion free.

Extensively {
The notion responsible-agent *is contained under* free-agent;
The notion man *is contained under* the notion responsible-agent;
∴ The notion man *is contained under* the notion free-agent.

Since conception and judgment are merely different forms of thought, it is perfectly competent to unite the two synonymous definitions of reasoning given above into one, and define thus: Reasoning is mediate thought. Again, as all thought is comparison: Reasoning is mediate comparison. Again, we found in Part Third that to infer is to derive one judgment from one or more others, and that immediate inference or illation does this directly, from a single antecedent. We now find that mediate inference does this indirectly, from two antecedent judgments having a common part; hence, we may define once more: Reasoning is mediate inference or illation.

A mediate judgment, when presented as in the examples given above, is called a Syllogism. What is subjectively a Reasoning, is objectively a Syllogism. Hence we define: A Syllogism is a reasoning fully and regularly expressed in language. What is meant by "regu-

larly" will hereafter more clearly appear. Another definition is: A Syllogism is an inference by which one proposition is derived from two others conjointly, the one being virtually contained in the others. Aristotle opens his Prior Analytics with this definition: "A Syllogism is an enunciation (λόγος, *oratio*) in which, from something laid down and admitted, something distinct from what we have laid down, follows of necessity."

Let us consider at once the import of this last phrase, "follows of necessity." The essence of a syllogism consists, not in the truth of the propositions laid down, nor in the truth of that which is inferred, but in the production of a new and distinct judgment, the truth of which cannot be denied without impugning those we have already accepted for true. In other words, the essence of the syllogism, and all that is actually declared by it, is the necessary consequence of the conclusion from the premises. This necessity flows from the necessary character of the primary laws of thought, to which the syllogism conforms and by which alone it is ultimately governed. It is frequently expressed in the conclusion by the addition of "must." For example,—

Since all metals are fusible,
And gold is a metal,
Gold must be fusible.

The common distinction, then, between demonstrative and moral or probable reasoning, lies wholly in the matter, not at all in the form. The form, or rather the process, by which we infer, is in all cases the same, and is in all cases, if correct, equally demonstrative, i. e., apodictic, necessary.

The affirmation of necessary sequence being the essence, it follows that the syllogism is really only one judgment, a single indivisible act of thought. Though apparently complex, though in a certain sense including three judgments, it does not affirm either of them taken separately, but only the necessary dependence of one on the others. It is a judgment concerning judgments, one affirming the relation of sequence, and may easily be expressed in a single proposition; e. g., That gold is fusible is an inference from the judgments that it is one of the metals, and that they are all fusible.

Another consequence of this doctrine is that Logic does not concern itself with the truth or falsity of the several propositions. One or all may be false, but, having granted the antecedents, the consequent must also be allowed, if the reasoning is sound, the syllogism regular.

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We may, however, note that the antecedents being true, the consequent is necessarily true. Also, what measure of doubt belongs to the antecedents, just that measure of doubt, no more, no less, belongs to the consequent. But should the antecedents be found false, it does not follow that the consequent is false; it is simply unproven, and may be established as true upon some other antecedents. For example,—

The natives of Italy were Greeks;
The Athenians were natives of Italy;
∴ The Athenians were Greeks.

Grant these antecedents, and the consequent must be admitted, for it follows of necessity.[1] But both antecedents are obviously false, yet the consequent is not false, for we can prove it from other antecedents.

Before entering upon another section, two additional remarks may be appended. A syllogism is an expression of one of the units of which the most elaborate argument is only the sum. The longest chain or most complex net-work of reasoning may be stated in syllogisms linked together. The links are quite similar. When we thoroughly understand one, in its characters and kinds, we understand all. It is true that an argumentation does not usually present the form of syllogisms; its steps are much abbreviated by elisions and variations from form; but however elaborate it may be, still it can be resolved into its elementary syllogisms, and these stated in regular form and consecution. Moreover, much of our common-place thinking and conversation, even many of our lightest witticisms, if closely analyzed and fully stated, will be found to resolve into syllogisms.

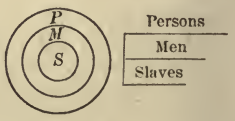
The other remark is but a repetition of one previously made. In the study of reasoning we, of course, are not advancing beyond complete system. The function of reasoning or mediate judgment is solely to make our concepts more clear and distinct, to ascertain their true relations to each other, and thus to fix their places in the hierarchies of our thoughts, which thereby approximate more and more to complete systems, the perfection of knowledge.

[1] See *Anal. Priora*, bk. ii, ch. ii; and, *per contra*, Esser, in Hamilton's *Logic*, p. 322. It may be well to say here that the doctrine of the Syllogism, as treated by Aristotle in the *Prior Analytics*, should be carefully read by every student of Logic. The best text of the Organon is probably that of Theo. Waitz, accompanied by his valuable Latin commentary; but it is to be regretted that the isagogue of Porphyry has not, at least in the edition I am using (Leipsic, 1844), been included. The translation and commentary of St. Hilaire is classical, and must not be neglected. For English readers Owen's translation and notes (Bohn's ed.), and Poste's text, translation, and notes on *De Soph.* (Macmillan, 1866), are excellent.

§ 2. The next step in the treatment of the syllogism, like that in our treatment of the proposition, is to view it as a mathematical or quantitative whole, and sever it by partition. The parts thus obtained by dissection are to be examined and named, and their relations indicated. We will then proceed to consider the kinds of syllogism.

The syllogism, as has been anticipated, consists of three propositions, two of which are called the antecedents, and the third, the consequent. The antecedents are also called the premises, and the consequent, the conclusion. To conclude (*con-cludere*) is to shut up together in the last proposition notions which stood apart in the first two. So also the word "syllogism" (*συν-λέγειν*) signifies a collecting together; as when Aristotle describes a conclusion as "a perfect syllogism of the extremes." The following is an example in the quantity of extension,—

All Men are Persons; $= M : \frown P =$ Major Premise;
All Slaves are Men; $= S : \frown M =$ Minor Premise;
$\therefore$ All Slaves are Persons; $= S : \frown P =$ Conclusion.



There are here only three terms or notions, "Slaves, Men, Persons." It is evident that these stand to each other in the relation of whole and part, Slaves being contained under Men, and Men being contained under Persons. Persons, then, is the term of widest extent (as in the symbols); Slaves, the term of least extent; and Men of intermediate extent. This M, which is called the Middle Term, is found in each of the premises, but not in the conclusion. The other two terms, which together are called the Extremes, are both found in the conclusion; separately they are called the Major Term and the Minor Term. Hence we may define as follows,—

The **Middle Term** (M) is the one with which each of the extremes is compared in the premises. It is also called the Argument.

The **Major Term** (P) is the term of greatest quantity, or the greatest whole. It is always (in extension) the Predicate of the conclusion.

The **Minor Term** (S) is the term of least quantity, or the least whole. It is always (in extension) the Subject of the conclusion.

The **Major Premise** is the premise containing the Major Term. It is usually placed first.

The **Minor Premise** is the premise containing the Minor Term. It is usually placed second.

Examples in the quantity of intension will now be given. We transmute the one above into this quantity, and add one other,—

All Slaves are human;	Silver is Metallic;	= S: — M	
All the human are Personal;	Metal is Positive;	= M: — P	= S: — M: — P
∴ All Slaves are Personal.	∴ Silver is Positive.	= S: — P	

The expression, for the sake of form and brevity, is permitted to be somewhat awkward. By "Positive" is meant electro-positive. In the graphic notation[2] on the right, the long pointed dash below is the copula of the extremes in the conclusion. This condensed form is read exactly like that standing just before it. The extensive syllogism can of course be expressed in a similar way, only the copulas are inverted. When we read in the direction that the copula points, i. e., extensively, it should be read "*is contained under*;" when we read in the direction opposite to that the copula points, i. e., intensively, it should be read "*comprehends*."

In changing the extensive syllogism into the intensive, the middle term continues intermediate, but the relative quantity of the extremes is inverted; the greatest part in extension (P) becomes the least part in intension, and *vice versa*. This is in accord with the law that extension and intension are in inverse ratio. In the example "Silver comprehends Metallic, and this comprehends Positive," S is obviously the greatest whole, and P the least. Hence, in intension the major term is the Subject of the conclusion, and the minor term is the Predicate of the conclusion. And hence, since it is usual to place the major premise first, the order of the premises is transposed. We have, then, for changing a syllogism of either quantity into the other, the following RULE: Transpose the premises, and invert in thought the meaning of the copula; i. e., instead of "*comprehends*," think "*is contained under*," and *vice versa*. For example,—

In Extension	All Metals are Positive elements; Silver is a Metal; ∴ Silver is a Positive element.	P — :M — :S

Aristotle's definition of the terms of a syllogism is so general that it will apply to either quantity, which renders it probable that, unlike his followers, he recognized both. "I call," he says in the first part of the Prior Analytics,[3] "the middle term that which is both itself in another and another in it; and which by its position lies in the mid-

[2] See Part 3d, iii, § 3.

[3] Ch. iv.

dle. The extremes I call both that which is in another, and that in which another is. I define the major extreme as that in which the middle is; the minor extreme, as that which is under the middle."[4]

Aristotle's method of stating the syllogism differs from ours. It is thus:

P inheres in, or is predicated of, all M;
M inheres in, or is predicated of, all S;
∴ P inheres in, or is predicated of, all S.

It will be observed that here the major premise (in extension) stands first. This brings the middle term, as he says above, into the middle position. Soon after his day logicians, preferring to state the propositions in their natural form with the subject first, transposed the premises, in order to keep the middle term in the middle position. We have, then, the—

Ancient order { All S is M; All M is P; ∴ All S is P.

This order was observed until the time of Boethius, who thought it more important to place the major premise first, returning in this respect to Aristotle, and these high authorities determined subsequent usage. Consequently in our method of stating the syllogism in extension, the only quantity recognized from the time of Aristotle until recently, the middle term does not have middle place.

There seems, however, no valid reason why the major premise should have precedence. It is said that it is more natural to begin our statement with the greatest whole. It may be so, but in the actual, practical expression of a reasoning we often find the order of all the propositions completely inverted, the conclusion being placed first as a *quæsitum*, or problem, or thesis, and the premises following in reversed order; as, "Silver is a positive element; for it is a metal, and all metals are positive elements." Is not this quite a natural order of statement? If so, then unquestionably, unless a more satis-

[4] *Anal. Pri.* i, iv. Ὁ μέσος καὶ αἱ ἄκραι. The middle term is the bridge between them. Properly, when we inquire after the *meaning* of a thing we are seeking the mean or middle term or notion. E. g., "What mean ye by this service?"—Ex. xii, 26. Meanness, as applied to our using means, has acquired a bad sense.

Can you imagine I so mean could prove,
To save my life by changing of my love?—*Dryden.*

The monkey using the cat's paw, is a proverbial specimen. Barring the bad sense, the middle term is the logical cat's paw.

factory reason can be adduced, we are justified in viewing the approved order of the premises as arbitrary, merely a matter of convention and custom.[5]

Kant takes a somewhat different view of reasoning and the syllogism. Reasoning is bringing a case under a general rule, and so determining it. In the syllogism the major premise is a rule, the assertion of a general condition, the Sumption (*Obersatz*). The minor premise is the cognition that the condition of the rule, somewhere or other, takes place; or, is that which brings a case under the condition of the rule, the Subsumption (*Untersatz*). The nexus of what is subsumed under the condition, with the assertion of the rule, is the Conclusion (*Schlusssatz*). Hence a syllogism is the cognition that a certain proposition is necessary, through the subsumption of its condition under a given general rule. Hereby we understand the conclusion *a priori*, not as manifested in things individual, but as universally maintained, and as necessary under a certain condition. And this, that all stands under the universal, and is determinable in universal laws, is the principle itself of rationality or of necessity.[6]

§ 3. In proceeding now to the consideration of kinds, we notice, first, the common division of reasonings into deductive and inductive. Deduction consists in drawing a less general or a particular truth from a general truth antecedently known. Induction consists in rising from particular facts to the determination of a general rule or law. It is evident, then, that the account which has just been given of reasoning and the syllogism relates exclusively to deductive thought. Many writers on Logic, accepting induction as a kind of reasoning opposed to deduction, attempt to subject the inductive process to syllogistic forms and laws. The results are not profitable nor commendable.

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[5] Likewise in the following the order of the propositions of the involved syllogism is completely reversed:

> "Qui melior servo, qui liberior sit avarus,
> In triviis fixum cum se demittit ob assem,
> Non video; nam qui cupiet metuet quoque; porro,
> Qui metuens vivet liber mihi non erit unquam."—Hor. *Epist.* i, 16.

The argument re-ordered may be stated thus:

> Whoever is fearful is not free.....Sumption.
> The miser is fearful.....Subsumption.
> ∴ No miser is free.....Conclusion.

[6] See *Logik*, §§ 56–58.

An examination of such views must be deferred. They are mentioned here only to say that it is at least very questionable whether the inductive process can properly be viewed as a species of reasoning at all, certainly not under the definitions of reasoning we have given. Without present discussion, it will be understood that by reasoning we mean the deductive process, and hold that the syllogism and its laws pertain exclusively to it.

Since the sumption of a syllogism is a general rule, or since the major premise contains notions of wide, often of absolute, generality, the question may have already arisen in the mind of the reader, Whence are they obtained? To say they are the conclusions of prior and wider reasonings may in most cases be true, but is an insufficient answer, for the same question recurs as to these. What, then, is the ultimate source of these generalities? We answer, it is either intuition or induction. By the former we know, for example, that "Every change is caused;" by the latter, that "The volume of gas is in the inverse ratio of the pressure." Sciences whose deductions are wholly from intuitive truths are called *a priori*, or pure, or demonstrative sciences; those whose deductions are from both intuitive truths and inductions are called *a posteriori*, or empirical, or inductive sciences.

The next division of syllogisms to be noticed is into intensive and extensive. This has already been sufficiently examined in the preceding sections, introduced there because needful in order to general definition, and to a complete view of the relations of the dissected parts. We are now prepared to make an estimate, briefly and once for all, of the importance of this distinction. Hamilton, to whom we are indebted for introducing it into the logical literature of our language, strenuously insists at great length that it is all-important, the two quantities of thought yielding two distinct kinds of reasoning. Reasoning in intension, he says, is the simpler and more natural form of reasoning; and in introducing it he claims to have "relieved a radical defect and vital inconsistency in the present logical system."

We cannot refuse to the modes thus distinguished the title of kinds; but how much in this case is it worth? The external difference consists wholly in transposed premises. But the order of the premises being merely conventional, any distinction founded thereon is entirely arbitrary and artificial, not real and natural, and hence goes for nothing. It is merely a convenient way by which we agree to indicate which quantity is intended. The other difference named in the rule is in the inverted meaning of the copula. This is not an ex-

ternal difference. In ordinary language the copula is wholly indifferent and ambiguous, and we can indicate its special meaning only by unusual substitutions. The slight grammatical difference which sometimes, but not always, occurs between substantive and adjective noun forms in the predicate cannot be regarded as a logical difference.

The difference, then, lies entirely in thought, and consists of that between the wholes of extension and intension, and of the reversed relation of parts and whole. That this constitutes a difference in kind we have granted, one which must be observed in an exposition of mental modes, and, we may admit, in a theory of thought, but it is of very small logical or practical consequence. For both modes are mediate inferences and through the same medium; both reach the same conclusion; the formal expression of both is the same; the supreme canon is in principle the same, requiring only verbal changes when expressly adapted to one or the other quantity; the general rules of the syllogism are the same for both, not requiring even a verbal change; the special rules are the same, requiring only a slight verbal change—the interchange of the words *major* and *minor;* hence no modification of the old logical doctrine is called for by the introduction of the intensive syllogism.

Moreover, when we consider that, without the slightest objective difference, one of these modes subjectively, and with the greatest facility, changes to the other, and that without further consequence, we ask, What is the worth of the difference between two things so completely and readily transmutable? Again, it is highly probable that the two quantities always actually coexist in thought as psychological correlatives, one being usually more obscure than the other. If so, their convertibility would rather indicate identity, being inconsistent with the opposition which belongs to kinds. And, again, we remark that Kant's admirable and philosophic view of reasoning and the syllogism does not distinguish the quantities. Finally, we often use both quantities successively in the same reasoning. For example,—

All of the metals are positive.....Intensive.
Silver is one of the metals.....Extensive.
∴ Silver is positive.....Intensive.

Can this be fairly objected to? Hamilton would denounce it as a hybrid; a senseless gymnastic, hopping from one quantity into another, and back again; possible, but stupid.[7] I cannot admit this,

[7] See *Logic*, p. 303.

and believe that only he who is riding a hobby would find it faulty.

From these considerations we may justly conclude that the distinction between extensive and intensive syllogisms is of very small, if of any, logical moment, and certainly very far from deserving the emphasis given to it by Hamilton, and repeated with passive sequacity by so many subsequent writers. We shall keep it in view only for the sake of more complete theory, and in illustrations use indifferently either quantity.

The following divisions of the syllogism are determined simply by the kind of its propositions. The general division is into—

The Categorical and the Conditional syllogism. We shall for some time continue to treat of the former exclusively. The consideration of the latter is postponed to a subsequent Topic.

Categorical syllogisms may be variously subdivided into—

1. The Simple and the Compound. The latter are deferred to a subsequent Topic. The simple will occupy us exclusively for the present.

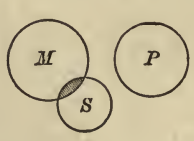
2. The Total and the Partial, or the Universal and the Particular. When any one proposition is particular, the syllogism is particular, having a particular conclusion. When all three propositions are universal, the syllogism is universal. The quantity of the proposition determines this kind.

3. The Positive and the Negative. When one premise is negative, the syllogism is negative, yielding a negative conclusion. The quality of one premise determines this kind. The two latter kinds, depending on quantity and quality, call for no further remark at present. They may, however, be here jointly illustrated by an example which has one premise particular, and one negative, yielding a conclusion which is both.

No murmurs are prayers.....(E)

Some sighs are murmurs.....(I)

∴ Some sighs are not prayers....(O)



Prayers.
Murmurs.
Sighs.
P — : M +, S +

Finally, categorical syllogisms are divided, according to the relative position of the middle term, into four Figures. These will be considered under the Topic next following. All examples thus far given are in the first figure.

Under the present Topic it remains to consider the Canon, and the General Rules of the categorical syllogism.

§ 4. The judgment whereof the syllogism essentially consists, the judgment that the antecedents necessitate the consequent, is determined by the three primary laws of thought. Since these, however, because of their wide generality, are not readily applicable, logicians have sought to express in a single special CANON the principle of syllogism, a Canon that is only a special statement of the three primary laws as governing the syllogism, and which may be used as an easy and direct test of its validity. The results of these attempts are not very satisfactory, the several forms of the Canon being each inadequate; but they are nevertheless useful. We will here state some of the most noteworthy:

1. "Part of a part is part of the whole." Remembering that marks are spoken of as parts of a concept, and species as parts of a genus, this axiom is obviously applicable to both quantities of thought, and to both wholes, the logical and the mathematical. Its generality, brevity, and simplicity render it perhaps the most useful form. It is, however, inadequate, being applicable only to affirmative syllogisms. A modified form, applicable only to the logical whole, is: "What is said distributively of a whole may be said of a part." If the reader will apply these forms to either of the foregoing affirmative syllogisms, the meaning will be sufficiently obvious; and it will also become evident that the Canon is only the essential judgment of the syllogism generalized in second intentions.

2. "*Contentum contenti est contentum continentis.*"—*Leibnitz.* Likewise applicable only to affirmative syllogisms.

3. "*Prædicatum prædicati est etiam prædicatum subjecti.*" A translation of Aristotle's first antipredicamental rule (*Categ.* iii). The following may be regarded as a free rendering of this excellent form:

4. "Whatever predicate is universally affirmed or denied of any middle term or part is also affirmed or denied of any subject contained under it."—*Burgersdyck.* Applicable, however, only in extension.

5. "*Quicquid de omni valet, valet etiam de quibusdam et singulis. Quicquid de nullo valet, nec de quibusdam, nec de singulis valet.*" These are the famous "*Dicta de omni et nullo*" of Aristotle, as drawn out by the Latin logicians from the Prior Analytics, Part 1st, i, 8.

6. "*Nota notæ est nota rei ipsius; et repugnans notæ, repugnat rei ipsi.*" This seems especially adapted to the intensive syllogism.

7. "What stands under the condition of a rule, that stands also under the rule itself."—*Kant.* See § 2, last paragraph.

8. "In so far as two notions (notions proper, or individuals) either both agree, or, one agreeing, the other does not, with a common third notion, in so far these notions do or do not agree with each other." This is Hamilton's "Supreme Canon for the Unfigured Syllogism," a form we will briefly consider in the sequel.

9. "What worse relation of subject and predicate subsists between either of two terms and a common third term, with which one, at least, is positively related, that relation subsists between the two terms themselves." This is Hamilton's "Supreme Canon for the Figured Syllogism."[8] He claims for it perfection of statement and absolute generality, it being the principle of syllogisms intensive and extensive, positive and negative, involving any of the eight Hamiltonian judgments.[9]

10. "Any notion may be replaced by an equivalent, or by its undistributed genus, or, if distributed, by any of its parts." We propose this, believing it to be a more general principle, and more truly expressive of the actual process of thought in reasoning than some of the preceding. It is simple and self-evident. For convenience in reference, we will call it the Canon of Replacement. Its view of the syllogism is somewhat peculiar. It considers the Sumption as declaring a relation between two notions; the Subsumption as declaring that some other notion is equivalent to, or a part of, one of these; the syllogistic judgment as being the substitution of that for this; and the Conclusion as setting forth the result. Thus, to take an old standard example, "All men are mortal;" but "Socrates is a man," i. e., he is one, a part of "All men." So, replacing "All men" by this part, we have therefore "Socrates is mortal." This Canon will apply not only to all reasonings in the logical whole, but also to those in the mathematical whole. For example:

A is equal to B;
B is equal to C;
∴ A is equal to C.

This most simple and most common mathematical syllogism, which Dr. Reid said could not be subjected to any of the approved logical

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[8] See *Logic*, p. 584; and *Discussions*, pp. 604, 605. See also the Table of the Eight Propositional Forms in Part 3d, iii, § 3.

[9] Notwithstanding the high pretensions of this Canon, it seems that Hamilton's own "Negative Moods" (*Logic*, p. 679), No. ii *a* and *b*, No. v *a*, No. vi *b*, No. vii *a*, No. viii *b*, No. xi *a* (*Ferio*), and No. xii *b* are in direct violation of it.

canons, and hence condemned the whole science and art of Logic, is obviously a very simple case when referred to the Canon of Replacement. Moreover, judgments often undergo easy modifications which are difficult to express in strict syllogistic form and bring under common logical rules, but which this Canon at once explains and justifies. For an example we take the famous logical puzzle proposed by the Port-Royal logicians, which they solve, not very clearly, in a page and a half of discussion; which Jevons says "cannot be proved by the rules of the syllogism;" and which most other writers omit to notice.[10]

The divine law commands us to honor kings;
Louis XIV is a king;
∴ The divine law commands us to honor Louis XIV.

Its solution by replacement is too obvious to call for remark, and seems to be the actual mental process by which any child will at once accept the conclusion.

§ 5. Aristotle's dicta are directly applicable only to syllogisms in the first figure. For this reason, and also because the application as a test is, in some cases, somewhat confusing, logicians have resolved the principle of the syllogism into a series of General Rules which are applicable to all figures; to which all sound reasonings must conform; and which, being quite simple and applied in succession, render the process of testing a syllogism easy, quick, and sure.[11] They are as follows:

1. **A syllogism has three, and only three, terms.** For if there be four, the two premises can have no common term. A good syllogism is a tripod. The following is a quadruped; verbally a triad, really a *Quaternio Terminorum:*

Light is contrary to darkness;
Feathers are light;
∴ Feathers are contrary to darkness.

[10] See *L'Art de Penser*, pt. iii, ch. ix; and Jevons' *Lessons in Logic*, p. 158.

[11] Hamilton (*Logic*, p. 215 sq.) reduces the six or eight Rules to three, with an acknowledged sacrifice of their generality, and with a sacrifice also, as it seems to me, of their perspicuity. His first Rule is merely our 1st and 2d stated in one compound sentence. But why condense them? The very intent is to evolve from the canon as many simple, explicit statements as are needed for a ready and easy test of the validity of any syllogism. Of course, we may condense them back to the canon itself, without displaying much ingenuity or obtaining any advantage.

2. **It has three, and only three, propositions.** For three terms give three pairs, and three only, without repetition. Apparently we have more in the following:

All beings that have nerves are sentient.....=A
All self-moving things have nerves.....=A
Worms are self-moving.....=A
∴ Worms are sentient.....=A

The reasoning is good, and the form logical; but we shall hereafter find that it is a Sorites, resolving into two syllogisms of three propositions each.

3. **One premise at least must be affirmative.** For if the middle term agrees with neither of the other two, we cannot infer through it whether or not they agree with each other. From these premises,

No marble is sentient.....=E
Some statues are not marble.....=O

we get no conclusion; however true it may be, they do not prove any statue not sentient. The following, however, yields a conclusion:

No man is entirely destitute of religious feeling.....=E
Many men are not true believers in God.....=I
∴ Many who are not true believers in God are not entirely destitute of religious feeling.....=O

But the minor premise is really an affirmative, the negative particle being treated as belonging to the predicate, which thereby becomes equivalent to "infidels," and constitutes the subject of the conclusion.

4. **If one premise is negative, the conclusion must be negative.** For if one term is denied to the middle, it must be denied finally to the other term which agrees with the middle by Rule 3. E. g.:

Few men weep.....=O
All men feel.....=A

We cannot conclude, "Some who feel weep." However obviously true it may be, these premises do not yield it. "Few" is essentially negative, and rightly construed gives us a negative sumption, yielding a negative conclusion; thus,—

Sumption,..... Most men do not weep.....=O
Now subsume, ... All men feel.....=A
Hence we conclude,.. Many who feel do not weep.....=O

5. **The middle term must be distributed at least once.** For if in each premise it is used in a partial sense, it may, in each, denote dif-

ferent objects, and so be equivalent to two terms, making four in all, in violation of Rule 1. From these premises,

Some of our citizens use profane language.....=I
Some of our citizens are refined gentlemen.....=I

we can conclude nothing, for the middle evidently refers to entirely different groups of persons. This logical fault is called the fallacy of *Undistributed Middle.* Sometimes it is not quite so very obvious; for example,—

A valid syllogism has three terms.....=A
This syllogism has three terms.....=A
∴ This is a valid syllogism.....=A

Here the middle is in each case the predicate of an affirmative, and hence is not distributed; and therefore the stated conclusion is unproven. Even when the portions of the middle are the *same*, a conclusion is not competent unless that fact be declared, which virtually makes the portion a total. For example,—

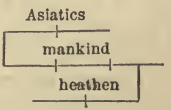
Some paper currency is legal tender.....=I
Government notes are paper currency.....=A

From these no conclusion is competent; but we may happen to know, and *think* it thus,—

All of a certain portion is legal tender.....=A
Government notes are that portion.....=A
∴ Government notes are legal tender.....=A

If, however, the undistributed middle term be so quantified that the sum of the two portions is more than the whole, a conclusion is competent. This Hamilton calls the "Ultra-total Quantification of the Middle Term." For example,—

Two thirds of mankind are Asiatics.....=I
Two thirds of mankind are heathen.....=I
∴ Some heathen are Asiatics.....=I
(At least one half are Asiatics, perhaps all are.)



One other example will suffice:

Very few men have never prayed.....=O
Nearly all men are far from being saints.....=I
∴ Many who are far from being saints have (not never) prayed..=O

The old Logic makes no provision for this exception to the rule; and it is manifest that the reasoning is mathematical rather than logical.

6. **An extreme particular in a premise must be so in the conclusion.** For if only *some* is premised, we cannot conclude *all;* we cannot argue from part to whole. The violation of this rule is called the fallacy of *Illicit Process.* It is called Illicit Major or Illicit Minor, according to the term to which the fault attaches. Here is an obvious example:

All birds are winged.....=A
A bat is not a bird.....=E
∴ A bat is not winged.....=E

The major term, "winged," is not distributed (i. e., is particular) in the premise, since it is there the predicate of an affirmative, but it is distributed (i. e., is universal) in the conclusion, since it is there the predicate of a negative, proposition. Hence there is an illicit process of the major term. The following, not quite so obvious, is an illicit process of the minor term:

Persons without imagination are not true poets.....=E
Good logicians are often without imagination.....=I
∴ Good logicians are not true poets.....=E

There are two useful rules which are deduced from those preceding, and might be appended as corollaries; but we will state them co-ordinately.

7. **From two particulars there can be no conclusion.** For if the premises be II, there is no distributed term for a middle, Rule 5. If they be OO, both premises are negative, Rule 3. If they be IO or OI, there is but one term distributed, the predicate of O; if this be taken for the middle term, then illicit major, since the negative conclusion required by Rule 4 distributes its predicate, the major term; if it be not so taken, then undistributed middle, Rule 5. E. g.:

Some students row well.....=I
Some study well.....=I
(No conclusion.)

Some students are not card-players.....=O
Some are not church-goers.....=O
(No conclusion.)

Some students do not waltz.....=O
Some "Germans" are students.....=I
(Nothing follows.)

8. **If one premise is particular, the conclusion must be so.** For a universal conclusion following

AI would require 2 distributed terms; there is but one;
AO " " 3 " " " are but two;
EI " " 3 " " " " " "
EO, both negative, Rule 3.

Aldrich, in close imitation of Petrus Hispanus, gives the following summary of his rules:

"Distribuas medium; nec quartus terminus adsit;
Utraque nec præmissa negans, nec particularis;
Sectetur partem conclusio deteriorem;
Et non distribuat, nisi cum præmissa, negetve."

II. FIGURE AND MOOD.

§ 1. Syllogisms are divided into Figures according to the position of the middle term. In the First Figure it is the subject of the major premise, and predicate of the minor. In the Second, it is the predicate of both premises. In the Third, it is the subject of both. In the Fourth, it is the predicate of the major premise, and subject of the minor. Thus:

Fig. 1.	Fig. 2.	Fig. 3.	Fig. 4.
M — P	P — M	M — P	P — M
S — M	S — M	M — S	M — S
∴ S — P	∴ S — P	∴ S — P	∴ S — P

sub præ - - - - - - - - - tum præ præ - - - - - - - - - tum sub sub - - - - - - - - tum præ sub.

This last line is a useful mnemonic, without other meaning. The notion of "Figure" was borrowed by Aristotle from Figures of Rhetoric, which are departures from the plain, literal forms of speech. On this analogy, there ought to be some one standard form from which all others are departures, and thence properly called Figures. Such standard form is the misnamed First Figure, which is the pure type of deductive argument.[1]

Each of these figures may claim to have its special Canon. Aristotle's *dicta de omni et nullo* are specially adapted to the first figure. It is easy to modify the phraseology so as to adapt them in turn to each of the others. But this cumbers us with four canons instead of one, and to no advantage. We will, then, let them go canonless, and subsequently show that the last three may be reduced to the first. There are, however, special rules governing the figures, deduced from the general rules of the syllogism, to which it is well to give some attention. They follow, illustrated by an example.

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[1] "Σχήματα, figuras syllogismorum, quæ dicuntur (Appuleius 'formulas' vocat), ab Aristotele appellatas esse Iul. Pacius putat, quia figuris geometricis adscriptis syllogismi ab eo illustrati sint. Equidem hanc vocem non tam a geometris petitam quam de ipso ordine terminorum accipiendam putaverim, quem σχῆμα appellari licebit, etiam si de geometricis figuris non cogitetur." (Waitz, *Com. on Organ.*, 26 b 33.) But Hamilton, *per contra*, maintains the opinion of Pacius.

Conspectus of Figure.

Example. *Special Rules.*

Fig. 1 (*sub præ*).

No man is perfect.....Major premise must be universal. (Else undistrib. middle.)
Some saints are men.....Minor premise must be affirmative. (Else illicit major.)
∴ Some saints are not perfect.

Fig. 2 (*præ præ*).

No perfect-one is a man....Major premise must be universal. (Else illicit major.)
Some saints are men.....One premise must be negative. (Else undistrib. middle.)
∴ Some saints are not perfect. (Hence the conclusion is always negative, Rule 4.)

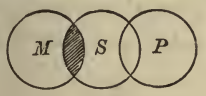
Fig. 3 (*sub sub*).

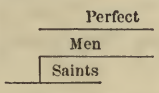
No man is perfect.
Some men are saints.....Minor premise must be affirmative. (Else illicit major.)
∴ Some saints are not perfect. Conclusion must be particular. (Else illicit minor.)

Fig. 4 (*præ sub*).

No perfect-one is a man....If either prem. is neg., maj. must be univ. (Else ill. maj.)
Some men are saints.....If maj. prem. is aff., min. must be univ. (Else undis. mid.)
∴ Some saints are not perfect. If min. prem. is aff., conclu. must be partic. (Else ill. min.)

These rules and their grounds should be thoroughly examined; but only those of the first figure need be retained in memory. All have reference to extension. To adapt them to the intensive syllogism, it is needful only to change the word "major" to "minor" and *vice versa*, wherever they occur. The symbolic notation of the example above (in extension) is the same for each of the four figures; the graphic notation is different for each of the figures; thus:





P —+—:M —,S (Fig. 1.)

§ 2. Quite a number of recent logicians insist that the variations of the syllogism by figure are arbitrary, simply serving to display the middle term in all possible positions. They endeavor to prove that reasoning in either of the last three is distorted and unnatural, and that the first only is the natural order of thought. Kant himself, in a little tract on the question, followed by Hamilton *in extenso*, contends that all reasoning is actually in the first figure; for, when perforce it is expressed in one of the others, the mind interpolates the converse of one at least of the propositions, and thus mentally reduces it to the first figure, which alone is pure and natural. This is possible to conceive, but perhaps impossible to prove. We readily

grant, however, that a reasoning which in the first figure is orderly and natural will, when reduced to another, appear distorted, awkward, and unnatural. Indeed, the example given above sufficiently illustrates the fact. But it seems that the same is true of the second and third; that there are reasonings which naturally appear in one or the other of these two figures, and that these, when reduced to the first, become harsh and disordered. We will briefly consider these two, deferring until later an examination of the fourth figure.

It is hardly to be questioned that the natural order of predication is that which predicates a greater of a less, as a genus of a species. How much better to say "Some scents are pleasant" than to say "Some pleasant things are scents." Now there is nothing in the nature of a negative proposition that determines the relative extent of its two terms; but if we happen to know that one is wider than the other, we naturally make that the predicate; and if it be the middle term, the reasoning will naturally fall in the second figure, because then it will be the predicate in both premises. E. g.:

The true apostles were not thieves;
Judas was a thief;
∴ Judas was not a true apostle.

By converting the major premise to "No thieves were true apostles," we get the first figure, but sacrifice the smooth natural order of statement as given in the second figure.

On the other hand, if what we know to be the narrower of the two terms of a negative proposition is the middle term, the reasoning will naturally fall in the third figure, because then it will be the subject of both premises. E. g.:

The apostles sought no temporal reward;
The apostles were zealous in their work;
∴ Some zealous persons did not seek temporal reward.

By converting the minor premise to "Some zealous persons were apostles," we get the first figure, but manifestly lose the smooth naturalness of the given expression.

So, then, we conclude with Thomson that, since in some cases natural reasons prescribe the second or third figure and reject the first, the distinction of these is not an arbitrary variation, but a true expression of the mental act.[2]

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[2] See *Outline*, § 95.

Let us append that while either of the four forms of the proposition may be concluded in the first figure, it seems especially suited to establishing general propositions; the universal affirmative A can be proved only in this figure. In its two affirmative forms the predicates are always thought as greater wholes than the subjects. But sometimes a previous thought, a special purpose in view, may determine us to prefer to make the greater whole the subject, and this also will often throw the reasoning in the second or third figure rather than the first. The second figure, whose conclusion is always negative, seems especially adapted for proving differences in things, and clearing obscure thought. Hence its principle—that if one term is contained under and another excluded from a third, they exclude each other—is called the *dictum de diverso.*[3] The third figure, whose conclusion is always particular, seems specially adapted for bringing in examples, and thus proving an exception to some universal statement. Its principle is that two terms which contain a common part, partially agree; or if one contains a part which the other does not, they partially differ. This is called the *dictum de exemplo.* E. g.:

Tweed was not an honorable man;
Tweed possessed high intellectual culture;
∴ Some one at least of high culture was not honorable.

This conclusion is the contradictory of "All of high intellectual culture are honorable," and overthrows it. Hence the third figure is well suited to disprove A, and also E.

The middle term in the example is individual. Such a case can occur only in Fig. 3; for in either of the others the middle term is once at least a predicate, and an individual cannot become a predicate. This alone establishes, not merely the naturalness and propriety, but the necessity, of this figure. Moreover we remark that while the middle term is essentially, and hence always, the medium of comparison, it is only in affirmative syllogisms of the first figure that it is necessarily of intermediate extent. But some of the logicians referred to above, as Bain and Bowen, involve in their objections to the second and

[3] Says Whately (*Logic*, p. 101), "The arguments used in the process called *Abscissio infiniti* will, in general, be most easily referred to this figure. The phrase is applied to a series of arguments in which we go on excluding one by one certain suppositions or certain classes of things from that whose real nature we are seeking to ascertain."

third figures the notion that the middle term ought to be always of intermediate extent. This is mere confusion of thought as to what is meant by "middle," and their objections are unsound.[4]

§ 3. The four figures of the syllogism are subdivided into Moods, upon the ground of the quantity and quality of the premises. The conclusion need not be taken into account, since its quantity and quality are determined by the premises. The method for determining the moods is as follows:

Relative to quantity and quality, we recognize four propositions, A, E, I, O. These, as premises, taken two at a time, yield sixteen possible combinations, exhibited in the following scheme:

AA	Figs. 1, 3, 4.	EA	Figs. 1, 2, 3, 4.	IA	Figs. 3, 4.	OA	Fig. 3.
AE	" 2, 4.	[EE]	3d Gen. Rule.	[IE]	6th Gen. Rule.	[OE]	3d Gen. Rule.
AI	" 1, 3.	EI	Figs. 1, 2, 3, 4.	[II]	7th " "	[OI]	7th " "
AO	" 2.	[EO]	3d Gen. Rule.	[IO]	7th " "	[OO]	3d " "

But not all these combinations will yield conclusions, i. e., they do not represent the premises of valid syllogisms. Those bracketed are to be eliminated as violative of General Rules (i, § 5). Eight—one half—remain as valid, since they accord with the General Rules. In reference to IE, we may remark that its conclusion must be negative, by Rule 4; the predicate of this conclusion, the major term, is therefore distributed; but the major premise I has neither term distributed, which violates Rule 6, giving illicit major.

Let us now inquire in which of the four figures each of these eight valid combinations may occur. We apply the Special Rules (§ 1), and find that EA and EI accord with all these rules, and therefore can appear in each of the four figures, as indicated in the scheme. The figures in which the others can appear are similarly ascertained and indicated. Upon counting, we find there are nineteen valid Moods of the Syllogism.

§ 4. The first figure exhibits four moods, AA, EA, AI, EI. Let us now annex to each of these the symbol of the conclusion it necessitates, and coin a word containing these three vowels in their order, as the name of that mood, thus: Barbara, Celarent, Darii, Ferio.

[4] "Major terminus appellatur in secunda figura qui medio proprior, minor qui remotior est ab eo." (Waitz, 26 b 37.) With Aristotle the relations of the terms, not their arbitrary position, fixes the figure. Cf. Trendelenburg, *Elem.* § 28.

The moods of the other three figures are treated in the same way, and the names of the nineteen moods thus coined are arranged in the following MNEMONIC HEXAMETERS, which the learner should carefully commit to memory:

Fig. 1 = Barbara, Celarent, Darii, Ferio *que prioris;*
Fig. 2 = Cesare, Camestres, Festino, Baroco[1] *secundæ;*
Fig. 3 = *Tertia* Darapti, Disamis, Datisi, Felapton,
Bocardo,[2] Ferison *habet. Quarta insuper addit*
Fig. 4 = Bramantip, Camenes, Dimaris, Fesapo, Fresison.

[1] —or Fakofo. [2] —or Dokamok.

These names of the nineteen valid moods are exceedingly convenient. By applying its name to any reasoning, we at once indicate its figure, and the quantity and quality of each proposition, and also, as will be seen now directly, its relation to other moods to which it may be reduced, and the method of reduction. Moreover, they constitute a test; for, since these are all the valid moods, whenever we have a simple syllogistic form to which none of these names is applicable, we know at once that the reasoning is false.

It may be well to mention here that had we taken the conclusion into account in developing the valid moods, we should have found in Fig. 1 two others, viz., AAI and EAO; in Fig. 2 two others, viz., EAO and AEO; and in Fig. 4 one other, viz., AEO. These are valid, indeed, but superfluous; for it will be observed that the conclusion in each is particular, although the premises warrant a universal. They are called the "Subaltern moods," or "Moods of a weakened conclusion." It is not needful to take them into consideration.

In noting the conclusions, it will be seen that each of the four judgments is proved in Fig. 1. Its four moods, however, are obviously reducible to two, the third and fourth being unessential varieties of the first and second. Thus:

Barbara or *Darii.*
All M is P;
All or some S is M;
∴ All or some S is P.

Celarent or *Ferio.*
No M is P;
All or some S is M;
∴ No S is P,
or Some S is not P.

Here is one positive and one negative form. Since all the other moods may, as we shall find, be reduced to one or the other of these, they are the two fundamental forms of all reasoning.

Again, in noting the conclusions throughout it will be further seen that—

A	is proved in	1	figure	and in	1	mood,	whose initial letter is	B.
E	"	3	figures	"	4	moods,	" " "	C.
I	"	3	figures	"	6	moods,	" " "	D.[1]
O	"	4	figures	"	8	moods,	" " "	F.[2]

[1] Except Bramantip. [2] Except Baroco and Bocardo.

Hence, says Aristotle, the proposition A is the hardest to establish and the easiest to overthrow; and O is the easiest to establish and the hardest to overthrow. In general, universals are more easily overthrown; particulars more easily established.

§ 5. We are now to consider Reduction. It is usually stated as of two kinds. First, then, Ostensive Reduction. A syllogism in any mood, except the first four, may be ostensively reduced to one or the other of these. The initial consonant in each name is the same as that of the mood in Fig. 1 to which it reduces. Or, more generally, equivalent moods have the same initial letter. We must except Baroco and Bocardo, or, rather, consider them replaced by their alternates Fakofo and Dokamok. The reduction is accomplished by substituting for one or both of the premises an immediate inference from it. Other consonants in the name of a mood direct us in doing this.

s indicates that the proposition symbolized by the vowel that precedes it is to be converted *simply*.

p indicates that the preceding proposition is to be converted *per accidens*. (Except in Bramantip, where it shows that, after converting *simply*, a universal is warranted by the premises. This is just the reverse of *per accidens*, which reduces quantity.)

k indicates conversion by *contraposition*.

f indicates *infinitation*.

m indicates that the premises are to be transposed (*mutari*).

The consonants b, d, l, n, r, t, are not significant, but are inserted merely for the sake of euphony, or for metrical quantity.

An exceptive remark is needful here. If in a given syllogism the premise requiring conversion in order to reduction is an individual proposition, then the reduction is not practicable; for an individual proposition cannot be converted. This consideration makes clear, not merely the propriety of figures other than the first, but their necessity, since many of our reasonings involving individual propositions cannot be expressed in the first figure.

The following examples will sufficiently illustrate the process:

Fig. 2, *Camestres*,	reduces to	*Fig.* 1, *Celarent.*
All P is M;		No M is S;
No S is M;		All P is M;
∴ No S is P.		∴ No P is S.

Cam- Every wicked man is discont'd;
es- No happy man is discontented;
tres. ∴ No happy man is wicked.

=

Ce- No discontented man is happy;
la- Every wicked man is discont'd;
rent. ∴ No wicked man is happy.

Fig. 3, *Darapti*,	reduces to	*Fig.* 1, *Darii.*
Da- All wits are dreaded;		Da- All wits are dreaded;
rap- All wits are admired;	=	ri- Some who are admired are wits;
ti. ∴ Some who are adm'd are dreaded.		i. ∴ Some who are adm'd are dreaded.

Fig. 2, *Fakofo*,	reduces to	*Fig.* 1, *Ferio.*
Fak- All murders are intentional;		Fe- No unintent'l things are murders;
of- Some homicides are not intent'l;	=	ri- Some homicides are unintent'l;
o. ∴ Some homicides are not murders.		o. ∴ Some homicides are not murders.

The ostensive reduction now explained could not, it was believed, be applied to the two moods named Baroco and Bocardo. Hence the old logicians devised for them what they describe as a second kind of reduction, the *Reductio ad impossibile.* It is intended as a test of the validity of reasoning from granted premises in these two moods.

B, the initial letter, shows, not that the reasoning is reduced to Barbara, but that Barbara is used in making the test.

c indicates that the proposition preceding it is to be omitted, and the *contradictory* of the conclusion substituted. This gives premises in Barbara, from which a new conclusion is drawn. E. g.:

Fig. 2, *Baroco*,	is tested by	*Fig.* 1, *Barbara.*
Ba- All murders are intentional;	(1)	Bar- All murders are intentional; (4)
roc- Some homicides are not intent'l;	(2)	ba- All homicides are murders; (5)
o. ∴ Some homicides are not murders.	(3)	ra. ∴ All homicides are intent'l. (6)

Here the conclusion drawn in Barbara (6) is false, because it contradicts a granted premise (2). Hence a premise in Barbara is false. But one of these (4) having been granted (1), the false one must be the one substituted (5). Now this false proposition being the contradictory of the original conclusion (3), that conclusion must be true, and this reasoning in Baroco valid. So also the following:

Fig. 3, *Bocardo*,	is tested by	*Fig.* 1, *Barbara.*
Boc- Most men do not weep;	(2)	Bar- All who feel weep; (5)
ar- All men feel;	(1)	ba- All men feel; (4)
do. ∴ Many who feel do not weep.	(3)	ra. ∴ All men weep. (6)

This process seems to be sufficiently simple and obvious. But the mood Bocardo was famous in the schools, and, even more than Baroco, was the opprobrium of the scholastic system of reduction. Says Hamilton, "So intricate, in fact, was this mood considered that it was looked upon as a trap, into which if you once got, it was no easy matter to find an exit. Bocardo was, during the Middle Ages, the name given in Oxford to the Academical Jail, or Carcer, for refractory students,—a name which still remains as a relique of the ancient logical glory of that venerable seminary." Perhaps the perplexity arose somewhat from the process being considered and named as a kind of reduction. Many logicians of the present day continue to speak of it as "indirect reduction." But obviously it is not a reduction at all, and to call it so is mere confusion. It is, as already indicated, only an indirect test of the validity of the reasoning when it occurs in these moods. And it may be well to add that all the other moods can be tested by the same process. This is elaborately but uselessly exhibited by Schuyler.[5]

But the test, even in the case of Baroco and Bocardo, is of no practical value, and is superfluous. We have inherited it from the old logicians, who, as has been said, supposed that these two moods could not be ostensively reduced. In this they were mistaken. Mark Duncan, as early as 1612, and after him Noldius, in 1666, showed that by the use of contraposition Baroco could be reduced to Ferio, and Bocardo to Darii. Noldius proposed to call the former Facrono, and the latter Docamroc. Whately called attention to this method, but did not observe a defect in the name Facrono, and rendered the other defective by omitting the terminal letter. Hamilton recognizes that the reduction may be made, but blunders sadly in the attempt to reduce Baroco, which his editor admits.[6] We have proposed the names Fakofo and Dokamok as alternates, or as substitutes, and have already exhibited in an example the reduction of the former to Ferio. The admission of these substitutes would not affect the metre of the mnemonic lines, and we could then dismiss from technical Logic this operose, indirect, and practically useless test *per impossibile.*

The Latin mnemonic hexameters, it must be confessed, are a marvel of ingenuity. De Morgan calls them "magic lines, more full of meaning than any others that ever were written." Hamilton calls them "cabalistical verses," and says that "taking them on their own

[5] *Logic*, pp. 75–77. [6] *Logic*, pp. 313–14.

ground, there are few human inventions which display a higher ingenuity." They were, so far as relates to the first three figures, the invention of Petrus Hispanus, already referred to as the author of the propositional symbols. He was a native of Lisbon, became Pope John XXII in 1277, and died the same year. The corresponding Greek lines are much less ingenious, as the names of the moods in them mark only the order, the quantity, and the quality of the propositions, not indicating any method of reduction, and, indeed, not even the equivalent moods. They were the invention of Nicephorus Blemmidas, who was nominated Patriarch of Constantinople. Which had priority in this invention is uncertain. It is curious, however, to note that these two logicians attained the two highest places, the one in the Roman, the other in the Greek hierarchy; but as the one had hardly begun to reign when he was killed by the fall of his palace, so the other, declining the nomination, did not enter on the office at all. The several works of the Pope and the Patriarch were for many centuries the text-books on Logic, the one in the Latin, the other in the Greek schools.[7]

But it may very properly be asked, why should we have reduction? Reasoning certainly does not become more cogent by being reduced to the first figure; but, says Bowen, it becomes more elegant and perspicuous. That depends on whether a given case naturally belongs in the first figure. If so, then this is true. But if the case naturally belongs in some other figure, then its reduction to the first renders it more or less awkward and obscure. The answer more usually given assumes that the system of reduction is a method for testing the validity of reasonings. As the dicta of Aristotle are directly applicable only to the first figure, instead of inventing other dicta for the other figures, we reduce them to the first, and then apply the *dicta de omni et nullo*. Thus we become assured of the validity of our reasoning, and any fallacies in it, which might otherwise escape notice, become at once apparent. This answer is clear, but unsatisfactory. It views Logic as an art. If such be the object of reduction, it is not worth an hour's study; for in actual argumentation this test is never used by the initiated, and the uninitiated never err for lack of it. The mind practically grasps with more ease an argument in its familiar condensed modes of presentation, and sees in them more clearly and certainly its validity or invalidity, than when expressed in these pro-

[7] Hamilton's *Logic*, p. 308.

lix scholastic forms. The answer we would prefer to give is as follows: Logic is a science. The system of reduction serves the purpose of showing that all reasoning is governed by the same principle, that these processes of thought, whatever shapes they may take naturally and spontaneously, are in all cases fundamentally one and the same. We are thus enabled to comprehend in a single grasp movements of intellect which otherwise would seem multifarious and perplexed. We attain that clear unity which is the end of all science. Our practice is improved by such investigation, but its direct object is not skill, it is knowledge.

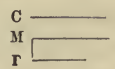
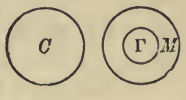
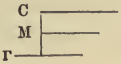
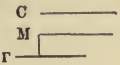
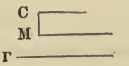
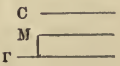
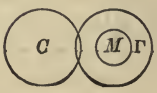
§ 6. In the mnemonic hexameters, moods of the same figure occur together. We present on the opposite page a scheme in which the equivalent moods are arranged together. Equivalent moods are those reducible to each other. Their names have the same initial letter. The three methods of notation are also exhibited. This scheme brings to light several important facts, among others the following:

Equivalent moods are not merely reducible to the same mood of Fig. 1, but are reducible to each other. That is, a syllogism in any mood may by reduction be expressed in any of its equivalents. This is evinced by the symbolic notation being similar for all equivalents. But a syllogism cannot be reduced to another mood not equivalent. Thus it appears that the variation by figure, as well as the order of premises, is in a sense unessential, accidental, external; whereas the variation by mood, which depends on the quantity and quality of the premises, is essential and internal. Hence it would seem to be logically more accurate to consider the syllogism as containing under it moods, the equivalents being the species, under which we find varieties in figure, and then we reach the individual moods which have received proper names. A subspecies also might be formed of those equivalent moods requiring only simple conversion in order to reduction. Such are absolutely equivalent, and appear in each of the four figures. They constitute groups in the scheme.

Moods which have the same initial letter, that is, equivalent moods, conclude the same formal judgment. Moods in B conclude A; those in C conclude E; those in D conclude I; those in F conclude O. The exceptions are Bramantip and Dokamok.

The linear and circular notation are symbolic. A different circular diagram may be made for each individual mood, the relative positions of the circles being varied so as in most cases to express the individual

Synopsis of Equivalent Moods.

Equiv. Moods.	Hᵉ· Graphic N.	Hᵉ· Linear N.	Euᵉ· Circular N.
Fig. 1. Barbara	C, —:M, —:Γ		
" 4. Bramantip	(Rejected)		
Fig. 1. Celarent	C: +:M, —:Γ		
" 2. Cesare	C: +:M, —:Γ		
" 2. Camestres	C: —,M: +:Γ		
" 4. Camenes	(Rejected)		
Fig. 1. Darii	C, —:M, —,Γ		
" 3. Disamis	C, —,M: —,Γ		
" 3. Datisi	C, —:M, —,Γ		
" 4. Dimaris	(Rejected)		
" 3. Dokamok (Bocardo)	C: +,M: —,Γ		Ditto
" 3. Darapti	C, —:M: —,Γ		
Fig. 1. Ferio	C: +:M, —,Γ		
" 2. Festino	C: +:M, —,Γ		
" 3. Ferison	C: +:M, —,Γ		
" 4. Fresison	(Rejected)		
" 2. Fakofo (Baroco)	C: —,M: +,Γ		
" 3. Felapton	C: +:M: —,Γ		
" 4. Fesapo	(Rejected)		

differences. This, however, presents no advantages. The linear notation, which is not thus variable, is on this account rather to be preferred. The graphic notation is not symbolic, but consists of arbitrary signs. It expresses all the accidental variations in external form, whereas the linear expresses only the internal, essential feature, i. e., the mood. The graphic, used in the scheme to express extension, may express also intension. In extension the copula points to the predicate, in intension to the subject; in general, the copula of the conclusion always points to the major term. In comparing the several notations, we must not forget, especially in case of moods containing *m*, that C and Γ are indifferent, and therefore interchangeable.

Arnauld, after detailing what Hamilton calls "the disgusting rules for reduction," pronounces them superfluous, and proposes to supersede them by one *General Rule for Reduction*, as follows: If the terms of the syllogism do not appear in the order required by the first figure, make them assume this order by any legitimate conversion, also transposing, if need be, the premises.

§ 7. We are now prepared to examine the Fourth Figure. Its legitimacy has been disputed by many logicians. Feeling it to be awkward, they reject it as an encumbrance, assigning various reasons. Hamilton hotly denounces it as "a monster undeserving of toleration, far less of countenance and favor."[8] He argues that it is unnatural and useless, because the premises are in intension while the conclusion is in extension, and that passing from one of these quantities to the other in the same syllogism is violative of the order of thought, and to no purpose. To this we object, first, that his assumption that the premises are in intension is grounded solely upon their order, which, we repeat, is arbitrary, and hence indicates nothing inherent in the reasoning. We object, secondly, that such alternations of quantity occur very frequently in the other figures, are often to good purpose, and in some cases seem essential (i, § 3). If so, we may grant they occur in Fig. 4, without furnishing a ground for rejecting it. Indeed, as has been said, these quantities cannot stand apart. Every logical judgment, every reasoning is in both at once, and their alternate predominance is not, in any important sense, a change of thought.

Other logicians have thought so well of Fig. 4 that it has withstood these attacks and taken deep root in the literature of Logic,

[8] *Logic*, p. 303.

so that every elementary treatise must give it place. Yet, truly, if it could be discarded without marring the symmetry of the science, without the loss of any essential doctrine or form, this would be a great stride towards simplicity. And it would seem not difficult to decide the question. The chief reason given for retaining it is that Figure requires this fourth variation to exhaust the possible forms; that Fig. 4 is essential to completeness, however rarely used or awkward. But this is true only if the order of the premises is essential. We have decided that the order is not essential, being merely conventional. It follows that the first three figures exhaust the forms; and that the fourth is the first with transposed premises, contrary to agreement, and hence ought not to appear.

The advocates of Fig. 4, however, point to its conclusion, which is not that which Fig. 1 should give, and claim that it implies an essential difference. The reply is not difficult. Let us consider the form

S is M;
M is P;
∴ P is S.

Here we readily see that the conclusion is not the one which the mind is naturally disposed to draw. It strongly inclines to conclude "S is P;" and in concluding "P is S," it is fully conscious of a revulsion. This it is that seems so awkward, and violative of that directness which should characterize the simple syllogism. The explanation is that the reasoner does mentally draw the conclusion "S is P," and so reasons in Fig. 1; and then immediately infers by conversion that "P is S." This is done tacitly, and almost unconsciously. But a slight reflection on the process leaves little doubt that the judgment "S is P" is mentally interpolated between the premises and the expressed conclusion. A concrete example will perhaps make this more clear.

Bram- All kings are men;
a- All men are mortals;
Direct— (a- ∴ All kings are mortals;)—tacit interpolation.
Indirect—ntip. ∴ Some mortals are kings—by conversion *per accidens*.

Camenes and Dimaris are entirely similar, the transposition of the premises and the simple conversion of the conclusion being all that is requisite to present them as Celarent and Darii. May we not rather say, they are Celarent and Darii (the order of the premises being non-essential) with the conclusion simply converted.

Fesapo and Fresison are each reduced to Ferio by converting both premises, leaving the conclusion intact. This reduction does not require the transposition of the premises. It is not probable, however, that the mind tacitly performs this double conversion when reasoning in these moods. It would rather seem that this process is similar to the above. Let us illustrate:

Fes- No ghosts are angels;
ap- All angels are spirits;
Direct— (e- ∴ No ghosts are spirits;)—tacit interpolation.
Indirect— o. ∴ Some spirits are not ghosts—by conversion *per accidens*.

This interpolated conclusion is an illicit process of the major term. But this the mind feels, and instantly restores the given quantity by converting *per accidens*. The case with Fresison is precisely the same. These two moods, then, are illegitimate.

We are therefore justified in concluding that the three legitimate moods of Fig. 4 are in reality those of Fig. 1 stated irregularly with transposed premises, and having an indirect conclusion which is an immediate inference from the actual and direct conclusion. The two illegitimate moods are, of course, to be condemned. Consequently Fig. 4, with all its moods, should be rejected from its usurped place in the logical system, and its legitimate forms should be classed with the irregular forms of the syllogism.

The fourth figure is not recognized by Aristotle, nor by any of his early followers. Averroës, in his *Commentary on the Organon*, attributes its introduction into Logic to Galen, who flourished a thousand years previously. But a critical examination of the extant logical writings of the physician discovers no trace of it. The Spanish Moor is therefore believed to have been mistaken. As it does not appear in any extant treatise of earlier date than the *Commentary*, its origin is altogether uncertain. We may confidently conclude, however, that it did not originate in ancient, but in early mediæval, times.

§ 8. In concluding this discussion of the simple Aristotelic syllogism, we will consider a charge that has been standing against it ever since the days of Sextus Empiricus, back to whom it may be traced. It alleges that the conclusion is already contained in one or both premises; that what is to be proved is therein assumed to be true; that the question is begged, and hence that by means of the syllogism we can make no real progress in knowledge. Thus it imputes uselessness and frivolity to the whole syllogistic theory, and pronounces its pre-

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tensions a sham. On this ground Stewart, Campbell, and a number of other thinkers have rejected Logic with some display of scorn. The charge is well and strongly stated by Mill thus:

"It is universally allowed that a syllogism is vicious if there be anything more in the conclusion than was assumed in the premises. But this is, in fact, to say that nothing ever was or can be proved by syllogism which was not known, or assumed to be known, before. It must be granted that in every syllogism, considered as an argument to prove the conclusion, there is a *petitio principii*. When we say,

All men are mortal;
Plato is a man;
∴ Plato is mortal,

it is unanswerably urged by the adversaries of the syllogistic theory that the proposition 'Plato is mortal' is presupposed in the more general assumption 'All men are mortal.' In short, no reasoning from generals to particulars can, as such, prove anything; since from a general principle we cannot infer any particulars but those which the principle itself assumes as known. This doctrine appears to me irrefragable."[9]

He says elsewhere, "From this difficulty there appears to be but one issue. Its refutation seems impossible on any theory which considers the syllogism as a process of inference." This only issue he expounds to be through his peculiar theory, which denies that the syllogism is an inference or proof, and views it as "the mere interpretation of the record of a previous process; the major premise as simply a formula for making particular inferences; and the conclusion as, not an inference from the formula, but an inference drawn according to the formula."[10] As we have not adopted Mill's theory of ratiocination, we need not state his reply to the objection which seems to him irrefragable. We therefore remain in the toils of this entanglement, and must make our exit, if possible, by other means.[11]

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[9] *Logic*, p. 139.

[10] *Examination of Hamilton*, vol. ii, p. 235.

[11] Mansel discusses the question ably in the Appendix to his edition of Aldrich, Note E. He directs his argument chiefly against Mill, showing in an *argumentum ad hominem* his inconsistency in these statements with his own logical principles. De Morgan examines the question briefly but skilfully under the head of "Fallacies of Petitio Principii," *Formal Logic*, pp. 257–59. Bain endorses and follows Mill's views,—*Logic*, p. 208 sq. See also Whately's chapter "On the Discovery of Truth," *Logic*, p. 262 sq.

Will Hamilton help us? In speaking of the usual order of the propositions in the formal syllogism, which he calls "the synthetic order," he says, "On this order the objection of *petitio principii* stands hitherto unrefuted, if not unrefutable, against Logic."[12] He entertains the odd fancy that the objection can be got rid of by merely writing the propositions in a different order, putting the conclusion first. This he calls "the analytic order," and insists that it is the true order in thought. This seems much like a solemn joke. Could he really think that the difficulty might be obviated by a juggling with an order of words? Truly, if a speaker starts with stating his conclusion, he cannot be said to have already admitted it in words. But has he not already thought it in a premise not yet expressed? Else how can his conclusion be a conclusion? Bowen, not seeing the joke, adopts and expands this reply of Hamilton as a serious and sufficient reply to the "unrefuted if not unrefutable" objection.[13]

We must help ourselves in this matter as well as we can. All logicians freely admit that there can be nothing in a valid conclusion that is not contained in the premises, i. e., in both premises, both taken together. The conclusion of a syllogism consists merely of a succinct and explicit statement of the relation of two notions, which relation is thought in their comparison in the premises through a third notion. It is universally allowed after Aristotle that a mediate reasoning is not three successive judgments as appears when written out to the eye, but that it is a single act of mind, a single judgment. Hence to admit that the premises contain the conclusion is pretty much the same thing as to admit that the conclusion contains itself. But to say that it is contained already, i. e., previously, in the premises is to mistake the nature of a reasoning. The premises as premises are logical, but not chronological, antecedents.

Now if the comparison be only apparently and not really mediate, if that which stands for the middle term is in fact identical with one of the extremes, it is evident that we have but two terms, and the conclusion is merely a repetition of this known relation. This is fallacy. This is to "beg the question." Herein is no progress. But if the medium be distinct and really that through which the relation of the other two notions is ascertained, then this is not to "beg the question," and there is progress.

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[12] Appendix to *Logic*, p. 623. See also *Discussions*, p. 604 (Am. ed.).

[13] *Logic*, p. 228 sq.

Let us remember that the premises and conclusion are correlatives, that neither can exist without the other. It is a very common case that a mind may be in full and familiar possession of two truths, but, never having thought them together, the consequence has never been thought, and is to this mind utterly unknown. It may have occurred as a question (*quæsitum*); but these two familiar propositions, which together necessitate it, not having been brought together, are not premises, and the *quæsitum* is not a conclusion. For example, everybody knows that young infants cannot talk, have no words, nor signs of thoughts that are not merely instinctive effects. Again, no one doubts that such infants really think. Yet many persons have never brought these truths together as premises of a conclusion. They may have questioned in their own minds the fact that can be inferred, but, being apart from these truths, it was a question merely. But bring the truths together thus:

Infants have no language;
But infants reason;

and is it not instantly seen that there is involved in this statement a new truth which we may explicate and state apart, thus:

∴ Some reasoning can be done without language.

Will any one say that nothing is proved here? Is there not a step forward in knowledge, an advance from the known to the unknown? Many persons, in view of this simple syllogism, would say, Why, of course, I might have known that, but I never thought of it. The two premises together contain the conclusion, but this is not to "beg the question;" they do not assume it, they produce it, a new truth distinct from either alone.

But it is said in one form of the indictment that the conclusion is contained in one of the premises alone, and that in stating it we merely repeat what is already said in this proposition apart, as in the example quoted above from Mill. The objection in this form has been greatly confirmed by the view that Arnauld takes of the syllogism. He says that every valid syllogism is governed exclusively by this principle: "One of the two antecedents must contain the conclusion, and the other show that it contains it."[14] This is very true, and a very ingenious and excellent view of the syllogism in the sense in which Arnauld intends the statement. But it is not, as is claimed, an

[14] *Port-Royal Logic*, pt. iii, chs. x, xi.

acknowledgment of *petitio principii*, nor can it be fairly construed to sustain the charge. The conclusion is contained in the premise in the same sense that any single notion is contained under one broader, its genus; but observe that the other premise is necessary to show this. I may have a good clear conception of a general rule, which on sufficient grounds I have accepted as universally true, but know nothing whatever of a multitude of cases to which it is applicable. We may say that the rule contains or includes these unknown cases, but I am not conscious of that until it is made to appear by bringing them in as minor premises, and then I progress in knowledge.

For illustration, when we say "All men are mortal," have we not already virtually said, by implication at least, "Plato is mortal." Not unless we have also said, or know, or thought that "Plato is a man," which is the minor premise. "Plato" may be a statue, or a book, or a town, or what not. I must first think that "Plato is a man" before, under this rule, I can say he is mortal. The bald truisms usual, for simplicity's sake, in logical examples lend countenance to the objection through the unwitting mental supply of the obvious minor premise. Yet a reasoning very similar to our *exemplum* was found needful by Paul and Barnabas at Lystra.[15] The people there knew very well the major premise, "No man should be worshipped." St. Paul supplied the needed minor, "We also are men, of like passions with you;" and the conclusion contained in these two premises was so obvious that it was left unexpressed. But let us take a case in which each premise is questionable:

No murderer hath eternal life;
All warriors are murderers;
∴ No warrior hath eternal life.

Here we have a major premise which some persons would deny, while admitting the minor; and many who would admit it would deny the minor. Hence, in the estimation of either class one of these premises may be affirmed without involving the affirmation of the conclusion.

Whately says that the object of reasoning is "merely to expand and unfold the assertions wrapped up, as it were, and implied in those with which we set out." Elsewhere he speaks of geometry as being all wrapped up in its definitions and axioms. I suppose this is tantamount to the statement that the conclusion is virtually contained in the premises. I do not object to Whately's metaphor, but say that

[15] Acts xiv.

knowledge thus wrapped up is merely virtual or potential, and to become actual knowledge its wrapping must be unwrapped. But is virtual or potential knowledge, knowledge at all? Is not real knowledge only that which is actual? Can it, indeed, be said to exist when not present in mind? Only in that very shadowy and questionable shape in which potential energy is said by the physicist to exist stored up in an inert, inactive mass. A keg of powder contains in it an explosion, i. e., potentially, but a spark is needed to realize it. So, if the major premise contains the conclusion, it certainly needs the minor to bring it about. A boulder on a mountain-top has stored up in it an immense quantity of potential energy. But it stays there very ineffectively until some minor starts it rolling down the steep, and this is necessary before we can have any experience of its force.

Very often, in our search after truth, a question clearly arises to establish which we have at hand the major premise, but, lacking the minor, we are utterly unable to reach a conclusion. Why is this if in affirming the major we have already affirmed the conclusion? Why not explicate it, and state it as established? For instance, an astronomer observes a new comet, and at once asks whether it will return again to our system. He knows full well that a celestial body moving in a hyperbolic orbit will not return; but from this major alone he can conclude nothing respecting the one in question. He laboriously and patiently sets to work to establish a minor. With minute pains he determines three or four points in the comet's orbit, and finally is enabled to affirm that its orbit is hyperbolic. Then, but not till then, the question resolves into the now established conclusion that this comet will not return. A large part of our thoughtful investigations is a search after, or rather an effort to establish, propositions to serve as minor premises under familiar general rules, in order to deduce thereby new truth.

Is it not progress in knowledge for one to deduce the consequences of new facts and laws obtained by observation and induction? Is not movement from the obscure and confused to the clear and distinct an advance, an addition? Is not a discovery of the true relations of our intuitive ideas and their systematic arrangement something gained, something new? All this is accomplished by deductive inference, and by it alone. The objection to the syllogism reaches too far to be sound. Were it so, then Euclid and Newton labored in vain.

Let us, finally, glance at the form of the argument that assails the syllogism. The eagle of the Libyan fable was slain by an arrow

feathered from its own wing. So the armory of the logician has been imagined to contain the fatal weapon of its own destruction. The empiricist has seized the syllogism, but, sheathing his own sword, he tries, like Giant Despair, to get his captive to commit suicide. Plainly he uses the syllogism to prove the syllogism useless. His argument is as follows:

Any reasoning that proceeds upon the assumption of its conclusion is *petitio principii;*
The Aristotelic syllogism, as is admitted by all logicians, proceeds thus;
∴ The Aristotelic syllogism is confessedly *petitio principii.*

Surely this is seething the kid in its mother's milk. But if it has proved its conclusion true, then this syllogism is itself a false reasoning, and therefore has not proved its conclusion true. A self-contradiction may well be dismissed. We remark, however, that, granting this reasoning to be sound, still the syllogism does not commit suicide, for *the minor premise is false.*[16]

At risk of being prolix, we must notice another phase of the objection which Mill confuses with the above. It is said, very ingeniously, that quite often the conclusion, so far from being deduced from the premise, is actually required to deduce the premise itself. Thus we do not know from "All men are mortal" that "Plato is mortal," but we must first know that "Plato is mortal" in order to know that it is really true that "All men are mortal." The objection here falsely assumes that to attain a universal proposition we must first know all the individual cases it includes. If this were true, then few, very few, universal propositions would be possible. But it is not true. We obtain a universal proposition, such as the one cited, not from an inspection of all cases, not by deduction, but by an induction from perhaps a single case, or, at most, from a very limited number. Once in possession of it, we proceed to bring other cases, hitherto unobserved, under it, and thereby draw new specific conclusions.

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[16] We should perhaps note that the usual vague and inaccurate sense of the phrase *petitio principii* has been accepted in this reply. Its proper meaning will be examined hereafter.

§ 9. **Praxis.** Supply the conclusion to each of the following pairs of premises. Prefix to each syllogism the name of its mood (§ 5). If not in Fig. 1, reduce it thereto. To Baroco and Bocardo apply also the test *per impossibile.* The regular order of the propositions is preserved throughout this section, the major premise, the one containing the predicate of the conclusion viewed in extension, being stated first.

1. Whoever possesses prudence possesses all virtue;
   Whoever possesses one virtue must possess prudence.—*Aristotle.*
2. Prudence has for its object the benefit of individuals;
   But prudence is a virtue.
3. No good action results in evil;
   Some alms-giving results in evil.
4. All abstract studies strengthen the intellect;
   Exercises that strengthen the intellect are profitable.
5. No science is capable of perfection;
   All science is worthy of culture.
6. No vicious conduct is praiseworthy;
   All heroic conduct is praiseworthy.
7. All pride is inconsistent with religion;
   Some pride is commended by the world.
8. No duty involves loss;
   To give freely is occasionally a duty.
9. All true philosophers account virtue a good in itself;
   The Epicureans do not account virtue a good in itself.—*Cicero.*
10. No one governed by passion is free;
    Sensualists are governed by passion.
11. All good reasoners are candid;
    Some infidels are not candid.
12. True poets are men of genius;
    Very unwise men have proved true poets.
13. No virtue is a natural quality;
    Every natural quality has God for its author.
14. Some kinds of anger are not unrighteous;
    Every kind of anger is a passion.
15. Some of our tax-laws are oppressive measures;
    All oppressive measures should be repealed.
16. No truth is worthless;
    Many truths are misapplied.

17. Some of the truths affecting human conduct are speculative;
All that affects human conduct is important.
18. No moral principles are animal impulses;
Nearly all animal impulses are principles of action.
19. All expedient acts are comformable to nature;
Nothing conformable to nature is hurtful to society.

Supply, in the following, any lacking proposition. Prefix the name of its mood. Write the linear and graphic notation of each.

20. All planetary bodies move in elliptic orbits;
Therefore the orbits of the asteroids are elliptic.
21. An inflated currency enables many persons to make rapid fortunes; hence, since this is promotive of national prosperity, one way to promote national prosperity is to inflate the currency.
22. He that is always anxious is never happy; but covetous men are always anxious.
23. Disgrace is never an infliction of nature; therefore natal deformity is no disgrace.
24. He that spareth the rod hateth his child; hence no loving parent spares the rod.
25. Since every partisan is prejudiced, and no prejudiced person can be a just judge, none of our reliable judges are partisans.
26. Whatever purifies the heart is a blessing;
But there are afflictions that purify the heart.
27. Sometimes very bad men attain high public honors; but bad men are always truly contemptible.
28. All men are liable to sorrow; hence some, at least, of those who are boasting of continuous prosperity may come to grief.
29. There are practically virtuous men who are necessitarians; it follows that while all necessitarians speculatively abolish the distinction between vice and virtue, some who do this are nevertheless, in practice, virtuous.
30. The connection of mind with matter is incomprehensible; but being most certain, there are things very credible which are beyond our comprehension.
31. Not every war is impolitic; but every one is ruinous; hence a ruinous procedure is, in some cases, good policy.
32. No virtue is contrary to the love of truth; but there is a love of peace which is opposed to a love of truth.

33. Nothing that must be repented of is desirable. Now many of our most intense enjoyments constrain repentance. Few of these, then, are truly desirable.
34. Prejudices are in no case compatible with perfection; yet some are innocent.
35. A fallacious argument is not a legitimate mode of persuasion;
A legitimate mode of persuasion sometimes fails to gain acquiescence;
∴ Not all those arguments are fallacious that fail.
36. Virtue is always attended by discretion; but there is a zeal without discretion.
37. No truth applicable to practice should be neglected; but any one may seem not to be practical; hence some seemingly unpractical truths should not be neglected.
38. None who have won enduring fame have ever lacked wisdom or industry;
Those failing in these requisites constitute the great majority of men;
∴ Few attain.

In the following miscellaneous reasonings the order of the propositions is still preserved, but the several propositions themselves are more or less irregular, and some are omitted. Bring the reasonings into syllogistic form, and prefix the name of its mood to each. If found defective, state what rule is violated.

39. Theft is a crime; yet some kinds were legal at Sparta.
40. Every virtue promotes general happiness; but exclusive self-culture does not; it has therefore no moral worth.
41. There is no growth without sunshine, and these flowers, being deprived of it, will not grow.
42. Who would offer a bribe would receive a bribe. Now, no one who would receive a bribe is fit for public office; hence no one fit for office buys votes.
43. Whatever is universally believed must be true. This may be said of the existence of God, which, therefore, must be a truth.
44. Some few men at least are truly honorable, yet all have imperfections; hence some are so who have imperfections.
45. The truly virtuous are the truly happy. The poor are often the one, and therefore the other.
46. No sin is excusable. Some faults are, and are therefore not sins.

47. Hard study strengthens the mind, but wearies the flesh; so that what wearies, strengthens.

48. Every candid man acknowledges merit in a rival;
Every learned man does not do so;
∴ Every learned man is not candid.

49. It is characteristic of theft to get, though not by gift, something for nothing; this gambling does, and thus is akin to theft.

50. A true evolution is caused wholly from within; but since very few beings, if any, have been exempt from adventitious causes, scarcely any, perhaps none, have been evolved.

51. Any disregard of moral order is wrong;
Every action disregards moral order whose moral quality is doubtful;
∴ Any action doubtful as to its moral quality is not doubtful as to its moral quality.

52. All do not strive, but all wish to succeed; hence not all strive who wish to succeed.

53. What is not in Scripture is not binding on conscience;
Since many ecclesiastical canons are not found therein, they may be disregarded.

54. Few men are entirely unworthy of respect;
Most men are unlearned;
∴ Some unlearned men are worthy of respect.

55. No one is rich who is not content;
No miser is content;
∴ No miser is rich.

56. Some Congressmen are evidently ignorant of political economy;
Such are unfit to legislate;
Hence some persons unfit for the position are sent to Congress.

57. Flesh and blood cannot inherit the kingdom of God;
Its heirs are human beings;
∴ Some of us shall not retain these vile bodies.

58. All imprudent acts are not vicious; all are, however, foolish; and so folly is not always vice.

59. No impenitent sinner can hope to escape the wrath of God, yet even the most hardened wish to escape;
Hence not all who desire it can hope for salvation.

60. Scarcely any of the ship's company could swim;
Yet of the numerous crew only a few perished;
∴ Some could not swim who nevertheless survived.

61. Some x is y; every y is not z; hence some x is not z.

62. Bacon was a notable statesman; and as he was a great philosopher, we infer that great philosophers are also statesmen.

63. Whatever is of practical use is worthy of attentive study;
Syllogistic reduction is of no practical use;
∴ Syllogistic reduction is not worthy of attention.

64. The ancient Greeks produced the greatest masterpieces of art;
The Lacedæmonians were ancient Greeks;
∴ They produced such masterpieces.

65. All prisoners are restrained from enjoying the common right of personal liberty;
But sailors on shipboard are not prisoners;
∴ They enjoy personal liberty.

66. Whatever causes intoxication should be prohibited;
The use of wine causes intoxication;
∴ The use of wine should be prohibited.

67. No sentient being is without a nervous system;
The sensitive mimosa is not sentient;
∴ The sensitive mimosa has no nervous system.

68. The man of strong will conquers his passions, and so does he that successfully resists temptation;
∴ Whoever does not yield to temptation possesses a powerful will.

69. All rational beings are accountable for their actions;
But many that suffer punishment are irrational;
∴ Many that suffer punishment are not accountable for their actions.

70. Suicide is simply one form of voluntary death; and voluntary death, in some form or other, has been embraced by many heroes and martyrs; so suicide is not always to be condemned.

III. QUANTITATIVES.

§ 1. The quantitative or mathematical judgment has already been considered at some length. It is now requisite to consider specially reasonings in the quantitative whole. Quantitative judgments are very common. We sometimes reason with them alone, and in other reasonings they intermingle with qualitative judgments. In neither case is such reasoning governed by the rules of the Aristotelic syllogism. The old Logic does not, I believe, recognize these judgments nor these reasonings; certainly not as distinct in kind, and governed by special laws. It would require all to be reduced to the Aristotelic syllogism, and brought in subjection to its rules. This is in most, perhaps in all, cases possible, but requires more or less violence. That is to say, the unity thus attained is not the result of analysis, showing that ultimately these kinds of reasoning are one in form and principle, but is attained by pressing the one into the mould of the other, and thus forcing it into an unnatural form. But the object of Logic being to exhibit the essential nature of thought in its original forms, it should recognize and treat these judgments or reasonings in the quantitative whole apart from others, and assign to them their special laws. Pure Mathematics proceeds almost exclusively in these quantitative forms; and the anatomical sciences, which are all essentially sciences of dissection and naming, deal, primarily at least, with quantities and sections, and not with qualities and kinds. Logic, as fundamental to all, should explain these processes, exhibiting the native manner of thought in all its forms.

When equivalence exists between two individuals, or between two aspects or thoughts of the same individual, the copula means "*is equal to*," or "*is the same as*," and may be expressed by the mathematical sign of equality. E. g., "The population of London is (=) double that of New York;" "The population of London is (=) one million." The principle governing reasoning with such propositions is the axiom "Things that are equal to the same thing are equal to each other." The first part of the Canon of Replacement (i, § 4), "Equivalent notions may replace each other," will be found to be more general in its application, and hence is preferable to the

axiom. The typical form of this syllogism of equivalence is the following:

$$A = B;$$
$$B = C;$$
$$\therefore A = C.$$

A concrete example in this form is as follows:

The density of the human body is the density of water;
The density of water is the density of air taken 817 times;
∴ The density of the human body is 817 times the density of air.

It will be observed that the middle term here is a standard of measure. And this gives occasion to remark the logical function of standards of measure of all sorts. They furnish the media through which we are enabled to compare quantities which cannot be immediately compared. The yard, the bushel, the pound, the atomic weight of hydrogen, the thermometer, barometer, electrometer, etc., supply us with middle terms through which to reason in our calculations.

The following is an example of the negative syllogism of equivalence, the only formal variation of which it is susceptible:

Selfishness is not the essence of virtue;
Duty is the essence of virtue;
∴ Duty is not selfishness.

We remark that all the terms in this particular example happen to be abstract. In general, then, abstract notions as well as concrete may be thought in the quantitative whole.

In the equivalent syllogism, the order of premises is obviously indifferent. The order of subject and predicate is also indifferent. That is, either term may be made the subject of thought, and the other the predicate, without other change. The distinction of major and minor terms, and consequently that of major and minor premises, does not exist, the terms being equivalent. The equivalent proposition is always and only simply convertible. The doctrine of Conversion, then, has no place relative to this syllogism. It follows, also, that Figure is of no moment, and is to be disregarded. Moods are reduced to two, the positive and the negative; for the quantification of every term is always total. Questions concerning distribution and non-distribution cannot, then, arise.

These eliminations render the logical treatment of this syllogism exceedingly simple. Perhaps from this simplicity it is, as well as from its clear intuitive exactness, that elementary mathematics is within the grasp of immature minds, even children often being able

to apprehend it quite thoroughly; whereas reasoning in the logical whole with the Aristotelic syllogism as the unit form requires more mental discipline and maturity. Hamilton impetuously declares "mathematics not a logical exercise."[1] It would perhaps be wiser to hold with Coleridge that "Mathematics is no substitute for Logic," and to consider mathematical studies as the proper discipline preparatory to logical studies.[2]

It will be well to observe that the distinction taken between logical and mathematical reasoning is not identical with the familiar distinction between moral reasoning and demonstration. Moral reasoning, better called dialectics, often occurs in the quantitative whole, and is then mathematical, yet it always involves more or less uncertainty. Demonstration is in many cases not quantitative or mathematical, but always carries with it certainty. The difference between these is that any dialectics involves to some extent empirical matter, and hence falls short of certainty; whereas demonstration is exclusively from intuitive principles, and carries their necessity along with it. This distinction, then, is not grounded on anything peculiar in the nature of the reasoning employed, which in all cases carries with it just the same approximation to certainty that belongs to the premises, but it is found in the nature of the premises themselves. According to its definition by Aristotle, demonstrative reasoning, producing scientific knowledge in the strictest sense, requires a conviction of the certainty of the propositions laid down.[3] His scholastic followers devised the following syllogism as a specimen of the "*Demonstratio potissima:*"

Omne animal rationale est risibile;
Omnis homo est animal rationale;
ergo, Omnis homo est risibilis.

Here is complete identity in the terms, and the reasoning may be readily construed in the mathematical whole; but its major premise is empirical, not intuitive, not *a priori*, and hence it falls short of demonstration. In moral reasoning we proceed from what is granted

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[1] See in *Discussions*, *Education*, Article 1st, "On the Study of Mathematics as an Exercise of Mind." See also an article in the *Athenæum* for Aug. 24th, 1850.

[2] The distinction drawn between mathematical and logical reasoning implies that the mathematical is not logical. The latter term, unfortunately, is used thus in a specific sense. In its general sense all reasoning is, of course, logical.

[3] *Anal. Post.* i, 2, 1. Aristotle treats of demonstration in the Posterior Analytics, especially in chs. i–xiii, drawing his illustrations from pure mathematics.

by the disputant; the *principia* must first be allowed. In demonstrative reasoning there is no concession; or rather there can be no disputant. Pure mathematics, which is strictly demonstrative, furnishes the clearest illustrations of quantitative reasonings.

§ 2. Let us, then, turn our attention to pure mathematics, and therein to synthetical geometry, to observe the application of quantitative reasoning, and to ascertain how truly and best to exhibit its logical process. We find that geometry makes some use of qualitative reasoning, as when it has proved of triangles in general, or of the genus, that the three angles are together equal to two right angles, it afterwards applies this truth to the several species of triangle—the equilateral, the isosceles, the scalene. We find, also, that it sometimes uses comparative syllogisms, but that by far the greater part of its mediate inferences are in equivalent syllogisms.

Geometry, which is the science of spatial magnitudes, supplies itself at the outset with a series of technical terms by means of definitions analyzing our complex notions of various magnitudes. It then lays down certain postulates concerning these. Thirdly, it states indiscriminately certain axioms. These are, however, of two kinds: 1st, Certain synthetical judgments, stating the self-evident properties of spatial magnitudes, such as "Two straight lines cannot enclose a space" (Ax. x); and, 2d, Certain analytical judgments, such as "Things equal to the same are equal to each other" (Ax. i). According to Kant, the first are geometrical axioms proper, and must be assumed as intuitively evident before any of the more complex relations can be known by demonstration. They constitute the ultimate premises from which the science proceeds, and are, therefore, its peculiar basis. Those of the second class express general conceptions of equality and inequality relative to magnitudes. They are derived from the Primary Laws of Thought as applied to quantity, and, corresponding to the Canon and general rules of the qualitative syllogism, govern, in a mode entirely similar, our inferences in the quantitative whole.[4]

It has, however, been usual for logicians to regard these analytical

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[4] Axiom 1st of Euclid (given above) is the Canon of mediate inference. Nos. 6 and 7 are merely modified statements of the same. The other analytic axioms, Nos. 2 and 3, 4, 5, which are deducible from it, are Canons of immediate inference, corresponding to "complex conceptions" (Part 3d, ii, § 5). E. g., As from "A horse is an animal," and "Whatever is young is strong," we immediately infer "A young horse is a strong animal," so under the axiom "The sums of equals are equal," we can immediately infer from $a=b$, and $c=d$, that $a+c=b+d$.

axioms, together with the synthetical, as ultimate premises in geometry, and, in exhibiting the logical analysis of a demonstration, to place one or the other as the major premise of nearly every syllogism. E. g.:

Magnitudes which are equal to the same are equal to each other;
Magnitudes equal to the adjacent exterior and interior angles of a triangle are equal to the same;
∴ They are equal to each other.

Magnitudes equal to the adjacent exterior and interior angles of a triangle are equal to each other;
The three interior angles and two right angles are equal to the adjacent exterior and interior angles;
∴ They are equal to each other.

All this is very true and formal, but very prolix and operose. Much in this way Mill exhibits the analysis of Euclid's Proposition v, bk. i;[5] and a similar analysis of the same proposition from certain old scholastic logicians may be found in Mansel's *Aldrich*.[6]

Now it is very possible to exhibit an analysis of arguments in the logical whole in the same manner, making one of the *dicta* of Aristotle the major premise of the syllogism; but both process and result would be cumbersome and artificial. It is far simpler, clearer, and more natural to treat geometrical reasonings as we treat qualitative reasonings. Let us take the analytic axioms as canons governing the form and authorizing the process, and develop the demonstration by a direct chain of quantitative syllogisms. If you ask me to justify my Canon, I do it, as I justify Aristotle's *dicta*, by deducing it from the Primary Laws. The above syllogisms would then reduce to the one following:

The interior angles of a triangle are equal to an adjacent exterior and interior angle;
But these are equal to two right angles;
∴ The interior angles are equal to two right angles.

The expression is rendered more facile by the use of a lettered figure, as is customary, whereby two or three letters take the place of a verbal description of a part. This method of exhibiting the logical analysis of a geometrical proof is not only far simpler, shorter, and more direct than the usual way, but it seems to me to correctly represent the actual mental process, which the other does not.

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[5] *Logic*, pp. 162, 163. [6] *Appendix*, Note L.

§ 3. "This simple reasoning," says Dr. Reid, "cannot be brought into any syllogism in mood and figure:

A is equal to B;
B is equal to C;
∴ A is equal to C."[7]

And hence this eminent philosopher rejected Logic. It is remarkable that Bain uses the following language: "Logicians are aware that the form 'A equals B, B equals C, therefore A equals C,' is not reducible to the syllogism. So with the relation of 'greater than' in the argument *a fortiori*. Yet to the ordinary mind these inferences are as natural, as forcible, and as prompt as the syllogistic inference."[8] He ought, then, to follow Dr. Reid, and give up Logic. Reid means to say that, taking the copula, according to approved logical rule, to be "*is*," and all that follows it to be the predicate, we have in this reasoning four terms: 1st, "A;" 2d, "equal to B;" 3d, "B;" 4th, "equal to C;" and this is unavoidable, so that this simple and unquestionably good inference is, according to the rules of your boasted Logic, the fallacy of *Quaternio terminorum!* Away with it!

The demand is to construe this quantitative reasoning as a qualitative syllogism subject to Aristotle's *Dictum de omni.*[9] A and B are presumed to be two different things. But how much of A is here thought? Only one mark, its *quantity*. And so of B. Hence the first premise becomes "The quantity of A is equal to the quantity of B;" "The cost of the museum is equal to the university debt;" i. e., these two quantities are equal. But the mere quantity of a thing is a pure abstraction, and the two quantities, taken apart from all other attributes, are, if absolutely equal, indistinguishable in thought, and therefore are to us the same. Hence the true interpretation of the thought, and its full and accurate expression is, "The quantity of A *is* the quantity of B;" "The amount of the cost of the museum *is* (*the same as*) the amount of the university debt;" $75,000 *is* $75,000, indistinguishably. A mere form of words cannot bind Logic, which postulates to interpret and express the thought. Now, with our proposition in this form, no difficulty remains; for we may transfer to the logical whole, taking the terms as coextensive, and yet think the subject as *contained under* the predicate. Our syllogism, then, is *Barbara*. But all this should not be required. The phrase "*is equal to*" is

[7] See Hamilton's *Reid*, p. 702. [8] *Logic*, p. 183.

[9] The treatment of this, and the cases discussed in the next section, by Mansel in his edition of Aldrich, Appendix, Note D, is quite unsatisfactory.

properly to be viewed as the copula interpreted. The same demand might be made to bring "A is contained under B," or "A is a kind or species of B," or "A has for one of its marks B," under the rule that the *is* is the copula, and what follows is the predicate. Then, upon the result, the demand might be repeated, and so *ad infinitum*.

So far of quantitative reasoning in the equivalent degree, misnamed the "positive" degree.

§ 4. Propositions in the comparative degree have for their copula "is greater than," or its correlative, "is less than," for which the mathematical sign of inequality may be substituted. The typical form of the syllogism is:

$$A > B;$$
$$B > C;$$
$$\therefore A > C.$$

Simply converting these propositions, we invert the meaning of the copula and read:

B *is less than* A;
C *is less than* B;
∴ C *is less than* A.

The planet Jupiter *is greater than* the earth;
The earth *is greater than* the moon;
∴ The planet Jupiter *is greater than* the moon.

The axiom governing this class of syllogisms may be stated thus: What is greater than a greater is greater still than the thing.[10]

What was said in § 1 respecting the elimination of Conversion, Figure, and Mood is to be applied also to syllogisms of comparison. We cannot, however, say as much for the simplicity of this reasoning. For, be it observed, the premises authorize more than the strictly logical conclusion states. This excess is usually expressed thus:

∴ By so much the more is A greater than C.

This sort of argument is called *a fortiori*, which may be understood to mean "by a stronger reason," and the conclusion expressed thus:

Therefore *a fortiori* A is greater than C.

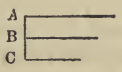
Such a conclusion can be reached only in the affirmative mood; so we may define the argument *a fortiori* to be a mathematical affirma-

[10] In pure mathematics this syllogism is used but rarely as compared with the syllogism of equivalence. We find, however, that Euclid demonstrates by aid of it Propositions vii, xvi, xvii, and others of his first book.

tive syllogism in which the premises contain less (or more) than the whole truth. Logicians sometimes distinguish between the reasoning *a minore ad majus*, and that *a majore ad minus;* but the distinction is superficial, since one is simply convertible into the other.[11]

Let us now examine analytically some miscellaneous examples. Our typical syllogism above may be analyzed thus:

A is as much as B (and more);
B is as much as C (and more);
∴ A is as much as C (and still more).



Here is a simple concrete example:

The tree is higher than the man;
The spire is higher than the tree;
∴ The spire is still higher than the man.

This may be re-dressed as follows:

The height of the tree *is greater than* the height of the man;
The height of the spire *is greater than* the height of the tree;
∴ The height of the spire *is still greater than* that of the man.

These propositions may be further analyzed, thus:

The height of the tree *is as much as* that of the man (and more), etc.

Very often we do not need the pleonastic conclusion; in which case the argument may be resolved thus:

The sea is broader than the lake;
The ocean is as broad as the sea (and more);
∴ The ocean is broader than the lake.

Here the second premise contains a surplus which is elided in thought. The syllogism may then be construed into *Barbara*, by taking for the middle term "what is as broad as the sea." It is evident that this treatment considers the judgments as compound, and views the reasoning as complex. Also, that both kinds of judgments of degree may occur in the same reasoning. Sometimes the judgments are triplex, as:

A includes B;
B includes C;
∴ A includes C.

[11] De Morgan gives a more elaborate analysis of this argument than others of our common authorities. See his *Formal Logic*, pp. 20–22.

The first premise says three things. It says that "A is greater than B," which is compounded of, 1st, "as much as," and, 2dly, "more;" also it says, 3dly, that "A partially coincides with," or "is the same as, B."

Not only do both kinds of judgments of degree occur in the same reasoning, but qualitative judgments also often combine with quantitative. For example,—

The sun is a star revolving about a remote celestial centre;
The sun is the centre of our system, controlling its secondaries;
∴ Our system revolves about a remote celestial centre.

The form is—

M is contained under P.....Qualitative.
M is the same as S.....Quantitative.
∴ S is contained under P.....Qualitative.

The Canon of Replacement is well suited to such cases. Nothing is more common in reasoning than to have the minor premise declare simply the equivalence of notions, one of which then replaces the other in the major premise to constitute the conclusion. The equivalence in such cases, however, amounts to identity, and should be read "is the same as."

We append a single example of reasoning from the mathematical whole to the part, as follows:

A minute *is a part of* a degree;
A degree *is a part of* a circle;
∴ A minute *is a part of* a circle.

§ 5. It is sufficiently manifest how readily, in a large number of cases, the quantitative syllogism may be converted into qualitative. There are, however, many cases when this cannot be done without great violence, and some perhaps wherein it is wholly impracticable. On the other hand, qualitative syllogisms may as often be readily transmuted into quantitative, sometimes by violence, sometimes not at all. The frequent practicability of this change may have been the origin of so many attempts of recent logicians, they not recognizing the fundamental distinction of these two wholes, to reduce all propositions to equations, proposing thereby to modify, or rather to supersede, the whole Aristotelic system. The best illustration of this perhaps is Hamilton's "Unfigured Syllogism,"[12] the Canon of which has already been given in i, § 4. He says that any syllogism whatever

[12] See Appendix to his *Logic*, p. 626; cf. *Discussions*, p. 604.

may be transmuted as in the following example, and find adequate expression in the unfigured form:

Darii, Fig. 1,	reduced to an	*Unfigured Syllogism.*
All patriots are brave;		All patriots and some brave men are equal;
Some who flee are patriots;	=	Some who flee and some patriots are equal;
∴ Some who flee are brave.		∴ Some who flee and some brave men are equal.

It will be observed that the change involves the quantified predicate. Hamilton says, "This form has been overlooked by logicians, while, in fact, it affords a key to the whole mystery of syllogism." Evidently it is only a forcing the qualitative reasoning into the quantitative mould, and making the expression needlessly awkward, in order to avoid even the mere appearance of figure. The innovation and the claim have been received with a just coldness by all except the most devoted followers of Hamilton.

§ 6. It is needful to observe, before closing, that there is another class of judgments, one which cannot be regarded as either qualitative or quantitative. These are *causal* judgments. Besides the two modes of thought we have discussed, there is that in which we think *events*, one as causing, bringing about, or determining another. With such judgments we syllogize, pursuing a train of causes and effects. The elementary form of this syllogism stands thus:

A causes B;
B causes C;
∴ A causes C.

This is not reducible without violence to any of the forms we have been considering, but logically it is quite similar to the quantitative syllogism. The copula is "*causes*," and, in converting, this is to be changed to the notion of effect. Obviously there is no more important reasoning in life or in science than that which follows the chain of cause and effect, fixing human responsibility, or explaining the facts of nature. But the logic involved does not seem to call for special discussion after what has been said of similar forms. It may be well to remark, however, that the copula is often absorbed in verb forms, as "A governs B," "A lifts B," "A excites B," etc. These, for simplicity's sake, may be allowed to stand in the place of the more formal copula, provided the causal relation is continuously maintained in the reasoning. Just that event, and no other, which was the effect of one must be the cause of the next, and so on in a chain throughout the series of propositions.

§ 7. **Praxis.** Name the class to which each of the following reasonings belongs. Supply any lacking proposition. Re-dress, if need be, analytically, and exhibit the copula. Explicate the several syllogisms that may be involved in one example. Construe two or three as qualitative:

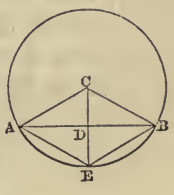
1. The favorite pupil of the Academy was Aristotle;
   Aristotle became the head of the rival Lyceum;
   ∴ Plato's favorite became his rival.
2. The author of *Athalie* was the greatest French dramatist;
   Racine was the author of *Athalie;*
   ∴ Racine was the greatest French dramatist.
3. The sting of death is sin;
   And the strength of sin is the law.—1 *Cor.* xv, 56.
4. John knew more than Peter;
   Peter knew more than Mark;
   ∴ John knew more than Mark.
5. Aristotle lived after Plato;
   Plato lived after Socrates;
   ∴ Aristotle lived after Socrates.
6. Virginia is one of the Southern States;
   The Southern States are a part of the Union;
   ∴ Virginia is a part of the Union.
7. All the vexations of this life, including the most petty, are not as numerous as its duties;
   Its duties are its delights;
   ∴ The vexations of life are less than its delights.
8. Lias lies above Red Sandstone;
   Red Sandstone lies above Coal;
   ∴ Lias lies above Coal.[13]
9. Wisdom is more precious than rubies;
   And rubies than gold;
   ∴ Wisdom is of yet higher value than gold.
10. A follows B;
    B follows C;
    ∴ A follows C.

[13] This example is given by Whately without remark. It has been a sore trouble to his successors. See Fowler's *Deductive Logic*, pp. 168–70, for what the head of Lincoln College, Oxford, thinks about it; and compare Dr. McCosh's summary treatment of it in his *Logic*, p. 144.

11. If God so clothe the grass of the field, - - - shall he not much more clothe you?—*Matt.* vi, 30.

12. The orbit of Venus is within that of the earth;
And this within that of Jupiter;
∴ The orbit of Venus is within that of Jupiter.

13. The radius perpendicular to a chord bisects the chord and the subtended arc. For in the right-angled triangles A D C and B D C, A C is equal to C B, since all radii are equal to each other, and D C is common; hence A D is equal to B D; for if two right-angled triangles have the hypothenuse and a side of the one equal to those of the other, the third sides are equal. (Prove also syllogistically the rest of the Proposition.)

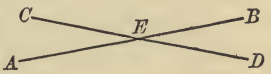


14. The dome is under the sky;
The altar is under the dome;
∴ The altar is under the sky.

15. Behold, the heaven and heaven of heavens cannot contain thee; how much less this house that I have builded.—1 *Kings* viii, 27.

16. To practise self-denial is to overcome temptation;
To overcome temptation is to conquer Satan;
∴ Self-denial is a mastery of Satan also.

17. If two straight lines cut each other, the vertical or opposite angles will be equal. For the angles C E A and A E D are together equal to two right angles, since the angles which one straight line makes with another upon one side of it are together equal to two right angles; and the angles A E D and D E B are together equal to two right angles for the same reason; therefore the two angles C E A and A E D are together equal to the two angles A E D and D E B. Take away the common angle A E D, and the remaining angle C E A is equal to the remaining angle D E B. In the same manner it can be demonstrated that the angles C E B and A E D are equal. Therefore if two straight lines, etc. Q. E. D.—*Euclid*, Prop. xv, bk. i.



18. Cocoanuts contain milk;
These barrels contain cocoanuts;
∴ These barrels contain milk.

19. Pilate's dictator was the servile mob;
    The multitudes cried with one voice, "Crucify him;"
    ∴ They who thus judged were the masters of the judge.
20. For if, when we were enemies, we were reconciled to God by the death of his Son; much more, being reconciled, we shall be saved by his life.—*Rom.* v, 10.
21. It were better to have no opinion of God at all than such an opinion as is unworthy of him; for the one is unbelief, the other is contumely; and certainly superstition is the reproach of the Deity.—*Bacon, Essay* xvii.

IV. COMPOUND AND DISGUISED FORMS.

§ 1. The reasonings thus far considered are simple. Under the present topic are to be examined a few varieties of compound or complex and disguised reasonings. The varieties are endless, and only some of the most important and illustrative can be here described. As preparatory to this, however, it is needful to give an account of certain irregularities which obtain in the ordinary statement of reasoning.

The deviation of propositions from strict logical form gives rise to a very common kind of irregularity. Simple propositions often take irregular forms; e. g., "It rains." Very common are inversions. Complex propositions are continually occurring in which there is a displacement of a clause. E. g., "In these sentences themselves the cases are exemplified which they state." The use of such propositions conceals or complicates the logical forms; but this may be more than compensated by the heightened rhetorical effect. A cause of still greater intricacy is the use of compound propositions. This we shall consider more fully in the sequel.

The order of the propositions being unessential, it is varied at will. E. g., "The fact that I defended him is proof that I held him innocent; for who would defend the guilty?" Here the major premise is implied by the question, and is stated after the conclusion. It is quite usual to state the conclusion first, followed by an illative, as *for*, *since*, *because*, *is proved by*, etc. E. g., "Not every passion is blameworthy; for anger is a passion, and there is a righteous anger."

Except in treatises on Logic, it is seldom that a formal syllogism occurs. In ordinary conversation, or even in avowed argumentation, its presence is apt to be an offence to the hearer or reader. He naturally expects to have some small share in the thinking; whereas the syllogism leaves him none, and charges him with a minimum of intelligence. The intelligent mind often, on the barest suggestion, catches a thought, and sweeps through a train of reasoning with marvellous rapidity and accuracy. Hence the more cultivated the hearer, the less need is there of elaborate statement. A hint, perhaps, is all that is required for cogent conviction; whence the old saw "A word

to the wise is sufficient." Besides, a logically formal statement would render the expression of almost any thought intolerably prolix. Brief expression is not only more pleasing and forcible, but often more clear. Unnecessary words do not elucidate, but obscure, thought. It is best, then, to use no more than are needful to convey the thought clearly and distinctly. For these reasons it is customary greatly to abbreviate expression. Essential propositions, such as are obvious, are elided; others are compounded or condensed in various ways, so that they rarely state the thoughts entire, nor, indeed, according to their actual order. The Enthymeme is the usual form of brief statement; and since reasonings so frequently appear in this guise, we will devote the rest of this prefatory section to its consideration.

It is customary, then, to abridge syllogisms; and since, in that case, some part of the reasoning is in the mind only, such statement is called an **Enthymeme** (ἐν θυμῷ), which is thus defined: An incomplete syllogism, one or two judgments being unexpressed.[1] We may, then, distinguish four orders of enthymemes, viz.:

1st. The major premise being unexpressed. E. g., Sirius is a fixed star; therefore it is self-luminous.

2d. The minor unexpressed. E. g., Prayers are often sinful; for whatsoever is not of faith is sin.

3d. The conclusion unexpressed. E. g., Enoch pleased God; but without faith it is impossible to please him (=whoever pleases God has faith).

4th. Only one proposition expressed. E. g., if we see on a tombstone "The memory of the just is blessed," the implied syllogism is sufficiently manifest. This form often occurs in the use of texts, proverbs, pithy sayings, and in witticisms. If some one, seeing me sorely vexed, should say, "The way of transgressors is hard," I am indignant, for the implied syllogism concludes me a transgressor, yet falsely, since it has an undistributed middle. Falstaff, when running from the battle-field, says, "The better part of valor is discretion," which also is a major premise. In the same scene he exclaims, in reply to Prince Hal, "Lord, Lord, how this world is given to lying!" —another major premise conveying what we call "an insinuation,"

[1] This, though an ancient view and generally accepted in Logic, is not the enthymeme of Aristotle. With him the enthymeme is a reasoning of a peculiar *matter*—from *likelihoods* and *signs*, συλλογισμὸς ἐξ εἰκότων ἢ σημείων. See *Anal. Prior.* ii, 27; *Rhet.* i, 2; also Hamilton's *Logic*, Lect. xx; *Discussions*, p. 153 sq. (Am. ed.); and Mansel's *Aldrich*, Appendix, note F.

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the implied conclusion.[2] The answer to a question is often indirect, i. e., a premise from which the doubtful proposition follows,—a very satisfactory mode of answer, since it furnishes also the ground of the opinion. E. g., "Is smuggling a crime?" Ans., "Whatever violates the rights of society is crime." Again, when the disciples of John asked our Lord, "Art thou he that should come?" he replied indirectly by giving them a minor premise, not, however, in words, but in acts. In that same hour he performed many miracles, and simply called their attention to them.[3] The message to Pilate from his wife furnishes an instance of a single word, "*just*," suggesting a major premise, while the conclusion is stated in the form of an exhortation: "Have thou nothing to do with that *just* man." The succeeding sentence conveyed a hint of arguments for the proof of each of the premises on which that conclusion rested.[4] A minor premise may stand alone. Paul closed his speech before Festus with, "I appeal unto Cæsar." The major to this minor is, "Every Roman citizen appealing unto Cæsar is entitled to certain immunities."[5] One of the propositions thus standing alone Aristotle calls an enthymematic sentence,[6] and quotes the following as an example: Ἀθάνατον ὄργην μὴ φύλαττε, θνητὸς ὤν. This may be rendered, "O mortal, cherish not immortal hate." But the participial phrase, more strictly rendered, is "Being mortal," and this constitutes a minor to the remainder, which is the conclusion. So it seems, in the common logical view, to be rather an enthymeme of the first kind.

The major premise is omitted more frequently than any of the other propositions, because it contains commonly a general rule, readily understood and fully admitted; whereas the minor premise is quite commonly a question of fact which needs to be stated and established in order to be subsumed. E. g., "A certain celestial body exhibits a proper motion among the stars, therefore it is a member of the solar system." The famous speech of Antony over the body of Cæsar consists of a series of enthymemes, the conclusions being only suggested.[7] This is high art before an audience whose favor is doubtful. When we permit the hearers to draw the conclusion, they then feel the argument to be somewhat their own, a feeling often more convincing than the logic.

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[2] *Hen. IV*, act v, sc. 4.
[3] Luke vii, 18–22.
[4] Matt. xxvii, 19.
[5] Acts xxv, 11.
[6] *Rhet.* ii, xxi, 6.
[7] *Julius Cæsar*, act iii, sc. 2.

§ 2. An **Epichirema,** or reason-rendering syllogism, is one that has attached to either premise, or to both, a supporting reason. That is to say, it is a syllogism having for a premise the conclusion of an enthymeme. This enthymeme may, of course, be expanded into a syllogism. A syllogism whose premise is the conclusion of another is called an "*episyllogism.*" One whose conclusion is the premise of another is called a "*prosyllogism.*" E. g.:

Episyllogism.		*Prosyllogism.*
Vice is odious;		Whatever enslaves is a vice;
Avarice is a vice; for it enslaves;	=	Avarice enslaves;
∴ Avarice is odious.		∴ Avarice is a vice.

The propriety of thus, in the progress of an argument, offering some reason or reasons in support of its doubtful propositions is apparent. By so doing we avoid the necessity of returning over the same ground; and by clearing doubts as we go along, we are not so likely to excite in the hearer the disgust that comes of suspense.

The oration of Cicero *pro Milone*, though not formally an epichirema, may be viewed as one on an extended scale, and an analysis of it stated thus:

It is lawful to slay one who lies in wait for us; for this is according to natural law; moreover, the laws of all other nations permit it; and, in addition, we have many precedents wherein our own law has justified it.

Clodius did lie in wait for Milo; for the known hostility of Clodius renders it probable; again, his equipment of deadly weapons indicates such a design; and, finally, the known murderous character of his attendants also evinces this purpose.

Therefore, It was lawful for Milo to slay Clodius.

It is more common for the whole effort of an advocate to be directed to the establishment of the minor premise, and long speeches have often no other object. This suggests that the arrangement of our criminal courts corresponds to, or rather presents the parts of, a syllogism. The judge expounds the law, which is the major premise in the case, and, being fully established, requires no proof. The prosecutor endeavors to prove the minor premise, that "The accused is guilty," which the jury decides. If "Not guilty," no conclusion follows. But if "Guilty," the minor is established. Now the judge,

in pronouncing sentence, formally draws the logical conclusion, which the sheriff practically executes. Thus:

The law says:—*Whoever is guilty of murder shall suffer death.* The judge expounds.
The prosecutor proves:—*The prisoner is guilty of murder.* The jury decides, Yea.
The judge sentences:—*Therefore he shall suffer death.* The sheriff executes.

§ 3. A **Sorites** (σωρός = a heap) is a chain of enthymemes, holding throughout the relation of prosyllogism and episyllogism. It is called by the Germans the chain syllogism (*Kettenschluss*). It can, of course, be expressed in either quantity, the intensive quantity being the common form. We give an illustrative scheme of the two forms.

SCHEME OF SORITES.

The progressive or Aristotelic form, in intension.

A — B
B — C
C — D
D — E
∴ A — E

The regressive or Goclenian form, in extension.

D — E
C — D
B — C
A — B
∴ A — E

Resolution.

A is B; a is c; a is d;
B is C; C is D; D is not E;
∴ a is c. ∴ a is d. ∴ A is not E.

Resolution.

D is not E; c is not e; b is not e;
C is D; B is C; A is B;
∴ c is not e. ∴ b is not e. ∴ A is not E.

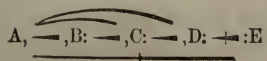
Example.

Some who are prosperous are avaricious;
The avaricious are intent on gain;
The intent on gain are discontented;
The discontented are not happy;
∴ Some who are prosperous are not happy.

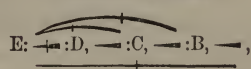
Example.

No discontented men are happy men;
All men intent on gain are discont'd men;
All avaricious men are men intent on gain;
Some prosperous men are avaricious men;
∴ Some prosperous men are not happy men.

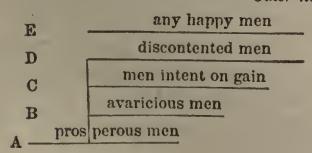
Notation in depth.

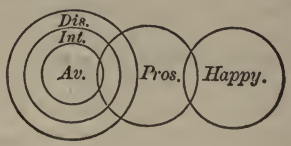


Notation in breadth.



Other notations in breadth.





The difference between these two forms is a question of order of premises merely, and therefore non-essential. We may agree, as in the case of the syllogism, to use the first form for intension and the second for extension; but it is an agreement, nothing more. Logicians have disputed which should be called progressive, and which regressive. It is merely a strife about words. Till Kant's time, the Aristotelic form was called regressive, and the Goclenian progressive. Kant reversed this. Afterwards, Jacobi restored it, followed by Krug and other German logicians. The influence of Hamilton, who follows Kant, has fixed in all recent English treatises the names as we give them. If we regard the Aristotelic form as expressive of intension, it ascends from fact to law, and might properly be called the ascending form. If we regard the Goclenian form as expressive of extension, it descends from law to fact, and might properly be called the descending form.

The following points should be particularly observed:

1st. The regular Sorites has as many middle terms, and hence resolves into as many syllogisms, as it has premises, less one.

2d. The first proposition is the only major premise that is expressed; all other premises are minors.

3d. Each unexpressed major is the conclusion of the preceding syllogism.

4th. Only one premise may be negative, and this must come last in intension and first in extension; else illicit process.

5th. Only one premise may be particular, and this must be the first in intension and the last in extension; else undistributed middle.

We also remark that in the scheme all the syllogisms are in Fig. 1. Sorites cannot occur in the other figures throughout. One step, however, may be in Fig. 2 or Fig. 3, but only one, and it must be either the first or the last. This, against Hamilton, is established by Mill.[8] But, in apparent contravention of this and others of the above rules, two sorites of different form may be connected into a continuous chain. The forms may be found in Schuyler's Logic.

The word "Sorites" was originally, and is still retained as, the name also of a special fallacy.

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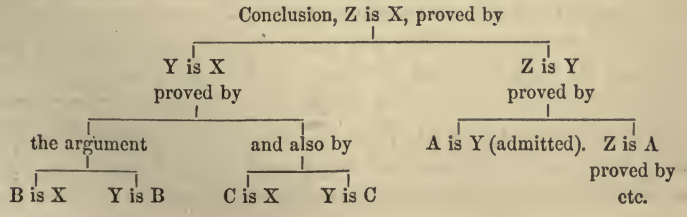
[8] See Hamilton's *Logic*, p. 619; and Mill's *Examination of Hamilton*, vol. ii, p. 226 sq. Mill, rather harshly, says: "If Sir W. Hamilton had found in any other writer such a misuse of logical language as he is here guilty of, he would have roundly accused him of total ignorance of logical writers."

§ 4. Our limited space will admit of only a few examples of the usual way in which actual arguments are abbreviated, cumulated, and compounded, and of the fact that all may be resolved into simple syllogisms. Such illustrations are needful, however, in order to confirm the preceding statements, and show with practical emphasis that the simple syllogism is truly the unit of elaborate reasonings.

The student of Logic should exercise himself in the reduction of select arguments to syllogistic statement. In most cases he will find this no easy task, a nice discrimination being requisite to discern and eliminate the merely rhetorical elements, and to bring out all the proof, much of which is often suggested rather than expressed. He is advised to begin by stating the ultimate conclusion, and then to seek for the premises on which it immediately rests. If a premise requires proof, regard it as a conclusion from prior premises, and search for them. Thus trace the reasoning backwards until the premises are reached with which the argument commences, not its statement, but its proof. For the conclusion is often stated first, and these primal premises last; they may occur in any order of statement.

"It will often happen that the same assertion will have been proved by many arguments, and then the inquiry into the truth of the premises will branch out accordingly. In mathematical or other demonstrative reasoning this will of course never take place, since absolute certainty admits of no increase; and if, as is often the case, the same truth admits of several different demonstrations, we select the simplest and clearest, and discard the rest. But in probable reasoning (wherein the premises are not absolutely certain) there is often a cumulation of arguments, each proving the same conclusion,—i. e., each proving it to be probable,—and from these we estimate the aggregate probability."

Whately, who makes these remarks, suggests that the student draw out the course of an argument in the form of a tree, or logical division, thus:



Our first example is drawn from Austin's *Province of Jurisprudence*. He states an argument of the intuitional school of moralists, which he combats, thus: "No opinion or sentiment which is a result of observation and induction is held or felt by all mankind. Observation and induction, as applied to the same subject, lead different men to different conclusions. But the judgments which are passed internally on the rectitude or pravity of actions, or the moral sentiments or feelings which actions excite, are precisely alike with all men. Consequently, our moral sentiments or feelings were not gotten by our inductions from the tendencies of the actions which excite them; nor were these sentiments or feelings gotten by inductions of others, and then impressed upon our minds by human authority and example. Consequently, our moral sentiments are instinctive, or are ultimate or inscrutable facts."

This argument consists substantially of a prosyllogism and an episyllogism, which, using for brevity the catch-words of the terms, may be stated thus:

Pro. Inductions are not held by all men alike;
Moral sentiments are held by all men alike;
∴ Moral sentiments are not inductions.

Epi. (All sentiments not inductive are instinctive;)
Moral sentiments are not inductive;
∴ Moral sentiments are instinctive.

The second example we take from the Epistle to the Romans viii, 28–30. "And we know that all things work together for good to them that love God, to them who are the called according to his purpose. For whom he did foreknow, he also did predestinate to be conformed to the image of his Son, that he might be the first-born among many brethren. Moreover, whom he did predestinate, them he also called; and whom he called, them he also justified; and whom he justified, them he also glorified."

This is evidently a polysyllogism, or sorites, and stated so nearly in strict logical form that redressing is needless. Another premise, quite obvious, is to be supplied at the close, thus: "And whom he glorified, they are they to whom all things work together for good." The strictly formal conclusion then would be: "Therefore, whom he did foreknow, they are they to whom all things work together for good." More freely and fully stated, it might read thus: "Therefore, whom he did foreknow, predestinate, and call according to his purpose, they,

loving God, are they to whom," etc. The apostle first affirms the conclusion and then details its proof.

The third example, which is rather a series of examples, we draw from Bacon, *Essay* xvi, "On Atheism:" "A little philosophy inclineth man's mind to atheism, but depth in philosophy bringeth men's minds about to religion; for while the mind of man looketh upon second causes scattered, it may sometimes rest in them, and go no further; but when it beholdeth the chain of them confederate and linked together, it must needs fly to Providence and Deity."

Here are two adversative enthymemes stated together. They easily explicate thus:

The mind, looking upon second causes scattered, may rest in them;
(The shallow philosopher does this;)
∴ The shallow philosopher may be atheistically inclined.

But The mind that comprehends the chain of causes linked must fly to Deity;
(The profound philosopher does this;)
∴ The profound philosopher must believe in God.

A little further on, Bacon says, "It appeareth in nothing more that atheism is rather in the lip than in the heart of man than by this, that atheists will ever be talking of that their opinion, as if they fainted in it themselves, and would be glad to be strengthened by the consent of others; nay, more, you shall have atheists strive to get disciples, as it fareth with other sects; and, which is most of all, you shall have them that will suffer for atheism, and not recant; whereas, if they did truly think that there were no such thing as God, why should they trouble themselves?"

Here the argument is cumulative, the same conclusion being supported by at least two sets of premises, thus:

(Those ever seeking to be strengthened by the consent of others do not believe heartily;)
Atheists are ever seeking this;
∴ Atheists do not believe at heart what is on their lips.

Moreover, (Those who at heart do not believe in God can have no occasion to trouble themselves to win disciples, or to suffer for opinion's sake;)
Atheists do find occasion thus to trouble themselves;
∴ Atheists do not believe heartily there is no God.

Again, further on, we find: "They that deny a God destroy a man's nobility; for certainly man is of kin to the beasts by his body;

and if he be not of kin to God by his spirit, he is a base and ignoble creature."

(What is of kin to beasts and not to God is base and ignoble;)
Man is of kin to beasts and not to God;
∴ Man is base and ignoble.

But (Who thus deny man's kinship with God make man ignoble;)
(They that deny a God must deny this kinship;)
∴ They destroy man's nobility.

"Atheism destroys likewise magnanimity and the raising human nature; for take an example of a dog, and mark what a generosity and courage he will put on when he finds himself maintained by a man, who to him is instead of a god, or *melior natura*,—which courage is manifestly such as that creature, without that confidence of a better nature than his own, could never attain. So man, when he resteth and assureth himself upon divine protection and favor, gathereth a force and faith which human nature in itself could not obtain; therefore, as atheism is in all respects hateful, so in this, that it depriveth human nature of the means to exalt itself above human frailty."

The above might perhaps be construed as in part an argument from analogy; but it would seem that the example of "a dog" serves rather to illustrate the wide universality of the major premise.

Any being, even a dog, maintained by *melior natura*, gathers a strength unattainable in its own;
(Man, maintained by divine protection and favor, has the confidence of a better nature than his own;)
∴ Man resting on Deity gathereth a force and faith otherwise beyond his reach.

Now by complex conceptions:

(Whatever deprives man of reliance on God deprives him of this means to exalt himself above human frailty;)
(Atheism deprives man of reliance on God;)
∴ Atheism deprives man of the means to exalt himself, and so destroys magnanimity and the raising human nature.

But, further, (Whatever deprives man of the means to exalt himself is hateful;)
Atheism does this;
∴ Atheism is hateful.

One of Whately's annotations on this Essay is as follows: "However imperfectly and indistinctly we may understand the attributes of God—of the Eternal Being who made and who governs all things—the 'mind of this universal frame,' the proof of the existence of a

Being possessed of them is most clear and full; being, in fact, the very same evidence on which we believe in the existence of one another. How do we know that men exist (that is, not merely beings having a certain visible bodily form—for that is not what we chiefly imply by the word 'man'—but rational agents, such as we call men)? Surely not by the immediate evidence of our senses (since mind is not an object of sight), but by observing the things performed, the manifest result of rational contrivance."

The prosyllogism is an equivalent syllogism.

Pro.—The proof that man exists is the argument founded on observed contrivance;
(The proof that God exists is the same argument;)
∴ The proof that God exists is the proof that man exists.

Epi.—(The proof that man exists is most clear and full;)
The proof that God exists is the proof that man exists;
∴ The proof that God exists is most clear and full.

For a fourth and final example, we take, from the Appendix to Whately's *Logic*, an analysis of the first part of Paley's *Evidences*, somewhat modifying and condensing the statement.

The ultimate conclusion is established thus:

A religion attested by credible miracles is from God;
The Christian religion is so attested;
∴ The Christian religion is from God.

Of these two premises, the minor was admitted, while the major was denied by unbelievers in ancient times; whereas at present the case is reversed. Paley's argument, therefore, goes to establish the minor, as follows:

All miracles attested by persons claiming to have witnessed them,
—who pass their lives in labors, dangers, and sufferings
—in support of their statements,
—and who submit to new rules of conduct,
—in consequence of their belief,
are worthy of credit;
Christian miracles are attested by such evidence;
∴ Christian miracles are worthy of credit: i. e.,
The Christian religion is attested by credible miracles.

The major of this syllogism, "That a story so attested is credible," is supported by two arguments: 1st, That it is *a priori* improbable that a false story would be thus attested, since no sufficient motive can be supposed. 2d, That it is *a posteriori* improbable, since no

other miraculous story whatever has ever been so attested; and hence, by subalternation, no false story of miracles ever has been so attested. The proof of this last proposition bifurcates; viz., concerning such stories as have been, or are likely to be, cited as parallels, it is proved either, "They are not so attested," or "They are not properly miraculous," being explicable, without questioning the veracity of the narrator, as hallucinations, etc.

The points of the minor premise of the syllogism are established by a series of arguments:

—That the early witnesses for Christianity suffered is proved:
1st. *A priori:* they were likely to suffer, since their doctrine was an offence, and regarded as foolishness.
2d. From profane testimony.
3d. From Christian testimony.
—That they suffered in support of their statements is proved:
1st. By that they had nothing else so to support except the claims of the new religion.
2d. By the testimony of historians, both Christian and profane.
—That they submitted to new rules of conduct, and
—That this was a consequence of their belief of their story, are similarly proved.
—That the miracles thus attested are what we call Christian miracles is proved:
1st. *A priori:* it is unlikely that the original story should have been lost and a new one taken its place.
2d. By incidental allusions of ancient writers, showing the stories of these witnesses and of our Scriptures to be the same.
3d. By the inherent credibility of our historical Scriptures.

This last is supported by a new series. It will be seen that much of the argument is cumulative, a number of reasons being cited to prove the same point. Also that in the latter part of the analysis each subsidiary argument can easily be turned into a syllogism by supplying an obvious major.

§ 5. Arguments are frequently stated in what at first glance appears to be a single simple syllogism, but which a slight inspection discovers to be compound, or at least to involve some essential deviation from rule. We will state and analyze a few representative examples.

When a conclusion is a compound proposition, it is evident that there must be at least one compound premise, and that the statement involves two or more syllogisms. E. g.:

The triumvirs were ambitious;
Cæsar, Pompey, and Crassus were triumvirs;
∴ Cæsar, Pompey, and Crassus were ambitious.

Here are obviously three syllogisms involved in one statement. If we substitute for the major term "friends," there are still three. But if we substitute "founded the empire," then there is but one, since the change makes all the propositions simple.

When the conclusion is simple, a compound premise involves a surplus of matter. E. g.:

Whatever revolves about the earth must present phases;
The moon alone revolves about the earth;
∴ The moon makes phases.

This compound minor premise resolves into "The moon revolves about the earth," from which the conclusion follows, and "What is not the moon does not revolve about the earth," from which no conclusion is competent, since it would give illicit major. Hence in this syllogism more is contained in the premises than can be collected in the conclusion.

But a compound exponible premise in other cases may yield a compound conclusion, which then collects all that was given. E. g.:

Justification comes by faith alone;
Our highest hope is justification;
∴ Our highest hope comes by faith alone.

This may evidently be resolved into two simple syllogisms in Barbara and Celarent. But this is not requisite; for we may treat the compound propositions in such case as if simple.

An exceptive subject has the effect to distribute, or rather to totalize, the predicate of the proposition, for one of its elements is negative. The following reasoning, therefore, though it may be construed as AAA, Fig. 3, is sound:

Only they who fail are scorned.....=**afa**
Only they who fail suffer.....=**afa**
∴ Only sufferers are scorned.....=**afa**

This conclusion is sound on condition that the middle is distributed in both premises. In the example, that it is so is sufficiently plain; but in common speech it is usual to make the distribution more fully

appear by a sort of reduplication, thus: "They, and only they, who fail are scorned."

In the following example also all three propositions are compound exponibles, but its solution is more intricate.

Except the evil-minded, all are truly happy;
But none are truly happy save the content alone;
∴ There are none, except the content, but those who are evil-minded.

Redressing this, we have as follows:

All but the evil-minded are truly happy;
None but the content are truly happy;
∴ None but the content are any but the evil-minded.

This is in form Camestres, but the matter is compound throughout, the conclusion doubly so. The whole explicates into four syllogisms, yielding the following conclusions:

1. The non-content are not non-evil-minded.
2. The non-content are evil-minded.
3. The content are non-evil-minded.
4. The content are not evil-minded.

When we explicate the syllogisms giving the two affirmative conclusions, we find an undistributed middle. Nevertheless, the compound conclusion from which they are evolved is competent, because the effect of an exceptive subject is to totalize the predicate. In this case, therefore, all contained in the premises is collected in the conclusion. Such an intricate form and analysis are, however, quite needless. Any one of the simple elementary syllogisms will be sufficient; for we may from its conclusion immediately infer, by infinitation, the other three.

§ 6. There is a class of disguised syllogisms which, from the various and unsatisfactory treatment these have received, seems to have been the bane of writers on logic. The premises are irregularly stated. They consist of simple propositions indeed, but require, in order to bring them under the common logical rules, the substitution of equipollent propositions, or else of one or more subsidiary inferences. In some cases the resolution is obvious; in others difficult. We cannot do better than to examine a few characteristic examples.

The following from Arnauld is pronounced by Jevons to be impracticable:[9]

The sun is a thing insensible;
The Persians worship the sun;
∴ The Persians worship a thing insensible.

Here are five terms; yet the reasoning is obviously very good. The Canon of Replacement is directly applicable, the conclusion being obtained by replacing, in the minor premise, "the sun" by its undistributed genus, "a thing insensible," as declared in the major premise. But even under the common logical rules the resolution is very simple. From the major premise we may immediately infer, by complex conceptions, "They who worship the sun worship a thing insensible," and we then have a perfectly regular Barbara.

The following would hardly puzzle a tyro:

Whoever probes a wound is on the verge of crime;
A wound is probed by the healer;
∴ The healer is on the verge of crime.

For the passive minor, substitute the active form immediately inferred, "The healer probes a wound," and we have again Barbara.

An example involving an immediate inference in opposition is as follows:

That riches are often a bitter curse is true;
And yet it is also true that most men desire riches;
∴ It is false to say that no men desire what is often a bitter curse.

The syllogism which is slightly disguised in this is the following Darii:

They who desire riches desire what is often a bitter curse;
Most men desire riches;
∴ Most men desire what is often a bitter curse.

This major premise is immediately inferred by complex conceptions; the conclusion, by opposition; for if E is false, then its contradictory, I, is true.

Finally, we recall the example formerly cited (i, § 4) as directly solved by the Canon of Replacement. Aldrich (p. 99) pronounces it a false syllogism on the ground that it has five terms, and therefore must be invalid. He is wrong; the reasoning is evidently very good.

The divine law commands us to honor kings;
Louis XIV is a king;
∴ The divine law commands us to honor Louis XIV.

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[9] *Lessons in Logic*, p. 158.

It is sufficiently evident that the middle term here is "king." This, then, is the true subject of the major premise, which, being redressed in a form that may be accepted as equipollent, gives:

All kings are of those whom the divine law commands us to honor;
Louis XIV is a king;
∴ Louis XIV is one whom the divine law commands us to honor.

The conclusion of this Barbara, again, is merely a similarly equipollent form for "The divine law commands us to honor Louis XIV."

The following treatment will to some readers be more satisfactory:

Louis XIV is a king;
—*by transference to the quantitative whole and inverting, we get:*
This king is Louis XIV (i. e., the one we are thinking of is Louis XIV).
—*by complex conceptions, we get:*
Whatever commands us to honor this king commands us to honor Louis XIV;
But (Whatever commands us to honor all kings commands us to honor this king;)
∴ Whatever commands us to honor all kings commands us to honor Louis XIV.
Now, The divine law commands us to honor all kings;
∴ The divine law commands us to honor Louis XIV.

§ 7. Logicians have distinguished, described, and named certain modes of arguing, some account of which may be fairly included under the present topic.

The *argumentum ad rem* is the direct proof of the main point in question.

The *reductio ad absurdum* indirectly proves an assertion by proving the absurdity of its contradictory. It is much used in geometry. It is sometimes called *argumentum per impossibile.* The refutation of an assertion may also be accomplished by an inverse treatment,[10] by proving its contradictory true. In discussions we sometimes hear the remark, "Your argument proves too much." If an absurd consequence be shown, then either its reasoning is illogical or a premise is false. The argument from effects is very similar. In a question of mere expediency—as, for example, the passage of a law for the suppression of intemperance—we might argue from effects, and, showing that they are likely to be evil, and that they had actually resulted in evil in analogous or entirely similar cases, we might thus prove the inexpediency of such a measure. Questions of duty should always, if possible, be determined *a priori*, without regard to consequences; but in

[10] See Part 3d, ii, § 8.

some cases duty can be determined only by considering the consequences of the contemplated line of conduct.

The *argumentum ad populum* is an appeal to such principles as are cherished by the people. This indirectly supports the point, and is legitimate if the principles are sound. But when the appeal is to passion or prejudice, it is a sign that the speaker himself lacks confidence in his other arguments.

The *argumentum ad judicium* is an appeal to the common judgments of mankind. We hear it often in conversation, in the phrases "Everybody says," "No one thinks," etc. The argument may possess great force. It is one of the strong supports of the Scottish doctrine of Natural Realism, hence called the philosophy of common-sense. Aristotle says, "What seems true to all, that we believe to be, and nothing is more worthy of credit."

The *argumentum ad verecundiam* is an appeal to authority, to some venerable institution, as an established religion, to antiquity, etc. E. g., with the scholastics it was a standing major premise, "*Stultum est dicere Aristotelem errare.*"

The *argumentum ad hominem* is arguing on the ground of an opponent. It is also called *argumentum ex concesso*. As all disputation must proceed *ex concessis*, we may accept an opponent's principles on which to base a counter-argument, though, perhaps, we may believe his principles false, our argument being directed against him personally, *ad hominem*. Even if we believe his principles sound, they may not be such as we would use in arguing with another, or with mankind generally. The conclusion we establish is frequently not the absolute and general one in question, but one merely relative and particular. We may no more than convict our opponent of inconsistency, ignorance, bad faith, or illogical reasoning. We then can claim a victory, but not possession of the territory. Such a course is often necessary in order to silence those who will not yield to fair general argument, or to convince those whose weakness and prejudices will not allow them to assign it due weight. Our Lord often used this method against the Jews. See, for example, Matt. xxii, 41–45.

The *argumentum a fortiori* has already been considered in iii, § 4. Its full form is: If A is greater than B, and B greater than C, still greater is A than C. This is essentially mathematical or quantitative. It may be described in general as the argument in which, from an admitted and less probable proposition, one depending on it, and more probable, follows *a fortiori*.

§ 8. **Praxis.** State of each of the following examples whether it is a simple enthymeme, or an epichirema, or a sorites. Write out the syllogisms implied in full logical form. In case of an epichirema, distinguish the pro- and epi-syllogism.

1. Blessed are the merciful; for they shall obtain mercy.
2. Cunning cannot be a virtue; for no virtue degrades.
3. It is I; be not afraid.
4. *Cogito, ergo sum.*—See Hamilton's *Metaphysics*, p. 258.
5. Every man should be moderate; for excess will cause disease.
6. Kings, having no equals, have no friends.
7. Suppose ye that these Galileans were sinners above all the Galileans, because they suffered such things? I tell you nay.
8. Will, since it often combats desire, as also it often yields to it, is not desire.
9. The flesh of ruminants is good for food, and these animals, since they have horns and cloven hoofs, belong to that class.
10. Man, inasmuch as he is naturally selfish, and is, moreover, liable to desires and passions which have no limits or power of restraint in themselves, needs the restraints of law.
11. Occasional turbulence, being the less of two evils, is preferable to rigid despotism.
12. What if a rule never is, and a principle always is, a law admitting no exception?
13. A wise man is never surprised, because he is never disappointed; and this is because he forms no expectations that are not placed upon the most certain basis.
14. Suppose a man to say, "I dislike all foreigners;" find a premise which, with this saying, would authorize the further assertion, "No foreigner ought to be liked."
15. Whatever tends to withdraw the mind from pursuits of a low nature deserves to be promoted. This classical learning does, since it cultivates a taste for intellectual enjoyments.
16. The Scripture narratives are trustworthy, because the writers had the means of knowing the facts; also, they evidently were sincere and candid; and, besides, the narratives are consistent.
17. All true patriots are friends to religion, religion being the basis of national prosperity; but, since their lives are not in accordance with its precepts, it follows that some great statesmen are not friends to religion.

18. Lithium is an element; for it produces an alkali, therefore is a metal, and hence an element.
19. I will not do this act, because it is unjust; I know that it is unjust, because my conscience tells me so; and my conscience tells me so because the act is wrong.
20. When the observance of the first day of the week as a religious festival in commemoration of Christ's resurrection was introduced, it must have attracted much attention; for it was a striking innovation. In this case, since attention would naturally lead to inquiry respecting the truth of the resurrection, the story would surely have been exposed as an imposture had it been one.

Put the following logical climax in the Goclenian form, and write the circular, linear, and graphic notation:

21. The prudent are temperate;
The temperate are constant;
The constant are unperturbed;
The unperturbed are without sorrow;
Those without sorrow are happy;
∴ The prudent are happy.—*Seneca, Epist.* 85.

Put the following in its opposite form, and write the notations:

22. Nothing which is indissoluble is mortal;
What has no composition of parts is indissoluble;
A spirit has no composition of parts;
A thinking substance is a spirit;
The mind is a thinking substance;
∴ The mind is not mortal.—*Plato.*

State each of the following as a regular sorites in either form:

23. We must increase the income-tax; for war has become a necessity, and we cannot go to war without money, which can be raised only by taxation. But the only tax which the resources of the country can bear is the income-tax, since it will fall on the richer part of the population.
24. A demagogue must hold the people in contempt; for, being a favorite with them, he must know how to manage them; therefore he understands their weaknesses, and his contempt must follow.

25. Riches are for spending, and spending for honor and good actions; therefore extraordinary expense must be limited by the worth of the occasion.—*Bacon, Essay* xxviii.

26. That defalcation is fraud, and therefore a crime, no one will deny, and neither this nor any other crime should go unpunished. But no one who acts with good intent should be punished. Now, all generous conduct is of this character, and it is generous to credit freely. But many failures in business are the consequence of free credit; so that not every one who fails is a defaulter.

Analyze the following arguments, stating the results either as simple syllogisms or as sorites:

27. No agent more effectually imitates the natural action of the nerves in exciting the contractility of the muscles than electricity transmitted along their trunks; and it has hence been supposed by some philosophers that electricity is the real agent by which the nerves act on the muscles. But there are many objections to such a view; and this very important one among the rest, that electricity may be transmitted along a nervous trunk which has been compressed by a string tied tightly round it, while the passage of ordinary nervous power is as completely checked by this process as if the nerve had been divided.—*Carpenter's Physiology.*

28. We are not inclined to attach much practical value to that analysis of the inductive method which Bacon has given us in the second book of the *Novum Organum.* It is, indeed, an elaborate and correct analysis. But it is an analysis of that which we are all doing from morning till night, and which we continue to do even in our dreams.—*Macaulay.*

29. Our intellectual part being common, the reason, also, in respect of which we are rational beings, is common. This being so, common also is the reason which commands us what to do, and what not to do; this being so, there is a common law also; hence we are all fellow-citizens; and hence members of the same political community; and therefore the world is in a manner a state.—*Marcus Antoninus.*

30. The general object which all laws have, or ought to have, in common is to augment the total happiness of the community; and, therefore, to exclude, as far as may be, everything that tends to

subtract from that happiness; in other words, to exclude mischief. But all punishment is mischief; all punishment is in itself an evil. Upon the principle of utility, if punishment ought at all to be admitted, it ought only to be admitted in so far as it promises to exclude some greater evil.—*Jeremy Bentham.*

31. Because the greatest part of men are such as prefer their own private good before all things, even that good which is sensual before whatsoever is most divine; and for that the labor of doing good, together with the pleasure arising from the contrary, doth make men for the most part slower to the one and prone to the other than that duty prescribed them by law can prevail sufficiently with them; therefore unto laws that men do make for the benefit of men it hath seemed always needful to add rewards, which may more allure unto good than any hardness deterreth from it, and punishments, which may more deter from evil than any sweetness thereto alloreth.—*Hooker, Eccl. Pol.*, bk. I, x, 6.
32. How did the barbarians reason in Acts xxviii, 3–6?
33. Prove syllogistically that O cannot be a premise in Fig. 1; that it cannot be the major in Fig. 2, nor the minor in Fig. 3. Also prove that in Fig. 2 the conclusion must be negative. Also that in Fig. 3 the conclusion must be particular.

Write out the syllogisms involved in the following irregular and compound forms, supplying any inference that may be lacking:

34. The French once more are endeavoring to establish a republic.
A republic is a representative government;
∴ The French once more are endeavoring to establish a representative government.
35. The value of money is merely a purchasing power;
Interest on money is only a reward of abstinence;
∴ Interest on money is not the value of money.
36. Now no chastening for the present seemeth to be joyous, but grievous; nevertheless afterwards it yieldeth the peaceable fruit of righteousness unto them which are exercised thereby.—*Heb.* xii, 11.
37. I give nothing, solely because I have nothing to give.
38. None are happy but the virtuous;
There are many rich men who are not virtuous;
∴ There are rich men who are not happy.

39. Whoever says, I love God, and hateth his brother, is a liar; for he that loveth not his brother whom he hath seen, how shall he love God whom he hath not seen?

40. They are out of the reach of their enemies who cannot be robbed of what they love;
He cannot be robbed of what he loves who loves God alone;
∴ They who love God alone are out of the reach of their enemies.

41. Every good pastor is ready to give his life for his sheep;
Now pastors in the present day who are ready to give their lives for their sheep are rare;
∴ There are in the present day scarcely any good pastors.

42. The Gospel promises salvation to the faithful;
Many whom the world condemns are faithful;
∴ The Gospel promises salvation to many whom the world condemns.

43. Every one desires happiness; but virtue (alone) is happiness; hence every one desires virtue.—*Arist. Eth.*, bk. iii.

44. Christianity obligates servants to obey their masters in those things only which are not contrary to the law of God;
But unlawful traffic is contrary to the law of God;
Therefore it does not obligate them to serve in an unlawful business, but forbids them so to do.

45. Only they who are not conscious of guilt are not subject to fear; thence it is that conscious hypocrites are always shy and timid, while the innocent are unsuspecting and self-possessed.

46. Gladstone, Disraeli, and Lord Derby are eminent statesmen;
But they are also eminent authors;
∴ In some cases literary success is not inconsistent with statesmanship.

47. (The commandment to sacrifice is greater than all others save one;)
To love is more than sacrifice;
∴ To love God is the greatest commandment.—See *Mark* xii, 28–34.

48. No man is to be punished for the crime of another (*Nemo punitur pro alieno delicto*);—*Legal maxim.*
Nearly all of our miseries are entailed on us by the crimes of others;
∴ Few, if any, of our miseries are punishments.

49. A true philosopher places his chief happiness in moral and intellectual excellence;
But there is no excellence without activity;
∴ A true philosopher places his chief happiness in moral and intellectual activity.

50. Put Cicero's episyllogism (§ 2) in form, and name the mood.

What names may be given to the following reasonings?

51. From a given point in a line only one perpendicular can be drawn. For if a second could be drawn, the angle which it would make with the given line would be a right angle by definition, and hence equal to that formed on the same side by the first perpendicular; for all right angles are equal. But one of these angles would be a part of the other, hence a part would be equal to the whole, which is impossible.
52. Those used by Demetrius in Acts xix, 23–27; and by the town-clerk in vers. 35–41.
53. Those used by our Lord in Luke xiii, 15–16; and in John x, 34–36.
54. Those used by Paul in Rom. v, 7–10.
55. That used by Eliphaz in Job iv, 17–19.

V. CONDITIONALS.

§ 1. Thus far only categorical forms have been considered. The common logical doctrine respecting conditional forms is now to be stated. Subsequently it may be inquired whether this doctrine needs modification, and to what extent.

A categorical judgment predicates absolutely. A conditional judgment affirms relatively to some prerequisite which constitutes a condition. Its forms are primarily distinguished according as the condition is expressed by means of an antecedent clause, or implied in a disjunction, or both. Thus:

- Judgments
  - Categorical ..... e. g., S is P, and S is not P.
  - Conditional
    - Conjunctive; e. g., If A is B, C is D.
    - Disjunctive; e. g., C is either D or non-D.
    - Dilemmatic; e. g., If A is B, C is either D or non-D.

By Boethius "conditional" (*con-dare, to put together*) is used as synonymous with "hypothetical" (*ὑπο-τιθέναι, to place under*), and this, having been usual with most logicians after him, is adopted here. Each of the three forms of conditionals, then, is also called generically a hypothetical. The word "supposition" (*sub-ponere*) is the Latin congener of "hypothesis," and synonymous with it. The dilemmatic proposition, because of its compound character, is also called the conjunctivo-disjunctive proposition.[1]

§ 2. A conjunctive hypothetical involves two clauses, one of which, expressing the condition, is regarded as the subordinate member, and is called the antecedent, the reason, the protasis (*προ-τείνειν, to stretch before*). The other, expressing the conditioned, is regarded as the principal clause or member, and is called the consequent, the apodosis (*ἀποδιδόναι, to give back*). Usually and formally the antecedent is written first, but inversions are quite common.

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[1] Hamilton uses "hypothetical" specifically, as synonymous with "conjunctive." Hence he terms the dilemmatic a hypothetico-disjunctive proposition. See *Logic*, p. 167. Whately, and, indeed, except Mansel, all the Oxford logicians, also Bain and others, use "hypothetical" as generic, and "conditional" as specific.

A complete enumeration of the conjunctive forms is as follows:

1(a)—If A is B, A is C; e. g., If man is responsible, he is free.
(b)—If A is B, C is A; e. g., If bliss has no anxieties, ignorance is not bliss.
(c)—If A is B, B is C; e. g., If rubies are clay, some clay is precious.
(d)—If A is B, C is B; e. g., If metals are fusible, gold is fusible.
2 ——If A is B, C is D; e. g., If the wise are virtuous, Socrates was innocent.

In each of the first forms there are but three terms, one being common to antecedent and consequent. In the second there are four. Either clause, or both, may be particular, or may be negative. The consequent in 1(b) must be negative, and in 1(c) must be particular. This distinction of forms, based on the number and order of terms, is, however, of no moment until we come to consider the ultimate analysis of hypotheticals. Until then we are concerned only with the conditional relation of the clauses, without regard to their terms. Neither does a partial identity of the matter of the clauses, nor their quantity and quality, affect the common logical doctrine now under consideration. In this view, the conjunctive proposition has but one form, expressed by any one of the above indifferently.

Upon grounds which will hereafter more clearly appear, a hypothetical proposition may be converted by taking the contradictory of the consequent as an antecedent, and the contradictory of the antecedent as a consequent. As this is similar to conversion by contraposition, it is called by that name. We may contrapone 1(c) thus:

If no clay is precious, some rubies are not clay.

Again: If most difficulties are conquerable, none should be unattempted;
Hence, If any should be unattempted, none are conquerable.

§ 3. Disjunctive propositions are those expressing that relation of two or more judgments in which one must be true; e. g., "Either C is D, or C is non-D;" usually abbreviated into "C is either D or non-D." Here neither D nor non-D is predicated absolutely of C; but one is affirmed of C on condition that the other is denied. In general, then, the condition lies in the opposition of the members. Hamilton defines disjunctives as conditionals having the condition in the predicate; but, e. g., "Either he or I am wrong."

A disjunctive proposition involves the principle, and is subject to the laws, of Division. It implies that we have divided an unnamed genus into co-ordinate species, and it affirms the alternative identity of an object with one of these. The opposed divisions are called the disjunct members, and their relation to each other the disjunction.

Disjunctive judgments, to be strictly logical, must make a complete disjunction; that is, the disjunct members must exhaust the divisum, and must be exclusive of each other. They are therefore contradictories. The characteristic of contradictory opposition is that the opposites cannot both be true and cannot both be false: i. e., one must be true and one must be false; hence, affirming either denies the other, and denying either affirms the other. The form is that already given; i. e.,

Either C is D, or C is non-D.
Either all wars are evil, or some wars are not evil.
Either the prisoner is guilty, or he is not guilty.

When the division is more than dichotomous, we have a series of disparate terms, exhaustive and coexclusive; e. g.,

C is either D, or E, or F, or - - - - - - - -.
Bodies are either solid, or liquid, or aeriform.

Disparates must always be reduced, for logical treatment, to contradictories by grouping them into two opposed members; e. g.,

Bodies are either solid or (liquid or aeriform =) fluid.
Angles are either right or (acute or obtuse =) oblique.

Less than all the members of a disparate series will not yield a disjunctive judgment, since they are not exhaustive. Thus, to say "Birds are shot either sitting or flying" is insufficient, for they may be shot running or swimming. Hence contraries, which are the extreme terms of a disparate series, cannot yield a disjunctive judgment. Thus we cannot say "Men are either black or white," for some are red, and therefore the statement is neither true nor logical.

We said above that, in logical strictness, the disjunct members must also be coexclusive. This is true, but often we make an imperfect division wherein the species are not coexclusive, but intersect, and constitute communicant species. Such a division will yield an imperfect or incomplete disjunctive judgment, which, as it is very common, it is needful to take into consideration. E. g.,

Jack is either a fool or a knave.

We affirm he must be one or the other, but it is also true that he may be both. These terms, then, are not contradictories. The judgment may be formulated thus:

Either C is D, or C is T.

Here D and T stand for communicant species. The principle governing this form is that one must be true, and both may be true:

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hence, denying one affirms the other; but affirming one, nothing follows. As this is the law of subcontrary opposition, we will, for convenience, distinguish these from contradictory and disparate disjunctive judgments as subcontrary disjunctive judgments.

Disjunctive judgments frequently appear in the form—

Either C is D, or M is N.
Either Richard III was a monster, or Shakespeare was wrong.
Either the patient has a fever, or the doctor errs, or I mistook him, etc.
Either Cæsar was ambitious, or Brutus was criminal.

Here the matter of the opposed clauses is entirely distinct. Such a judgment is not directly disjunctive, for there is no immediate opposition between the opposed clauses. The opposition is mediate; thus,—

Either Richard III was a monster, or he was not a monster;
But If he was not a monster, Shakespeare was wrong;
Hence, Either Richard III was a monster, or Shakespeare was wrong.

The alternative, then, is declared, not between members that are directly opposed, but between one of these and the necessary consequence of the other.

Both conjunctive and disjunctive judgments always affirm, are always positive, never negative. If we say "C is neither D nor E," e. g., "The boy is neither smart nor good," this is not to declare an alternative, but is merely a double denial.

§ 4. Conjunctivo-disjunctive propositions are, as the name indicates, compounds of the two preceding forms, and hence involve no new principle. They may be defined as conjunctives having a disjunction in the consequent, or in the antecedent, or in both; or, inverting the formula, they are disjunctives having a conjunction in one or in both members. Their forms, which are certainly subject to great apparent variations, are usually represented as numerous and intricate. If there are but two disjunct members, the proposition is called dilemmatic (δι-λῆμμα, *a double assumption*); if three, trilemmatic; if four, tetralemmatic; or if more than two, polylemmatic. Ordinarily, however, the adjective "dilemmatic" is applied to all indiscriminately.[2]

[2] To avoid a common confusion, we will use only the adjective when speaking of propositions, restricting the use of the nouns "dilemma," "trilemma," etc., to certain syllogistic forms hereafter to be described.

In § 1 we gave the following abbreviated form as representative:

If A is B, C is either D or non-D.

This may now be expanded to—

If A is B, either C is D, or C is non-D.

Again expanding we have—

Either if A is B, C is D; or if A is B, C is non-D.

This now appears as a double hypothetical having the same antecedents and disjunct consequents.

Now let us consider that a difference in the matter, or in the quality of two clauses, e. g., "C is D," and "C is non-D," makes them distinct clauses, quite properly represented by "C is D," and "E is F;" for a partial identity does not modify, except in ultimate analysis, the logical treatment of such propositions. Also, that a separate representation of contradictory and subcontrary opposition may be omitted, with the understanding that every formal statement represents either. Again, since disparate members must always be grouped, for logical treatment, into two contradictory members, and since the trilemmatic and polylemmatic forms are of this nature and subject to this rule, they also may be omitted in a classification of forms that is grounded on the relation of clauses rather than of terms.

Under these provisos we may represent very easily an exhaustive statement of the conjunctivo-disjunctive forms, thus:

1, **Simple,** (a)—Either if A is B, C is D; or if A is B, E is F;
—having antecedents identical and consequents disjunct.

" (b)—Either if A is B, C is D; or if E is F, C is D;
—having antecedents disjunct and consequents identical.

2, **Complex,** —Either if A is B, C is D; or if E is F, G is H.
—having antecedents disjunct and consequents disjunct.

The following are concrete examples of these forms:

1 (a)—If Socrates was innocent, Anytus was either deceived or perjured.
If we go to war, we must either contract a debt or increase taxation, or indemnify ourselves at the enemy's expense.

1 (b)—If man is either well or ill deserving, he is a moral agent.
If my king is moved, or if he is covered, or if I capture the attacking piece, I am checkmated at the next move.

2—— If the prisoner knew the consequences of his act, he was criminal; or if not, he was insane.
Either if education is popular, compulsion is unnecessary; or if it is unpopular, compulsion will be resisted; or if the people are indifferent, compulsion will be fruitless.

§ 5. Upon the basis of the conditional propositions now described, logicians have erected a system of syllogizing corresponding in terminology with the categorical system. There are then four kinds of conditional syllogisms.

The Conjunctive hypothetical syllogism is one which has for its major premise a conjunctive proposition, the minor premise and conclusion being the affirmation or denial of its component clauses. It claims for its canon the Law of Sufficient Reason, modified thus: Every assertion must have a ground or reason for its support.[3] This is explicated into two axioms, as follows:

1. Asserting the reason asserts the consequent.

2. Denying the consequent denies the reason.

The converse of neither axiom, it is said, is true. Denying the reason does not deny the consequent, and affirming the consequent does not affirm the reason; for it may follow from some other reason. To do either is fallacy. But we shall find exceptions to this.

The double axiom gives rise to two so-called moods. The form of the conjunctive syllogisms in these two moods, and their names, are as follows:

Modus Ponens (*constructive*)	If A is B;	then	C is D;	Modus Tollens (*destructive*).
	But A is B;		But C is not D;	
	∴ C is D.		∴ A is not B.	

If the people are industrious, wealth increases;

Ponens.— They are industrious; | Wealth is not increasing;—Tollens.

∴ Wealth is increasing; | ∴ The people are not industrious.

The major premise, or sumption, is always universal and affirmative. Either or both of its clauses may be particular or negative; but the total proposition always universally affirms that the antecedent necessitates the consequent.

It will be observed that the major premise only is conditional, both the minor premise and the conclusion in each of the two moods being categorical. If the conclusion, or both it and the minor premise, were also hypothetical, the reasoning would not be conditional, but categorical. This paradox will be illustrated subsequently.

In the conjunctive syllogism there are only three propositions, but there may be four terms, as in the given example. All the terms occur in the major premise. Hence, unlike the categorical syllogism, the minor premise introduces no new term, and the conclusion may have nothing in common with it.

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[3] See Part 1st, ii, § 7.

There are three Rules deduced from the axioms, as follows:

1. The subsumption in **Ponens** agrees with its corresponding clause in quality, but may differ in quantity.
2. The subsumption in **Tollens** disagrees with its corresponding clause in both quantity and quality.
3. The conclusion in **Ponens** agrees, and in **Tollens** disagrees, with its corresponding clause in both quantity and quality.

These disagreements in Tollens are because the only true denial is by the logical contradictory. Let us, however, remember that when the subject is individual, as in the previous example, contradiction is merely a change of quality. In illustration of the rules, we may take the following:

	If any nation prospers, all are benefited;		
Ponens {	All are prospering;	----------	} Tollens.
	or Some are prospering;	Some are not benefited;	
	or This one prospers;	or That one is not benefited;	
	∴ All are benefited.	∴ None are prospering.	

Negative clauses, one or both, do not offend these principles. The following is in strict conformity:

	If A is not B, then C is not D;		
Ponens, *asserts*,—	A is not B;	C is D;	—Tollens, *denies*.
(*constructive*.)	∴ C is not D.	∴ A is B.	(*destructive*.)

If we contrapone a major premise, we shall find that this reduces the moods the one to the other. Hence they are fundamentally the same, which might be inferred from the common origin of the two axioms evolving the two moods.

In *reductio ad impossibile* it is quite usual to state the argument hypothetically, and then the tollent mood applies, perhaps so obviously as to be left unexpressed; e. g., "If we say we have not sinned, we make God a liar."

A number of objections in detail might be urged against this scheme of syllogizing. We will be content at present with pointing out a couple of exceptions. Whenever one clause is the infinitated form of the other, as "If A is B, A is not non-B," or if the antecedent is the sole condition of the consequent, then the converse of each axiom *is* true; i. e., denying the antecedent denies the consequent, and affirming the consequent affirms the antecedent. As these forms

not unfrequently occur, the rules should provide for them. For example,—

If silver is legal tender, it cannot be refused in payment of debts.
If not even one was saved, then all were lost.
If he was present, you certainly saw him.
If force is expended, an equivalent force is generated.
If the scheme is perfect, no exception will hold.

Hamilton gives the following as the Canon of the hypothetical (i. e., conjunctive) syllogism: "Two or more propositions being thought as indeterminate in quality, but as in quality mutually dependent, the determination of quality in the one infers a determination of the corresponding quality in the other."[4] But this, on the other hand, is true, in its full generality, only of the above excepted contradictory forms. Nevertheless, Hamilton goes on to say, "This Canon embodies and simplifies the whole mystery of hypothetical syllogisms, which have been strangely implicated, mutilated, and confused by the logicians." He then proceeds to an elaborate critical discussion. Mill says, "There is a great quantity of intricate and obscure speculation, in our author's Lectures and their appendices, relating to disjunctive and hypothetical propositions."[5] But Mill, like a true critic, proceeds only in the tollent mood, leaving others to reconstruct as they may.

§ 6. The Disjunctive hypothetical syllogism is one having for its major premise a disjunctive proposition, and having the disjunction resolved in the minor premise and conclusion.[6] According to Hamilton, it is governed by the axiom of Excluded Middle,[7] affirming that of two contradictories one must be true and the other false. The

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[4] *Logic*, p. 602.

[5] *Examination of Hamilton*, vol. ii, p. 225.

[6] "Our Hypothetical and Disjunctive Syllogisms may be reduced to the class of Conditional Syllogisms. The Hypotheticals should be called, as they were by Boethius and others, *Conjunctive*, in contrast to the co-ordinate species of *Disjunctive*. Hypothetical, as a name of the species, ought to be abandoned.

The Conjunctive are conditional, inasmuch as negation or affirmation is not absolutely asserted, but left alternative, and the quality of one proposition is made dependent on another. The Disjunctive are conditional, inasmuch as a notion is not absolutely asserted as subject or predicate of another or others, but alternatively conjoined with some part, but only with some part, of a given plurality of notions, the affirmation of it with one part involving the negation of others." (Hamilton's *Logic*, p. 600.)

[7] Rather, of Duality. See Part 1st, ii, § 4.

disjunct members being contradictories, we may, through affirming one, deny the other, and *vice versa*. This yields two moods, each of which is double. They have the following names and forms:

Modus Tollendo Ponens.	C is either D or E (=non-D); C is not D; ∴ C is E.	C is D; ∴ C is not E;	Modus Ponendo Tollens.
	or	or	
"	C is not E;	C is E;	"
"	∴ C is D;	∴ C is not D.	"

All men are either justified or under condemnation;

To. Ponens.—Some are not justified; ∴ Some are under cond.	Some are justified;—Po. Tollens. ∴ Some are not under cond.	
or	or	
" This one is not under cond. ∴ He is justified.	That one is under cond. " ∴ He is not justified.	

The sumption (the whole proposition, not the clauses) is universal and affirmative; the subsumption may vary. The conclusion must have the same quantity as the subsumption, and the opposite quality.

We shall hereafter find that every contradictory disjunctive judgment may be reduced to two conjunctives. Each conjunctive yields two moods, and the four forms thus arising correspond with the four disjunctive forms.

Negative clauses call for no modification of the rules; e. g.:

Either C is not D, or E is not F;
Po. Tollens. — But C is not D, ∴ E is F.

If the major premise presents three or more disjunct members (disparate terms), they must be reduced to two contradictories before an inference can be drawn. E. g.:

Sciences are either pure, inductive, or mixed;
To. Ponens.— Astronomy is neither a pure nor an inductive science;
∴ Astronomy is a mixed science.

A formal illustration is as follows:

Either C is D, or E is F, or G is H.

Now let the first clause be contradictory of the other two taken together. Then:

To. Ponens.—C is not D; ∴ Either E is F, or G is H.
" Neither E is F, nor G is H; ∴ C is D.
Po. Tollens.—C is D; ∴ Neither E is F, nor G is H.
" Either E is F, or G is H; ∴ C is not D.

This doctrine of the disjunctive syllogism is inadequate, as it considers only the contradictory forms, not embracing the subcontrary forms. The modification needed is that the latter conclude in the ponent moods, but not in the tollent. E. g.:

All afflictions are either punitive, or tentative, or disciplinary;
To. Ponens.— Job's afflictions were neither punitive nor disciplinary;
∴ They were tentative.

" David's were not tentative;
∴ They were either punitive or disciplinary (perhaps both).

Israel's, we cannot say, were punitive, and therefore not tentative nor disciplinary; for they were not only punitive, but also both tentative and disciplinary.

There is a syllogism founded upon contraries, called by Arnauld the "Copulative Syllogism." A thing may be either or neither of two contraries, but cannot be both. Hence we may reason thus:

Ye cannot serve God and Mammon;
Ye serve Mammon;
∴ Ye do not serve God.

We may have all the disjunct members affirmed of something, or something affirmed of them. This gives rise to other forms. Thomson calls the disjunctive syllogism above described "complex," and gives the following as examples of "pure" disjunctive syllogisms:

Every body is solid, liquid, or aeriform;
Solid, liquid, and aeriform bodies are elastic;
∴ Every body is elastic.

All sciences are either pure, inductive, or mixed;
Astrology is neither;
∴ Astrology is not a science.

But manifestly this reasoning is not conditional, but categorical. There is no resolution of the disjunction. Its members constitute the middle term, totally affirmed in the first, totally denied in the second. The forms are strictly Barbara and Camestres.

§ 7. The Conjunctivo-disjunctive, or dilemmatic, proposition being a compound form, the syllogisms founded on it are also compound, and often very intricate. Before proceeding to syllogize, we should be careful to see that the antecedent is a true condition of the consequent. This is not the case, for example, in "If an egg is good, it will either sink or swim." Again, we should see that the disjunction is exhaustive. In this example, "If the habit of virtue is of any value,

it must ensure either pleasure, wealth, or fame," the disjunction is not exhaustive, for virtue may be valued on other grounds. We should further ascertain whether the disjunct members are contradictory or only subcontrary. We may then proceed to syllogize, viewing the proposition either as a conjunctive or as a disjunctive.

If we reason on the conjunction looking on the disjunct members as a single clause, the syllogism is governed by the principles explained in § 5; thus:

	Either if A is B, C is D;	or if A is B, E is F;	
PONENS.—	But A is B;	Neither C is D, nor E is F;	—TOLLENS.
	∴ Either C is D, or E is F.	∴ A is not B.	

If the apostles taught falsely, they were either deceivers or deceived.

PONENS. —	They did teach falsely;	They were neither deceivers nor deceived.	—TOLLENS.
	∴ They were either deceivers or deceived.	∴ They did not teach falsely.	

These are plainly hypothetical syllogisms, one in each mood. The latter, the tollent form, is given by Kant and Wolf as a dilemma, and by Wallis and Mansel as a disjunctive syllogism.[7] But certainly it is neither; for it simply denies the antecedent through denying the consequent; it neither introduces a disjunction nor resolves one. This tollent form is sometimes called *cornutus*, or the horned syllogism (*bi-cornis*), "because in the sumption the disjunctive members are opposed like horns to the assertion of the adversary; with these we throw it from one side to the other in the subsumption, in order to toss it altogether away in the conclusion. Such a syllogism is very easily abused for the purpose of deceiving, through a treacherous appearance of solidity, and from terrifying a timorous opponent by its horned aspect."—*Krug.*

It should be remarked that while the particles "either—or" are disjunctive, the corresponding negatives "neither—nor" are not so, but are conjunctive or total. They do not exclude one on condition of the inclusion of the other, but they exclude both or all. They directly deny both clauses, and consequently deny the existence of the disjunction. This is not resolution, but annihilation.

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[7] See Mansel's *Aldrich*, p. 109, note. He says this form of reasoning is sometimes called a Dilemma, but it is a perversion of the Dilemma proper, and introduces no distinction whatever; being merely a common disjunctive syllogism, as is shown by Wallis himself (iii, ch. 19). It is, in fact, the *enumeratio*, not the *complexio*, of Cicero.

Now, on the other hand, if we reason on the disjunction involved in the conjunctivo-disjunctive proposition looking on the conjunctive statement as a single clause, the syllogism is disjunctive, and governed by the principles explained in § 6. Thus, adopting for convenience the abbreviated form, we have:

If A is B, either C is D, or E is F;
To. Ponens.— But C is not D;
∴ If A is B, E is F.

If Socrates was innocent, Anytus was either deceived or perjured;
But Anytus was not deceived;
∴ If Socrates was innocent, Anytus was perjured.

By denying that Anytus was perjured, we have another To. Ponens. If the disjunct members are, as in this example, contradictories, they yield also two forms in Po. Tollens, or the destructive mood.

The conjunctivo-disjunctive syllogism, now explained, although it has a dilemmatic sumption, is not properly a dilemma at all. The logics are full of confusion here, and often mistake it for the dilemma. Thomson, following Hamilton, distinctly so names it, and rejects the dilemma proper from Logic.[8]

§ 8. The Dilemma (or trilemma, etc.) is a conditional syllogism having a double (or triple, etc.) conjunctive premise, and a disjunctive premise. Neither one of its propositions is dilemmatic, or conjunctivo-disjunctive. In the conjunctivo-disjunctive syllogism treated conjunctively, as explained in the first part of the preceding section, the disjunct members of the sumption are either affirmed both together or denied both together. But they may be affirmed or denied disjunctively. In this case the existing disjunction is declared in the subsumption, and (in the complex forms) in the conclusion. It is therefore needless to state it in the sumption, which then appears merely as a double conjunctive. Thus the dilemmatic proposition is distributed; one of its essential features, the conjunction, appearing in the sumption; the other, the disjunction, appearing in the subsumption and conclusion. It is, indeed, indifferent, as the definition above implies, as to which of these shall be called the sumption and which the subsumption; but it is usual to place the double conjunctive first, and call it the sumption or major premise.

[8] So also Bain. See *Logic*, p. 121 sq.

The Dilemma presents three distinct and inconvertible forms, as follows:

1. Simple constructive: If A is B, C is D; and if E is F, C is D;
   Ponens.— But either A is B, or E is F;
   ∴ C is D.

2. Complex constructive: If A is B, C is D; and if E is F, G is H;
   Ponens.— But either A is B, or E is F;
   ∴ Either C is D, or G is H.

3. Complex destructive: If A is B, C is D; and if E is F, G is H;
   Tollens.— But either C is not D, or G is not H;
   ∴ Either A is not B, or E is not F.

It will be observed that the subsumption in each form declares a disjunction between certain of the components of the conjunctive sumption; that in the simple form the conclusion is categorical; and that in the complex it declares a disjunction between the other components of the sumption.

A single concrete example from *Demosthenes de Corona* must suffice. It is in the complex constructive form, as follows:

If Æschines joined in the public rejoicings, he is inconsistent; if he did not, he is unpatriotic;
But either he did, or he did not;
∴ Either he is inconsistent, or he is unpatriotic.

The form of the sumption in this example may be expressed thus:

If A is B, A is C; and if A is not B, A is D.

Here the first term of each of the clauses is the same, and the antecedents differ only by the negative. Nevertheless, the form is complex, corresponding to No. 2; for the clauses all differ from each other either in matter or in quality.

There cannot be both a simple constructive and a simple destructive dilemma. Denying the consequents in No. 1 gives—

If A is B, C is D; and if E is F, C is D;
But C is not D;
∴ A is not B; and E is not F.

This, however, is merely a double conjunctive syllogism in Tollens.

The following, much more than the last, has an appearance of being the simple destructive form, corresponding to No. 1. It is given as such by Fowler, and copied from him with approbation by McCosh:

If A is B, C is D; and if A is B, E is F;
But either C is not D, or E is not F;
∴ A is not B.

But if this be examined, it will be found to be No. 1 contraponed, and then treated in Tollens. Now, to treat a proposition in Ponens, and then to treat its contraponed form in Tollens, is to do the same thing. The reasoning in both cases is the same. Hence this form cannot be accepted as additional to those given, it being the same as No. 1, and only slightly disguised by a rearrangement, after contraposition, of the letters in alphabetical order. Truly, it is a simple destructive dilemma, which is to say that a simple dilemma may appear either in the destructive or in the constructive form; but, since these are essentially the same, we should not reckon both; and this is the statement which we are now supporting.

Whately,[9] endorsed by Mansel,[10] says, "There cannot be a simple destructive dilemma," that "the destructive is always complex." This is true under his definition of the dilemma, as "A syllogism having a conditional (i. e., conjunctive) major premise with more than one antecedent, and a disjunctive minor." But this limitation of the major premise is purely arbitrary, and the definition is too narrow. The truth is that under the proper definition there may be either, but not both.

In disputation an adversary is sometimes "caught on the horns of a dilemma." If he meet it by another with an opposite conclusion, he is said to "rebut the dilemma." Aristotle thus illustrates it: "An Athenian mother said to her son, Do not engage in public affairs; for if you do what is just, men will hate you; and if you do what is unjust, the gods will hate you. This the son rebutted by the following retort: I ought to enter into public affairs; for if I do what is unjust, men will love me; and if I do what is just, the gods will love me." Both these are in the complex constructive form, and each is followed by an implied categorical syllogism. The first dilemma originated with Antisthenes the Cynic, who proposed by it to excuse himself from meddling with politics.

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[9] *Logic*, bk. ii, ch. iv, § 5.

[10] Note in *Aldrich*, p. 108.

§ 9. **Praxis.** Specify to what class each of the following judgments belongs, put it in logical form, and distinguish its members. If conjunctive, into which of the five forms does it fall? Contrapone four examples. If disjunctive, is it contradictory or subcontrary, mediate or immediate? If a polytomy, reduce to a dichotomy. If dilemmatic, is it simple or complex? Formulate with letters. If the proposition is defective, say wherein.

1. Wherever there is smoke, there is fire. (Wherever=If in any place.)
2. If a government is well constituted and skilfully administered, it is promotive of the industry and wealth of its subjects.
3. If I err, it is because I am human; for to err is human. (Is this good reasoning?)
4. I will not let thee go, unless thou bless me. (Unless=if not.)
5. Until the night come, we must work.
6. Is any among you afflicted, let him pray.
7. Lear is at the hut or the palace. (Real difference.)
8. Hiawatha left his hut or wigwam. (Nominal.)
9. If the rebellion be not crushed, the king will be dethroned.
10. If virtue is voluntary, then vice is.—*Aristotle, N. Ethics*, bk. iii.
11. Punishment is intended either to repress crime or to reform the criminal. (Perhaps both.)
12. Neither flattery nor threats could prevail.
13. If ye were Abraham's seed, you would do the works of Abraham.
14. Whenever the sun and moon attract in the same line, the tides are at a maximum. (Whenever=If at any time.)
15. Either if this is a judgment, it affirms or denies; or if it is a question, it does neither.
16. Ye shall not eat of it, neither shall ye touch it, lest ye die.
17. If ye eat, ye shall die. Though ye eat, ye shall not die. (Though =even if. The concessive clause, introduced by "though," etc., grants the protasis of a sentence whose apodosis is denied by the principal clause. The above means "It is not true, or it does not follow, that if ye eat, ye shall die.")
18. Though deep, yet clear; though gentle, yet not dull.
19. It has not been decided whether the war will continue or not.
20. If Cæsar lives, he will either rule or ruin.
21. Those who slew Cæsar are either patriots or parricides.
22. The sun moves round the earth, or the earth moves round the sun.

23. Although Homer sometimes nods, nevertheless is he the greatest of poets.
24. *Aut amat aut odit mulier; nihil tertium.—P. Syrus.*
("A woman loves or hates; she never thirds it.")
25. If you have failed to nourish the poor, you have destroyed them. (*Si non pavisti, occidisti.*—From *Arnauld.*)
26. If the heart is right, the actions will be.
27. The solitary is either a beast or a god.—*Aristotle, Polit.* i, cap. 1.
28. The world will not be happy until either kings become philosophers, or philosophers become kings.—*Plato's Repub.*
29. If the foot-marks were made by the prisoner, he must have worn shoes too small for his feet.—*Ad abs.*
30. Virtue is teachable if it is knowledge.
31. If man is either well or ill deserving, he is a moral agent.
32. If the square described upon one of the sides of a triangle be equal to both the squares described on the other two sides of it, the angle contained by these two sides is a right angle.—*Euclid*, Prop. xlviii, bk. i.
33. There could be no choice, were there no difference.
34. Nor pain, nor grief, nor anxious fear,
Invades thy bounds.
35. This elegant rose, had I shaken it less,
Might have bloomed with its owner awhile.

If an example among the following is merely a proposition, syllogize from it. If an incomplete syllogism, complete it and name the kind and mood. If a conjunctivo-disjunctive, or a dilemma, classify it, and formulate with letters. If defective, say wherein.

36. If any objection that can be urged would justify a change in the established laws, no laws could reasonably be maintained.
37. Mahomet was either an enthusiast or an impostor;
He was an enthusiast;
∴ He was not an impostor.—*Gibbon.*
38. Corn will be dear if the crops are bad, and they seem likely to be so.
39. A government cannot be at the same time despotic and the licenser of a free press;
But the English government permits a free press;
∴ The English government is not despotic.

40. If man cannot make progress towards perfection, we must believe him to be either an incapable brute, or already divine.

41. I could, with justice, be accused of acting contrary to my law, only if I maintained that Muræna purchased the votes and was justified in doing so. But I maintain that he did not buy the votes; therefore I do nothing contrary to the law. — *Cicero pro L. Muræna*, cap. iii. (Ramus cites this as bad reasoning. Was he right?)

42. Unless matter can move of itself, its first motion must have been given it by a spiritual being. But matter cannot move itself; therefore, etc.

43. If pain is severe, it will be brief; and if it last long, it will be slight; hence it should be borne patiently.

44. If the system of the universe is not the best possible, we must suppose that the Creator did not prefer a better, or that he knew none better, or that he could not create a better. But we can entertain neither of these suppositions, for we should thereby limit his goodness, his intelligence, or his power. Therefore the system of the universe is the best. (Thomson, *Outline*, § 109, quotes this as a trilemma. McCosh, *Logic*, p. 150, likewise. Are they right?)

45. Whether Logic be regarded as a means of mental discipline or as a practical guide in reasoning, it ought to be studied. But it is both. Hence—what?

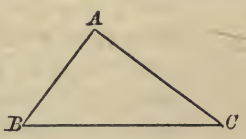
46. If this man were wise, he would not speak irreverently of Scripture in jest; and if he were good, he would not do so in earnest;
But he does it either in jest or in earnest;
∴ Either he is not wise, or he is not good.

47. If the books in the Alexandrine Library be in conformity with the doctrines of the Koran, there is no need of them; if adverse, then also they should be burned.

48. If classical education is worth the cost, either it must be pre-eminently fitted to develop the mental powers, or it must furnish exceedingly valuable information. But neither alternative can be maintained, and so classical education is not worth the cost. (This is given by Bain, *Logic*, p. 122, as a dilemma. Right?)

49. If any satisfactory theory could be framed to explain the establishment of Christianity by human causes, such a theory would have been proposed before now; but no such theory has ever been proposed; hence, none can be framed.

50. The greater angle of every triangle is subtended by the greater side, or has the greater side opposite to it.



Let A B C be a triangle, of which the angle B is greater than the angle C; then the side A C is likewise greater than the side A B.

For if A C be not greater than A B, it must be either equal to or less than A B. But A C is not equal to A B, because then the angle B would be equal to the angle C, the angles at the base of an isosceles triangle being equal, by Prop. v: but it is not; therefore A C is not equal to A B. Neither is A C less than A B; because then the angle B would be less than the angle C, the greater side of every triangle having the greater angle opposite to it, by Prop. xviii: but it is not; therefore A C is not less than A B. And it has been shown that A C is not equal to A B; therefore A C is greater than A B. Wherefore the greater angle, etc. Q. E. D.—*Euclid*, Prop. xix, bk. i.

51. The ancients were in genius either superior to the moderns, or inferior, or equal.—See *Hamilton's Logic*, p. 234.

52. If the world existed from eternity, there would be records prior to the Mosaic; and if it were produced by chance, it would not bear marks of design. But there are no records prior to the Mosaic, and the world does bear marks of design.

∴ The world neither existed from eternity, nor is it the work of chance.

53. If the prisoner is to be legally discharged, either the magistrate must refuse to commit, or the grand jury reject the bill, or the petit jury acquit, or the governor exercise the prerogative of pardon. But neither can the magistrate refuse to commit, nor the grand jury reject the bill; hence—what?

54. If a man cannot be virtuous, he must be either unable to know what is right, or unable to will what is right. But he is not unable to know what is right, for he is intelligent; nor unable to will what is right, for he is free.

55. If there be no future life, then either virtue receives its due reward in the present world, or there is no perfect government administered among men, neither of which is admissible.

56. The hope of immortality is either a rational expectation or an illusion; but that belief cannot be an illusion which all the most enlightened peoples have adopted.

57. If Christ be preached that he rose from the dead, how say some among you that there is no resurrection of the dead? But if there be no resurrection of the dead, then is Christ not risen; and if Christ be not risen, then is our preaching vain, and your faith is also vain. Yea, and we are found false witnesses of God, because we have testified of God that he raised up Christ: whom he raised not up, if so be that the dead rise not. For if the dead rise not, then is not Christ raised: and if Christ be not raised, your faith is vain; ye are yet in your sins. Then they also which are fallen asleep in Christ are perished. If in this life only we have hope in Christ, we are of all men most miserable.—1 *Cor.* xv, 12–19.

58. A system of government which extends to those actions that are performed secretly must be one which refers either to a regular divine providence in this life, or to the rewards and punishments of another world;

Every perfect system of government must extend to those actions which are performed secretly;

∴ No system of government can be perfect which does not refer either to a regular divine providence in this life, or to the rewards and punishments of another world.—*Warburton's Divine Legation.* See vi, § 5.

59. There are two kinds of things we ought not to fret about,—what we can help, and what we cannot. (From this, form a dilemma.)

60. We must either gratify our vicious propensities or resist them; the former course will involve us in sin and misery, the latter requires self-denial; therefore we must either fall into sin and misery, or practise self-denial.

VI. ANALYSIS OF CONDITIONALS.

§ 1. A categorical proposition declares a relation between two terms unconditionally; that is, no condition being expressed. A conditional or hypothetical proposition involves an express condition. The latter is a complex sentence, consisting, in the conjunctive form, of a principal clause, the apodosis or consequent, and of a subordinate clause, the protasis or antecedent or condition.

The question whether conditional propositions and syllogisms may or may not be reduced to categorical forms is much discussed by logicians. "By Kant and his followers the hypothetical proposition is described as representing a form of judgment essentially distinct from the categorical; the latter being thoroughly assertorial, the former problematical in its constituent parts, assertorial only as regards the relation between them. Two judgments, each in itself false, may thus be hypothetically combined in a single truth; and this combination cannot be reduced into categorical form. The hypothetical syllogism, in like manner, is a form of reasoning distinct from the categorical, and not reducible to it, being based on a different law of thought, namely, the Logical Principle of Sufficient Reason, *a ratione ad rationatum, a negatione rationati ad negationem rationis valet consequentia.*" [1]

But observe that the question is not properly whether the categorical and hypothetical forms are convertible. Logic as the Theory of Thought has no concern with what may or may not be done with forms. The question proper to Logic is this: Does thought hypothetically expressed differ from that categorically expressed; and if so, what is the specific difference? In other words, Are there two processes of thought, as there are two kinds of propositions, and do we need a distinct system of syllogizing to explain how we necessarily think when matter is thought hypothetically? We propose now to show that while there are two spheres of thought, its process is one; that all reasoning has essentially the same form, having the Aristotelic syllogism for its formal unit. We propose to discover the true rela-

[1] See Kant's *Logik*, § 25 and § 76; and Mansel's *Aldrich*, Appendix, Note I.

tions of categorical and hypothetical thinking, and to show that they do not differ logically, but only psychologically.

In attempting this, we will first point out a psychological distinction between two spheres of thought; then consider the propositional use of hypotheticals, and the syllogisms arising therefrom; and then advert to the common logical doctrine of conditionals. Herein we hope to confirm the general doctrine of this Treatise, that thought is of only two logical kinds, immediate and mediate, and that of the latter the Aristotelic syllogism is ultimately the universal form.

§ 2. Thought is either of the real or of the ideal. Real thought considers its matter as existent, and affirms or denies of it categorically. Ideal thought considers its matter as merely logically possible, and affirms hypothetically, that is, in a suppositious mode. This matter may or may not really exist; but thought posits merely its ideal existence, and, limited only by self-contradiction, proceeds to evolve logically conceivable consequences. So even when the matter is known to be real, the mind may choose rather to view it ideally, thought readily transferring it from one sphere to the other. Thus when I say

Plato is a man, therefore he is mortal,

I think the matter real, and draw a real conclusion. But when I say

If Plato be a man, then he is mortal,

I think the matter ideally, making a supposition without regard to fact, and on this hypothetical statement I reason to an equally ideal conclusion.

What distinguishes ideal from real thought is precisely what distinguishes hypothetical from categorical judgments. Thus far we have used the words "conditional" and "hypothetical" as interchangeably synonymous. But the former is opposed to "categorical" in the characteristic that it formally expresses a condition of the principal thought; the latter in the other characteristic that it expresses ideal, suppositious thought, and not real declared fact. The words should be used accordingly.

It is manifest that the distinction between categorical and hypothetical judgments as real and ideal is not logical, but psychological. This will still more plainly appear when it is shown that thought in the real and in the ideal sphere is logically the same; that is, governed by the same laws, assuming the same forms, analyzing into the same principles, and hence indistinguishable on logical grounds.

These two mental moods, the real and the ideal, are formally expressed by the two grammatical moods, indicative and subjunctive. It would seem that by a language scientifically constructed and expressing accurately the mind of the speaker, these moods would always be sharply discriminated. But perhaps in all of the more refined languages, notably in our own, there has been a strong tendency to obliterate the subjunctive forms, and to substitute the indicative to express ideal thought. In hypothetical propositions, which are all essentially ideal, the indicative has largely usurped the place of the subjunctive.

It is quite common for grammarians to characterize the subjunctive mood as expressive of doubt or uncertainty. But this is inept, for its past tenses never express doubt, and its present tense is entirely consistent with full conviction, the doubt in this case, so far as the expression implies it, being altogether formal or rhetorical, and not actual. It should be observed that the real and ideal are modes of cognition, of intellectual apprehension; whereas belief and doubt are feelings, modes of self-consciousness. These coexist with cognitions, but are very widely separated from them in psychological analysis. If, then, they are not to be made the basis of a psychological distinction between modes of thought, much less should they be made the basis of a logical distinction. Any uncertainty attending a premise modifies in no way whatever the character of our reasoning. We do not reason one way when we are in doubt, and in another when we are certain. In all cases reasoning proceeds apodeictically, the deduction is necessary, not more so in demonstration than in dialectics. An uncertainty in a premise is carried along, and attaches to the conclusion, without being itself increased or diminished. The doubt affects not the reasoning, nor the reasoning the doubt. Hence we must here set entirely aside any consideration of the feelings of certainty, degree of belief, doubt, etc.; and especially have care not to confuse these feelings with the intellectual moods real and ideal.

The indicative mood, then, properly deals with the real. It declares concerning facts as facts. It has moreover, perhaps under the influence of doubt, taken upon itself to express, what properly belongs to the subjunctive, ideal thought. The present subjunctive deals with a subjective ideal which is objectively contingent. It expresses a supposition of a fact,—the ideal; one which may or may not become a fact,—the contingent. The past tenses of this mood have, in *usus loquendi*, come to express a supposition contrary to fact,—an ideal, not

contingent, but unreal. The psychological distinction between real and ideal thought is thus profoundly embedded in language.[2]

It will be useful to illustrate this matter by some divisions, taken in a grammatical rather than a logical view. In the development of our language, the tenses of the subjunctive have moved forward in time, so that usually the present tense expresses future time; the imperfect tense, present time, etc. The present tense has not, however, ceased to express present time. E. g., "If the book be in this room, it may be found." Perhaps more commonly now it would be said, "If the book *is* in this room," etc., which, though indicative, is equally ideal and contingent. Considering, however, the step forward in time as established, we find three phrases of ideal subjunctive thought:

1st. The ideal and contingent future; both the protasis and apodosis being suppositions lying in the future;

(a) Future from the standpoint of the present; e. g.:

If he repent, he should be forgiven.
Should he come, he would be welcome.
Only were you to wax fat, should I love you more.
I tell you that, if these should hold their peace,
the stones would immediately cry out.

(b) Future from the standpoint of the past; e. g.:

I told you if you were to do this, I would reward it.

2d. The ideal and unreal present; it being implied that neither the protasis nor apodosis really exists; e. g.:

If he were here, I would tell him. I would if I could.
Were the question definite, it should be answered.
The moon would be always full if it were self-luminous.

If all the year were playing holidays,
To play would be as tedious as to work.—*Shakespeare.*

3d. The ideal and unreal past; wherein likewise the real existence of both protasis and apodosis is impliedly denied; e. g.:

If he had been present, I should have seen him.
Could the fort have held out, the city would not have been taken.

Oh, had your fate been joined with mine,
As once this pledge appeared the token;
These follies had not then been mine,
My early vows had not been broken.—*Byron.*

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[2] Too deeply to be uprooted or disturbed by Schelling's *Philosophy of Identity*, declaring the absolute identity of the real and the ideal, of being and thinking.

Besides these fundamental forms of the pure and strict subjunctive, there are a number of mixed forms, as follows:

Past tense combined with the present; e. g.:

Had he been prudent, he were now living.
Were these his companions prudent, he had not lost his life.

Subjunctive protasis with indicative apodosis; e. g.:

If this be judged treason, still will I maintain it.

The same in the concessive relation (see v, § 9, Ex. 17); e. g.:

Though hand join in hand, the wicked shall not be unpunished.

The same in the iterative relation, equivalent to a general rule; e. g.:

If (at any time, or whenever) the centres of the sun and moon be in the same line with the centre of the earth, there must be an eclipse.

The subjunctive with the imperative; e. g.:

If love be rough with you, be rough with love.—*Shakespeare.*
If thou be the Son of God, command that these stones be made bread.

The subjunctive with the potential; e. g.:

Had you seen the city before it was razed, you might have thought it indestructible, and could not have foreseen its fate.

A comparative construction with an ellipsis of the apodosis; e. g.:

He brags as (*he would brag*) if he were of note.—*Shakespeare.*

Any special examination of these mixed forms must be omitted; we only observe that, being mixed, the principles governing their elements govern them.

§ 3. The conjunctive hypothetical, then, is an ideal form of speech expressing either the contingent or the unreal. The protasis is a subordinate clause related to the apodosis, in the contingent forms, either as a qualifier or as an antecedent condition. This indicates a double use that is made of these hypothetical forms in thought. They are either propositions containing a qualified term, or they are propositions declaring an inference. We will first consider the qualified propositions.

Looking on the contingent forms, we observe that very often the sole purpose in the mind of a speaker using this form is to declare an ideal truth. It is a mere proposition, one not intended to offer a reason, but to state a judgment. In such case, since the mind passes readily from the ideal sphere to the real, and *vice versa*, these prop-

ositions may generally be easily reduced to categorical forms. The protasis being viewed, not as a condition, but rather as a qualification, explicative or limitative, we may redress the four forms (v, § 2) thus:

1 (a) If a house be undermined, it will fall;
i. e., A house undermined will fall.

(b) If virtue is voluntary, vigor is not a virtue;
i. e., Vigor is not voluntary virtue.

(c) If mere rhyming is poetry, poetry is easily written;
i. e., Poetry that is mere rhyming is easily written.

(d) If carbon will burn, the diamond will burn;
i. e., The diamond, being carbon, will burn.

What the hypothetical here states ideally, the categorical equivalent states as a real fact. This difference is psychological and grammatical, not logical. The hypothetical proposition is grammatically a complex, but logically a simple, sentence. The generality of a universal statement must not be confused with the ideality of a hypothetical. When we say "A house undermined will fall," and "An injurious deed, if it be unintentionally committed, is not a crime," the former is stated as a real fact, having a potential, if not an actual, existence. It is general, not ideal. The latter is both general and ideal.

Each of the foregoing examples may be taken as a general rule, and stand as the major premise of a syllogism, or it may be viewed as a specialized statement, and used as a minor. Other cases are sometimes only particular, and fitted to become only minors. The following, cited by Fries[3] as an example of a hypothetical not reducible to categorical form, is general or particular according as we interpret it:

If Caius is disengaged, he is writing poetry.

It may be construed as a universal statement meaning—

Caius, whenever disengaged, is writing poetry,

thus expressing iterative relation, "At any or every time that," etc. But it may also be construed as a particular statement meaning—

Caius, being disengaged, is now writing poetry.

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[3] *System der Logik*, § 62. "Es ist sogar fehlerhaft, indem man behauptet, in jeder hypothetischen Regel, die nur ein Subjekt hat, könne man die beiden Prädikate in eine kategorische Regel verbinden; z. B., 'Wenn Caius frei von Geschäften ist, so dichtet er.' Im Allgemeinen, wenn der ganze Vordersatz oder sein Subjekt mit dem Prädikat verbunden und nicht nur sein Prädikat der Grund im Satze ist, so geht diese Veränderung gar nicht. Noch willkürlicher sind die Veränderungen, wenn die hypothetische Regel zwei verschiedene Subjekte hat." See also Mansel's *Aldrich*, p. 239.

To illustrate various expression, one other example of this transference from the ideal to the real will suffice:

Were he to repent, he would be forgiven.

The apodosis is affirmed in case that the contingency expressed by the protasis become a fact. The whole is an ideal lying in the future.[4] Transforming the proposition, the ideal becomes real, the affirmation categorical:

He, repenting, will be forgiven.

These considerations recall a remark formerly made that an adjective word, phrase, or clause is the result of a previous judgment. We shall find hereafter that in the hypothetical proposition viewed as an inference, it is the middle term that becomes the adjective qualifier. Hence every categorical proposition having a qualified subject may be easily converted into an ideal statement; as follows:

A soft answer turneth away wrath.
If an answer be soft, it turneth away wrath.

The examples so far contain only three terms. Because they are reducible to simple categoricals, therefore, says Thomson, they are not true hypotheticals; the proposition of four terms, since it cannot be so reduced, is the true hypothetical. But let us test this:

If the wise are virtuous, Socrates was innocent;
i. e., The wise Socrates, who was virtuous, was innocent.

If a government is well administered, the people are prosperous;
i. e., If a government is well administered, it has prosperous people;
i. e., A well-administered government has prosperous people.

If there are spots on the sun, the needle is disturbed;
i. e., The needle is disturbed whenever there are spots on the sun.

That in some cases it is difficult, or even impracticable, to make such reduction is only because some connecting media, not contained in the

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[4] Let us note that the event of his repenting, being contingent, is doubtful, but not that he would in that case be forgiven. This may be said of Satan himself; but then the doubt could hardly exist, for we feel quite sure he will never repent, and hence, on that ground, will never be forgiven,—a near approach to the unreal statement. Burns, however, with bizarre tenderness, felt, or affected to feel, otherwise, doubting also the forgiveness even in the event of repentance:

"But fare you weel, auld Nickie-ben!
Oh wad ye tak a thought, an' men',
Ye aiblins might—I dinna ken—
Still hae a stake—
I'm wae to think upo' your den,
Ev'n for your sake!"

proposition, are obscure or unknown. But this does not differentiate these propositions as of another kind. Nor is it, as Thomson says, that conjunctives of four terms are always causal. Attributives of four terms, as the first example above, are very common, as well as reducible causals of three terms. Moreover, there is no reason why, in deductive Logic, the causal should be distinguished from the attributive judgment. In logical deduction all judgments are thought attributively, and cause and effect, so far as they are conceived in thought, stand to each other in the relation of reason and consequent. Objective cause becomes subjective reason.

§ 4. Let us consider now the contingent forms as propositions declaring an inference. The conjunctive proposition, as a whole, affirms a relation between the two subordinate propositions of which it consists. It expresses a judgment respecting judgments. It is logically a simple sentence. The apodosis is the subject, and the protasis the predicate.[5] The protasis (*προτείνειν*) is so named, and usually written first, because, as we shall hereafter show, it is in reality a premise, and hence the logical antecedent.

And what is the relation the conjunctive declares? This relation is invariable. It is the relation of consequence. The proposition declares that one judgment is consequent on, or follows from, another. Let it now be particularly observed that the affirmation is not only simple, but categorical; i. e., this relation is affirmed unconditionally. E. g.:

If virtue is knowledge, it is teachable.

Now strip this proposition of its hypothetical dress, and we have—

That virtue is teachable *is* an inference from the judgment that it is knowledge.

This is purely categorical. But not less is it categorical when presented in its hypothetical dress. The relation of the clauses is real.

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[5] The logicians generally invert this statement. But the subject, properly, is that of which something is said, which evidently here is the apodosis; e. g.:

If history is reliable, the latter days are the better days.

Here we are talking about the latter days being the better days, and we say quite simply that its truth is conditioned on, or follows from, the reliability of history. This relation of subject and predicate in conjunctives would be a little plainer if the usual form were, as in the present sentence, inverted, and stated thus:

C is D, if A is B;
The flowers will bloom, if the sun shines.

That the conjunctive proposition, having one clause as its subject, the other as predicate, and declaring the relation of consequence, is a simple categorical will perhaps be a little clearer if we look into the matter of the proposition, and consider wherein lies its material truth or falsity. When we say

If man is responsible, he must be a free agent,

we do not affirm the reality of his responsibility nor of his free agency. These are treated here as ideal. But we do affirm the real connection, the necessary coexistence of the two. Indeed, the force of the word "must" in the example is to declare the necessity of this consequence. That the conjunction of the two clauses, the dependence of one on the other, is all that is affirmed is still more manifest when we consider that the truth of this affirmation is entirely independent of the truth of the clauses. E. g.:

If the Koran came from God, Mohammed was the prophet of God.

The truth of this statement is indisputable, yet we hold both of the clauses, considered apart, to be false.

A false hypothetical is said to be one having a false condition. This, however, does not mean that the protasis is false, but that the affirmation of consequence is false, that the given condition is not the condition. Hence it would perhaps be better to say that a false hypothetical is one affirming as consequent what is inconsequent. E. g.:

If Moses was a lawgiver, he was very meek.

Here we may admit each clause separately to be true, but the proposition as a judgment respecting judgments is false; the one does not follow from the other. The concessive clause, granting a protasis, denies the consequence, thus pronouncing the hypothetical false; but it does more, it denies the apodosis also. E. g.:

(If our outward man perish, the inward man must fail.)
"Though our outward man perish, the inward man is renewed."

Since, then, the truth and acceptance of a conjunctive proposition lie wholly in the correctness of this single unconditional declaration of sequence, it is manifest that the statement, as a whole, is a simple categorical affirmation of this relation.

In the previous section it appeared that the conjunctive hypothetical in its first propositional use makes simply an ideal statement, and that the sole difference in thought between it and the correspond-

ing categorical judgment is that the former is ideal, the latter real; a difference that is non-logical. It now appears that the same hypothetical in its second propositional use, as declaring the relation of inference, is, in that regard, categorical and real. In a former chapter it was pointed out that the syllogistic judgment in the categorical or Aristotelic syllogism simply and solely declares consequence. Wherein then is the distinction between this and the hypothetical expression of inference? None appears beyond this, that in the syllogistic judgment the inference is from matter that is pronounced real, whereas in the hypothetical judgment the inference is from matter that is ideal. This difference, we repeat, is psychological, not logical. So far, then, we find no ground to justify a logical discrimination between categorical and hypothetical thought.[6]

Before advancing in our analysis, two remarks are worthy of place. First: We have pointed out the subject and predicate of the conjunctive; where is the copula? Many logicians call the conjoining particle, united with the verb "*to be*," the copula. Thus, say they, in conjunctives it takes, among others, these forms: "If —— then is ——;" "When —— then is ——;" "Where —— there is ——." In disjunctives these forms: "Either is —— or is ——;" "—— is either —— or ——." This is a confusing, and in Logic an improper, use of the word "copula." Let us rather say that the appearance of the copula in the conditional forms is grammatically inadmissible, but that it is implied by the conjunctive and disjunctive and illative particles.[7]

We remark, secondly, that while the common characteristic of the conjunctive and syllogistic judgments is, as above indicated, the affirmation of the sequence of dependence, it is not at all peculiar to

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[6] It may be well to note that immediate inferences are easily expressed ideally; e. g., "If ignorance is degrading, then something that degrades is ignorance." This is merely conversion *per accidens*.

[7] The word "*if*," which is the most usual grammatical characteristic of the conjunctive, is classed as a conjunction. But it was originally a transitive verb, having for its object the clause following it. As explained by Horne Tooke, in *Diversions of Purley*, it is the Anglo-Saxon *gif*, the imperative, second person singular, of the verb *gifan*, to give. Its original meaning, then, is "grant," "allow," "admit," "suppose," but is now equivalent to "provided that," "in case that," "should it be proved that," or "it follows from —— that." Thus:

If a man love me, he will keep my words.

That is, grant this premise, and the conclusion must follow.

them to be propositions respecting propositions. Like other things, propositions have a variety of attributes. In a conjunctive the attribute predicated is that of being an inference from another proposition. But this is only one of many attributes that may be predicated. E. g.:

That the whole is greater than a part is a mathematical axiom.
My belief is that with God time is an eternal now.
It is obvious that propositions may be either term of a predication.

§ 5. When in thought we use the protasis merely as a qualifier of a term of the apodosis, it is quite evident that we may reason from such judgments as premises whether reduced to categorical form or not. The only difference is, that when the judgments are in hypothetical form, we are then reasoning in the ideal mood; when reduced to categoricals, we think the matter as real. When, on the other hand, we view the propositions as declaring an inference, we may likewise reason from them as premises, and this judgment being categorical, its matter is real. We may understand "*if*" as representing the copula "*follows from*," and present a typical form, thus:

C is D if A is B;
E is F if C is D;
∴ E is F if A is B.

The order of nature is the product of benevolent design,
if it tends to promote moral good;
It must have had an intelligent and beneficent author,
if it is the product of benevolent design;
∴ It must have had an intelligent and beneficent author,
if it tends to promote moral good.

This, evidently, is Barbara. Returning to the usual form, the following example is only in part hypothetical:

If the using of credit is a demand for goods,
all forms of credit affect prices;
But bills of exchange are a form of credit;
∴ If the using of credit is a demand for goods,
bills of exchange affect prices.

This, also, is Barbara. We call attention to its easy solution by the Canon of Replacement.

It is manifest, then, that, so far, we discover no new principles, and hence need no new rules or forms. These examples may properly be called Conjunctive Hypothetical Syllogisms, and so distinguished from

the purely categorical forms; but the difference is evidently not a logical difference.

Let us at once extend the view to other forms of hypotheticals. The disjunctive proposition, which, as we shall hereafter show, is compounded of conjunctives, and therefore subject to the same treatment, may, however, be considered as a simple categorical affirmation, either predicating alternatives, or predicating a mark of alternatives. So, then, we may have Aristotelic syllogisms formed of disjunctives, and such are true Disjunctive Hypothetical Syllogisms. E. g.:

Memory is either circumstantial or philosophic;
Also it is either voluntary or spontaneous;
∴ In this case, what is either voluntary or spontaneous
is also either circumstantial or philosophic.

This is Darapti. The following consists partly of disjunctives. It is evidently Aristotelic; but its reduction to strict logical form, thus determining its mood, is quite a complex process. Its solution by replacement is, however, obvious and easy:

Desires are either spontaneous or voluntary;
But whatever is voluntary has moral quality;
∴ Desires are either spontaneous, or they have moral quality.

Since the Dilemmatic proposition is a compound, a conjunctivo-disjunctive, it is subject to the same view, and we may have Aristotelic syllogisms involving it. E. g.:

If a ruler makes an entirely unselfish use of despotic power,
he must be either a saint or a philosopher;
But saints and philosophers are rare;
∴ Those rulers who so conduct themselves are rare.

There are, of course, Enthymemes, comprising hypotheticals. E. g.:

If matter is essentially inert, there must be a higher
moving power, and this implies a governing will.

So, also, we may have Epichiremas, comprising hypotheticals. E. g.:

If government has a right to enforce the laws,
and without this it could not subsist,
then it has a right to use military force against its own citizens,
for in extreme cases this may be requisite;
If so, then government has a right to inaugurate civil war,
since civil war is the likely result of such use
of military power, counter to the right of revolution;
∴ If a state has a right to enforce its laws,
it has the right to inaugurate civil war
for the suppression of revolution.

A series of hypothetical syllogisms formed of conditional propositions may be abridged into a Sorites. E. g.:

If the Scriptures are the word of God, they should be clearly explained;
If they should be clearly explained, they must be diligently studied;
If they must be diligently studied, an order of men must be devoted to them;
∴ If the Scriptures are the word of God, an order of men must be devoted to them.

This is purely Aristotelic reasoning. Had we affirmed—

But the Scriptures are the word of God;
∴ An order of men must be devoted to them—

the forms would be mixed, the last step being the so-called hypothetical syllogism, in the ponent mood. Finally, we may construct a Sorites consisting of disjunctives, wherein the reasoning is strictly Aristotelic. The following is partly of this character, and involves a prosyllogism:

Every science is either pure or inductive;
A pure science, since it treats of the necessary forms
either of thought or of imagination,
is either logical or mathematical;
A mathematical science is either exact or worthless;
The science of probabilities is neither logical nor exact;
∴ It is either inductive or worthless.

The reasoning in all these cases turns upon the categorical affirmation of sequence alone. Hence it is strictly of Aristotelic form, comes under its moods, and is subject to its Canon and rules. Logic, then, cannot distinguish these as kinds of reasoning, as different forms of thought.

§ 6. But the conjunctive proposition, viewed as declaring an inference, implies within itself a reasoning. The affirmation of sequence is a characteristic common to it and the syllogistic judgment. The protasis is a condition or logical antecedent of the apodosis; in other words, it is a premise, and the apodosis is a consequent, or conclusion.

Now, whether a conjunctive is thought thus, or merely as a qualified proposition, can, in general, be ascertained only by considering the matter and the context. In pure Logic it is, of course, undetermined. Let us illustrate:

If air is pure, it is wholesome.

This, probably, in the minds of most persons who do not receive it upon mere testimony, is a direct induction from observation or ex-

perience, and, though capable of being construed syllogistically, is with them a simple judgment, not expressive of any reasoning whatever, but equipollent with—

Pure air is wholesome.

But in this example,

If the moon has no atmosphere, it has no twilight,

there would seem to be a reasoning implied; the apodosis being necessitated by the protasis standing under some general rule, such as:

Atmosphere is essential to the phenomenon of twilight.

The reasoning thus implied may be expressed in full as follows:

(An orb that has no atmosphere has no twilight;)
now, If the moon has no atmosphere,
it follows that The moon has no twilight.

We have, then, in this given condition or protasis an ideal minor premise, yielding an ideal conclusion, the apodosis. It is manifest, therefore, that the contingent conjunctive hypothetical proposition declaring an inference is a simple Ideal Enthymeme.[8]

It has already been indicated that we may reason in the ideal sphere of thought as well as in the real, and that the principles are precisely the same. We may pass from the one to the other; from the real to the ideal in every case; from the ideal to the real, if we have ground. In the last example we have a major real, and pass to an ideal minor and conclusion. We may readily transfer a reasoning totally from the real to the ideal. Thus, it is easy and proper to say

If all men are mortal,
and If Plato is a man,
then Plato is mortal.

This throughout all of its propositions is purely ideal.

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[8] A varied but quite correct view of the conjunctive hypothetical is as follows:

It is merely an affirmation of necessary sequence. But upon what does this sequence depend? Upon the existence (in the Form 1 a) of an unexpressed major, under which, as a general rule, the ideal minor would come as a special case. To affirm the sequence is only to affirm indirectly this major; to prove it is to establish an unexpressed premise. For example:

If virtue is knowledge, it is teachable.

Do you admit this? Yes. Then that is merely to say that you admit "All forms of knowledge are teachable." Hence this hypothetical conjunctive affirms a mental judgment, which, taken as a major, would necessitate the consequent.

CALIFORNIA.

Let us now follow the several conjunctive forms (v, § 2), and, regarding them as ideal enthymemes, explicate the syllogisms implied. It should be observed that the unexpressed premise in each case consists of terms not common to the two clauses.

1 (a), If A is B, A is C; e. g.,		If man is responsible, he must be free.
(B is C)		(The responsible must be free;)
If A is B	(**Barbara.**)	If man is responsible,
then A is C.		Then man must be free.

1 (b), If A is not B, C is not A; e. g.,		If bliss has no anxieties, ignorance is not bliss.
If A is not B		If bliss has no anxieties,
and (C is B)	(**Cesare.**)	(And ignorance has anxieties,)
then C is not A.		Then ignorance is not bliss.

One clause, at least, must be negative, else undistributed middle.

1 (c), If A is B, B is C; e. g.,		If rubies are clay, some clay is precious.
(A is C)		(Rubies are precious;)
If A is B	(**Darapti.**)	If rubies are clay,
then B is C.		Then some clay is precious.

The apodosis must be particular; else illicit minor.

Variations in quantity and quality in the above will yield the other moods of the several figures.

1 (d), If A is B, C is B; e. g.,		If the metals are fusible, gold is fusible.
If A is B		If the metals are fusible,
and (C is A)	(**Barbara.**)	(And gold is a metal,)
then C is B.		Then gold is fusible.

We might have expected this to yield Fig. 4. Bramantip, but its direct resolution into the first figure confirms the rejection of the fourth. In 1(a) the minor premise is given; in 1(d) the major.

We reach now the second form, having four terms, and hence no common term. For the sake of symmetry we rearrange the letters.

2. — If B is C, A is D; e. g.,		If the wise are virtuous, Socrates was innocent.
(A is B)		(Socrates was wise;)
If B is C	(**Sorites.**)	If the wise are virtuous,
and (C is D)		(And the virtuous are innocent,)
then A is D.		Then Socrates was innocent.

It is evident there is no new principle involved here. The proposition is an ideal enthymeme. Supply the mental premises, and it falls at once into an established form.

In this last example all the requisite middle terms are given. Clauses may, however, be logically so remote from each other that several, perhaps many, intermediate links must be supplied to complete the chain. This it may not be easy to do, unless the unexpressed media are obvious. It is the part of the speaker or writer to furnish these to us. He may, at the outset, as preparatory, lay down his chief premise hypothetically, connecting it at once ideally with his ultimate conclusion, and then proceed to supply the media. E. g.:

> If the desire for distinction is an essential stimulus to industry,
> then communism is antagonistic to the progress of civilization.

Here arguments might be needed to establish the antecedent, and perhaps a long series to show that it necessitates the consequent. So, also, we might say, "If the tenth proposition of Euclid is true, then the one hundredth is true also."

As an actual example of the matter before us, we will quote a passage from Locke.[9] He is speaking contemptuously of the Art of Logic and of the syllogism, saying, "God has not been so sparing to men to make them barely two-legged creatures, and left it to Aristotle to make them rational." He then tries to show that logical forms are worse than useless, being confusing. The passage is curious as an effort to overthrow that which it uses, and therefore unwittingly acknowledges. He says, "To infer is nothing but by virtue of one proposition laid down as true to draw in another as true." This he illustrates by the following example: "If men shall be punished in another world, then men can determine themselves." He then remarks, "What is it that shows the force of this inference, and consequently the reasonableness of it, but a view of the connection of all the intermediate ideas that draw in the conclusion? . . . The mind, seeing the connection there is between the idea of men's punishment in another world and the idea of God's punishing; between God's punishing and the justice of the punishment; between justice of the punishment and guilt; between guilt and a power to do otherwise; between a power to do otherwise and freedom; and between freedom and self-determination, sees the connection between men and self-determination. Now, I ask whether the connection of the extremes be not more clearly seen in this simple and natural disposition than in the perplexed repetitions and jumble of five or six syllogisms?" It is

[9] *Essay on the Human Understanding*, bk. iv, ch. xvii, "Of Reason."

very clear that, in decrying logical form and showing us the "simple and natural" way, he has developed the hypothetical enthymeme into a progressive sorites, stated so nearly in strict logical form that redressing is needless.

It has now been shown that all the reasoning founded on or implied in the contingent hypothetical is thought strictly in the form of the Aristotelic syllogism. The only distinction is that one is ideal, the other real. We now add that viewed as a conditional proposition, apart from its ideality, it differs from the categorical only in that the latter does not *express* a condition. But, in fact, every logical proposition is a conclusion conditioned on its premises; so all reasoning is conditional reasoning. The conditional character may not appear in expression, but it belongs to all thought. It adheres to every possible judgment except the primitive or intuitive; but then this is not thought. A judgment truly unconditional neither requires nor is susceptible of proof; it cannot appear as the conclusion of a syllogism. Or, in other words, every syllogism is a conditional judgment in which the premises are the antecedents and the conclusion the consequent. So, then, the distinction between categorical and conditional, which did not originate with Aristotle,[10] is a mere accident of expression, ought never to have been introduced, and ought to be dismissed from Logic. The distinction between categorical and hypothetical propositions belongs to psychology, is of no logical moment, and ought also to be discarded.[11]

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[10] See Part 3d, i, § 7, note.

[11] "Of the truth or falsehood of propositions, in themselves, Logic knows nothing, and takes no account; all in Logic may be held true that is not conceived as contradictory. In reasoning, Logic guarantees neither the premises nor the conclusion, but merely the *consequence* of the latter from the former; for a syllogism is nothing more than the explicit assertion of the truth of one proposition, *on the hypothesis* of other propositions being true in which that one is implicitly contained. A conclusion may thus be true in reality (as an assertion), and yet logically false (as an inference).

In a certain sense, therefore, all logical inference is *hypothetical*, hypothetically necessary; and the hypothetical necessity of Logic stands opposed to *absolute* or *simple* necessity. The more recent scholastic philosophers have well denominated these two species the *necessitas consequentiæ* and the *necessitas consequentis*. The former is an *ideal* or *formal* necessity; the inevitable dependence of one *thought* upon another, by reason of our intelligent nature. The latter is a *real* or *material* necessity; the inevitable dependence of one *thing* upon another because of its own nature. The former is a logical necessity, common to all legitimate *conse-*

§ 7. The hypothetical forms expressing the ideal unreal are now to be considered. These are always in the past tenses of the subjunctive mood. In *usus loquendi*, the meaning which they convey is always to deny the reality (hence "unreal") of the thought, and, thus, is always indirect. There seem to be of these also two uses,—either indirectly to declare a fact, or indirectly to declare an inference.

We exemplify at once the first use:

Were he king, he would tyrannize.

That is to say, he is not king, and he does not tyrannize.

If it were not so, I would not say it.

That is to say, It is so, and I do say it. Thus the ideal case supposed is denied as a fact, which is to posit its opposite. The apodosis makes its statement contrary to, or in spite of, the real fact, which is thus indirectly declared.[12]

But this denial of the supposition is not the ultimate purport. Yet more indirectly, these propositions convey quite another meaning. They seem to be a rhetorical or grammatical or linguistic device for saying something emphatically, quite aside from and beyond what the words directly express, and which it would perhaps be difficult to state directly. For example:

Were he here, I would tell him.

It is clearly implied that he is not here, and I do not tell him. But to state these patent facts is not the object of my saying this. The purport seems to be *to declare my state of mind*, perhaps to justify myself in reference to some matter in question. So, briefly, and indirectly, but quite emphatically, I mean to affirm that I am so disposed and determined in this case that no circumstance whatever prevents my action now, except the obvious one of the absence of the object of action; my mind is fully made up, all questions settled, and there is no other external fact I know of to hinder me from thus and so; now you know what I think and feel and will about it. Observe

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quence, whatever be the material modality of its objects. The latter is an extra-logical necessity, over and above the syllogistic inference, and wholly dependent on the modality of the matter *consequent*. This ancient distinction, modern philosophers have not only overlooked but confounded."—Hamilton, *Discussions*, p. 146.

[12] The past tense of the subjunctive in the subordinate clause of a categorical proposition has the same force of denial. E. g., "I would I were a boy" implies that I am not a boy.

that the denial implies more than we first stated. In full it should be, "He is not here, and *therefore only* I do not tell him." This is the sole condition, the *only* reason why I do not tell him. In my present disposition, then, there is none.

Just so in the former example, "If he were king, he would tyrannize," the meaning is "He is not king, and for this reason only he does not tyrannize," thus declaring indirectly his tyrannical disposition. Again, "If it were not so, I would not say it," affirms my truthfulness. In the following example,

> Were this beam not rotten, it would serve,

we think of the beam as rotten and unserviceable, but mean primarily and chiefly to affirm the suitableness of all its other unnamed qualities. In the trite proverb

> If wishes were horses, beggars would ride,

we say something indirectly, rhetorically, about beggars' vain longings, but still more indirectly, in the application of the saw, we mean to rebuke extravagant aspirations. In Macbeth's speech,

> If it were done when 'tis done, then 'twere well
> It were done quickly,

he means not merely, That a deed outlives itself, must give us pause; but rather he means to justify his hesitation. Gay's couplet,

> How happy could I be with either,
> Were t'other dear charmer away!

is a palimpsest of enamoured distraction. So far the unreal present. In the unreal past, we have,

> If thou hadst been here, my brother had not died,

which is indirectly a strong expression of confidence in superhuman power and love. Might, could, or should, in the apodosis, modifies the meaning, referring the matter to possibility, ability, or duty.

Propositions of this sort must be treated logically with reference to the primary, fundamental, unexpressed meaning, and not to the ostensible ideal statement, nor to the negation of fact, which are secondary, and, taken apart from the primary though more indirect intent, are generally senseless. They are, then, to be viewed and interpreted as simple categorical judgments.

Turning now to the unreal proposition declaring an inference, we find it presents a further peculiarity. Let us recall that the denial of

a consequent or conclusion denies an antecedent, one or more of the premises that necessitate it. Then let us consider the following:

Whoever talks so must be crazy;
Diogenes talks so;
∴ Diogenes must be crazy.

Any one having this reasoning in mind may prefer, for emphasis perhaps, to state it indirectly. By expressing ideally a denial of this mental conclusion, he denies ideally the fact of his minor, a denial of a granted fact, and hence professedly false, and thus indirectly he affirms his conclusion. Thus:

Were Diogenes not crazy, he would not talk so;

meaning, But since he does talk so, therefore he must be crazy. So by this device, this custom of language, we mean to declare the opposite of our words. Our reasoning is consciously and intentionally unreal, and goes to establish its opposite.

For further illustration we renew our old familiar example,

If Plato be not mortal, he is not a man.

Here the matter is stated ideally as a mere contingency, as formally questionable. But in

If Plato were not mortal, he would not be a man,

the matter is stated as absolutely unreal, thereby declaring emphatically, without saying so, that

Since Plato is a man, he must be mortal.

But in the following affirmative example we express ourselves more fully, the purposed conclusion being distinctly stated:

No rain has fallen; for if there had, the ground would be wet.

It will be observed that this is essentially the *reductio ad absurdum*, as is sufficiently manifest in the following examples:

If ignorance were bliss, 'twere folly to be wise.
Were all the prosperous happy, then some discontented would be happy. (See example in iv, § 3.)

These conclusions are evidently self-contradictory and absurd. Hence the contradictory of their antecedents is true. Again:

Were Christianity not from God,
it would not have been accompanied by credible miracles;
Were its miracles unworthy of credit,
they would not have been attested in the manner
in which it has been proved they were. (See the argument in iv, § 4.)

The formal *reductio ad absurdum*, appropriately called the method of indirect demonstration, is conveniently, elegantly, and usually stated in these ideal unreal forms. For an instance refer to iv, § 8, ex. 51.

We conclude that the ideal unreal form of the hypothetical proposition, when declaring an inference or offered as proof, reasons indirectly. By an ideal denial of an unexpressed conclusion, it denies an unexpressed but unquestionable premise, which denial, being absurd, impliedly affirms the truth and reality of that conclusion.[13]

§ 8. It is needful now to revert to the form commonly known in Logic as the hypothetical conjunctive syllogism (v, § 5). Aristotle ignores all forms of the so-called conditional syllogism. In one place in his *Analytics*, however, he describes the process now known as the hypothetical syllogism, but denies that it is a syllogism.[14] He was right. Conditional syllogisms were nevertheless introduced into Logic by his immediate successor in the Lyceum, Theophrastus, were accepted by his rival Eudemus, and were adopted by the Stoics. They have received the sanction, in one way or another, of nearly all logicians down to the present time. Especially were they endorsed and developed by Boethius, and his great authority has given them a permanent place in Logic. Still there has been a continual wrangle about the details of the system, betraying a deep dissatisfaction, although their right to be considered special modes of reasoning has hardly been questioned. The admiring commentators of Aristotle have generally felt it needful to apologize for the hiatus which his disregard of them makes in his *Analytics;* excepting, however, Saint-Hilaire, who, in his translation of the

[13] I have nowhere seen a development of the matter contained in this and the previous sections, nor, indeed, of the views presented throughout this general discussion. Hardly a hint is to be found in our Logics. Arnauld in a single sentence speaks of the enthymemic character of conditionals. Mansel (*App. to Aldrich*, p. 240) writes two sentences in which the doctrine glimmers. The most explicit statement I have encountered is from Titius (*Ars Cogitandi*, ch. xii), as follows: "Conditionalis seu hypotheticus nihil aliud est quam enthymema vel sine majore vel minore." "Syllogismus disjunctivus est enthymema sine majore." "Sequitur nullum peculiare concludendi fundamentum vel formam circa syllogismos conditionales occurrere, nam argumentationes imperfectas, adeoque materiam syllogismorum regularium illi continent." My own views were worked out before this caught my eye, but it seems they are not altogether new.

[14] *Anal. Prior.* i, 32, 7. "If because man exists, it is necessary that animal should be; and animal existing, that there should be essence; then, because man exists, essence must necessarily be. But this is not yet syllogistically inferred, for the propositions do not subsist as we have said they should."

Organon, insists that they are therein recognized.[15] Emboldened by this generally admitted silence of Aristotle, let us question their title, and judge whether the Stagyrite did his work only by half.

A number of modern writers on Logic, recognizing hypothetical syllogisms as distinct modes of reasoning, endeavor in various ways to show that they may be reduced to Aristotelic forms. But are they reasonings at all? We recall that deductive inference is of two kinds, mediate and immediate. In mediate inference we determine the relation of two notions through a third, the middle or medium. A syllogism is the formal expression of this mediate process, and hence a middle term is its essential feature. Now hypothetical syllogisms, so called, contain no middle term. Therefore they are not syllogisms, not expressive of reasoning at all. Inspect the following:

	If law prevails, our rights are secure; = *Major Premise.*
Modus Ponens.	But law does prevail; ..... = *Minor Premise.*
	∴ Our rights are secure..... = *Conclusion.*

There is no term here with which the two terms found in the conclusion are compared in the premises. There are in all four terms, and all found in the so-called major premise. The so-called minor introduces no new matter, and has nothing in common with the conclusion, as necessarily occurs in the true syllogism.

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[15] "Aristote n'a pas omis davantage les syllogismes hypothétiques, dont on a voulu faire honneur encore à ses élèves Théophraste et Eudème. Les syllogismes hypothétiques sont ce qu' Aristote appelle les syllogismes d'hypothèse, de convention. Il en avait traité tout au long dans un ouvrage que le temps nous a ravi, mais que lui-même mentionne dans le Premiers Analytiques, i, 44, 4."—*Logique d'Aristote*, Préface, p. lx. St.-Hilaire then proceeds to a discussion. See, also, tome iv, top. i, 8, 9. He has against him, however, Waitz (see *Comment.* on *Anal. Prior.* i, 44) and Hamilton (see *Discussions*, p. 151). For references to other authorities, see Hamilton's *Logic*, p. 613, note; and Grote's *Aristotle*, p. 243, note.

In the passage above referred to by St.-Hilaire, Aristotle promises to treat at some future time of Syllogisms from Hypothesis, but more probably the treatise was never realized, as there are no extant references to it. Against St.-Hilaire it can be proved that by Syllogisms from Hypothesis Aristotle meant the various forms of the *Reductio ad impossibile*, and not at all what are now known as Hypothetical Syllogisms. Moreover, the historical fact already stated, that Theophrastus changed the Aristotelic sense of the term "categorical," which was simply "affirmative," to the sense opposed to "hypothetical," is evidence that he, and not his master, was the inventor of the hypothetical system. I have not seen the point mentioned, but the change seems clearly to indicate that Aristotle had no such opposed term, and that Theophrastus found a special need for one to mark a new distinction.

Impressed by the absence of a middle term, Kant declared these pseudo-syllogisms to be forms of immediate inference. Now, immediate inference is merely from a given judgment to infer directly, i. e., without a medium, a different judgment. Let us inspect the above example presented in a slightly different form:

If Law prevails, then our rights are secure.
Law prevails, then our rights are secure.

Now here is an absolute iteration of thought, stated first as suppositious, then as assertorial. The subject is the same. The predication is the same. The second judgment, then, is not different logically from the first, and therefore this cannot be an immediate inference.[16] Another example, to vary the forms:

If my debtors are honest, they will repay me;
(1) My debtors are honest, they will repay me;
(2) Some are honest, some will repay me;
(3) This one is honest, he will repay me.

In 2 and 3 there is a diminution of quantity. Is not either of these an immediate inference from the major premise by subalternation? No; for subalternation concludes that "some are," because "all are;" which is not here the case, since we may be able to affirm 2 or 3, when 1 (all) is not true.[17]

If, then, these forms are not inferences of either kind, what are they? Three views are possible. First, they are forms of speech indicating *a transfer from the ideal to the real* mode of thought. It has been already observed that we cannot pass from the ideal to the real without some ground. We may say, ideally, If law prevails, a certain consequence follows; but whether law does really prevail, or not, is not determined by anything in that proposition. We must seek ground for the affirmation elsewhere; and when discovered, then, but not until then, can we pass to the real, and assert—Law prevails. Now, by virtue of this discovered ground we can declare the conclusion, already stated ideally, to be real also—Our rights are secure. The discovered ground may not be sufficient to establish the reality

[16] Let us be reminded that progress from doubt to certainty is a change in conviction, in degree of belief, in feeling, but is not a change in thought.

[17] In the treatment of these forms Hamilton wavers. In his *Lectures* he accepts the old doctrine. In his latest note (*Logic*, p. 603) he almost reaches the point of rejecting them, saying, "If inferences at all, they are immediate, and not mediate." See also his note in *Discussions*, p. 151.

of more than a part; if so, we can conclude the reality of a part only —Some of my debtors will repay me. Here it appears that the hypothetical conjunctive proposition is the ideal enthymeme; that the so-called conjunctive syllogism is not a syllogism at all, nor expressive of reasoning or inference of any kind; that it merely reiterates the enthymeme as real; that it indicates a transfer from the ideal to the real on unexpressed grounds; that it is simply a formal mode of announcing the ideal premise established as real, in whole or in part, and the consequent reality of the conclusion. The reasoning implied is purely Aristotelic, and is duplicated in the two enthymemes.

A second view considers the conjunctive proposition as merely an affirmation of sequence, its second propositional use. In this view the so-called syllogism consists of three propositions. The conjunctive affirms the necessary coexistence of the other two judgments, or, better, it affirms only a consequence from one to the other. One of these affirms categorically the existence, in whole or in part, of one fact. The other infers the existence of another fact.

x is; but if x is, y is; then y is.

Here again is an enthymeme. In this view, however, the enthymeme lies solely in the two categorical judgments, but is strengthened by a distinct affirmation of their necessary sequence. The reasoning, then, lies not at all in any inference from the hypothesis to the assertion, but wholly in the relation of the two categorical judgments as premise and conclusion. This reasoning is purely Aristotelic.

A third view is that the conjunctive proposition affirms indirectly an unexpressed major premise.[18] In this view the so-called hypothetical syllogism affirms the three real propositions of a categorical or Aristotelic syllogism. It is not now an enthymeme, unless the indirectness of the major be held to bestow this character, and not the slightest ground appears on which to distinguish it as a special form or mode of reasoning.

It follows that the axiom of Sufficient Reason[19] is an entire superfluity in Logic. The three Primary Laws, and the rules evolved from them, are all-sufficient; for every case of a violation of the axioms of Reason and Consequent will be found, on developing the enthymeme, to be a violation of one or another of the general rules of the Aristotelic syllogism. Hamilton latterly suspected that the Platonico-Leibnitzian Law was out of place in Logic, and Mansel definitely

[18] See *supra* foot-note 8.

[19] See Part 1st, ii, § 7.

reached this conclusion. There can be no doubt that it should be relegated to the realm of Metaphysics, whence it was drawn.[20]

§ 9. It remains to indicate more explicitly that disjunctive and other compound conditional propositions are merely enthymemes. We here speak of disjunctives as compounds, and it is easy to show that they are so. Every disjunctive having two subcontrary members consists of two hypotheticals, which may be explicated thus:

C is either D or E; e. g., God is either loved or feared;
yields If C is not D, C is E; e. g., If God is not loved, he is feared;
and If C is not E, C is D; e. g., If God is not feared, he is loved.

When the opposition is contradictory, as in "God is either trustworthy or untrue," the analysis yields four hypotheticals, the two others being:

If C is D, C is not E; e. g., If God is trustworthy, he is not untrue;
If C is E, C is not D; e. g., If God is untrue, he is not trustworthy.

Now, the disjunctive proposition being merely a double or quadruple hypothetical, it follows that what has been proved of hypotheticals is true of it. Moreover, it is easy to show that the so-called disjunctive syllogism is merely a reiteration of the enthymeme expressed by one or another of these constituent hypotheticals. Thus:

C is either D or E explicates into

(What is not D is E;)		(What is not E is D;)
If C is not D;	—and—	If C is not E;
then C is E.		then C is D.

These two simple syllogisms in Barbara or Darii correspond to the *Modus Tollendo Ponens.* In case of contradictories, we have also:

(What is D is not E;)		(What is E is not D;)
If C is D;	—and—	If C is E;
then C is not E.		then C is not D.

These two latter syllogisms in Celarent or Ferio correspond to the *Modus Ponendo Tollens.* It appears, then, that the disjunctive proposition condenses or involves in one compound statement two or four hypothetical enthymemes; and that the pretended disjunctive syllogism is merely a restatement or explication of some one of these enthymemes either as ideal or as real.

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[20] See Hamilton's *Logic*, p. 62, and note; also p. 251. See also Mansel's *Aldrich*, note p. 235; and *Prolegomena Logica*, p. 193.

The conjunctivo-disjunctive proposition is an acknowledged compound, and the dilemma is obviously made up of conjunctives and disjunctives. It is needless to trace the principle through these intricate forms. It may be well, however, to observe that the former is merely a disparate disjunctive proposition, one member of which has been reduced to the conjunctive form. E. g.:

Man must be either capable of progress, or a brute, or a divinity.
If man is incapable of progress, he must be either a brute or a divinity.

§ 10. Ought not, then, these conditional forms, these pseudo-syllogisms, to be banished from Logic? By no means; for they are true, natural, and very common modes of expressing thought, and hence call for logical analysis and treatment. Nothing is more common than for a reasoner at the outset to state hypothetically his premise and conclusion. This he does for the sake of clearness, and to show whither he is tending. E. g.:

If the prisoner was sane, then he is responsible for his act.

His first argument may be to show the necessity of the sequence herein declared. As accusing counsel, he next endeavors to establish this antecedent minor, perhaps by showing the deliberation of the agent, his consistency, his motives, etc., etc.; and, it may be, he brings in medical evidence. When the argument is complete, he closes by declaring categorically:

The prisoner was sane, therefore he is responsible for his act.

Hence Hamilton, in one place, proposes to call the various conditional forms "preparations for argumentation."

Again, many of these conditional forms present exceedingly condensed expressions of reasonings through which the mind darts with rapidity, and unless the thinker is familiar with their analysis, he is in danger, especially in the more intricate dilemmatic forms, of paralogism, or of being imposed upon by sophism. Hence these were favorite forms with the Greek Sophists, and indeed are still preferred by all who wish to make the worse appear the better reason. On the other hand, their condensation gives to a just argument weight, and logical and rhetorical force. They should, then, be discussed, not only as subjects of analysis, but also because of the practical advantage resulting from their close examination.

It is clear, however, that their nomenclature ought to be changed. The unfortunate misapplication of the terms "syllogism," "major and

minor premise," "mood," etc., etc., and the attempt to enunciate rules and methods of reduction parallel to, but distinct from, those of the true syllogism, has filled Logic for centuries with confusion and error. But so deeply rooted in logical literature, and so widely spread is this false system and terminology, that the needed correction can be made only by the highest authority.

It is a great satisfaction, however, to say that the omission by Aristotle of any treatment of conditionals, so far from calling for apology, may be adduced as an evidence of the profound and thorough character of his Analytics. Logicians should respect the silence of the master, and when its significance is not clear, it would be well and modest to imitate it.

To sum up: There are but two kinds of deductive inference, the immediate and the mediate. The analysis of Aristotle is limited to these kinds. The various forms of conditional propositions are essentially hypothetical conjunctives, or ideal enthymemes. There is no such thing as conditional reasoning distinct from categorical; but all conditional is categorical, and all categorical is conditional. The so-called conditional syllogisms are not syllogisms at all, nor inferences of any kind; but are mere reiterations of the enthymeme as real. They do not, therefore, require a distinct system of rules and forms, but rightly take their places under the Aristotelic system, which is an exhaustive analysis of deductive thought.

PART FIFTH.—OF FALLACIES.

I. DISTRIBUTION.

§ 1. The Primary Laws of Thought, whose consequences have been expounded in the foregoing pages, are derived from, or formulated in accordance with, the ultimate original constitution of mind. They are necessary; that is, their contradictories are inconceivable, they cannot be doubted or questioned by the human mind. It follows that mental processes and results in strict conformity with them are equally necessary in the same sense. But these Laws are not necessary in the sense that they must perforce be obeyed. Mental processes do not necessarily conform to them. They declare how we must think, if we think consecutively; but they are not inviolable. Our thoughts are not determined in their course, like the planets, by inexorable forces. The planet has no choice. Laws of thought are impressed upon our mental constitution just as laws of health are impressed upon our physical constitution. The latter we may consciously or unconsciously disregard, but the inevitable consequence is disease; the former we may likewise disregard, but only to incur the deadlier consequence of error and folly.

A System of Logic, a Theory of Thought, is complete on its positive side, in showing how we do and must think, if we think correctly and fruitfully. But this cannot be, without contemplating at the same time the possibility of error, and modes of incorrect thinking. The Law of Relativity declares that every notion has its opposite, that the notion of truth implies the notion of error, that the notion of correct, regulated thought implies the notion of incorrect, unregulated thought. If all objects were white, and of the same shade, none would be distinguishable. Hence the scholastic maxim: *Contrariorum eadem est scientia.* We cannot consider the observance of a law apart from its violations; the one implicates the other. When good reasoning is exhibited, bad reasoning must be conceived as at least possible, else the good cannot be conceived as good. "According

to old definitions," says De Morgan, "bad reasoning is a reasoning, *syllogismus sophisticus* is one kind of syllogism, and in a certain old book the fruits of demonstration are, science, opinion, and ignorance, the latter derived from *bad* demonstration, what we would now call *no* demonstration." Hence, all along through the present treatise, it has been necessary, in showing the methods of correct reasoning, to glance at the incorrect. Examples violating the rules have frequently been given. But as our view has been steadily fixed on the positive side of the theory, the negative side, or incorrect thinking, has been very imperfectly developed. To the satisfactory completion of our task it is needful now that we take a comprehensive and systematic view of the violations of the Laws of Thought.

If any further justification were needed for adding to our treatise a discussion of Fallacies, it might be found in the valuable practical results following the study of them. It contributes greatly to a habit of clear and logically consecutive thought, that one be familiar with the various dangers that threaten it, with the slips to which it is inclined, with the snares which environ it. Error, seen to be error, is harmless; it is only when in the guise of truth that it is dangerous. But error, thus disguised, abounds, and a practical skill in detecting and exposing it is of inestimable value. So important is this considered, that, while Logic might justly confine itself to very simple illustrations of the violations of its rules, it is customary to extend the examination to quite intricate and difficult cases, and to consider many varieties of error.

Moreover, if it can be shown, as we progress, that all kinds of fallacious thinking are at bottom violations of established logical rules, it will go far to confirm the doctrine of this treatise, that the Aristotelic syllogism is the unit of all mediate thought.

§ 2. Bacon was the first philosopher who attempted a systematic enumeration of the various sources of human error.[1] He made of them a quaint classification into four genera, under the significant name of "Idols" (εἶδος, *an image*), in the sense of illusions, described as if presented in a magic mirror. He says: "I do find, therefore, in this enchanted glass four idols, or false appearances, of several distinct sorts, every sort comprehending many subdivisions." These he enumerates as follows:

[1] *Novum Organum*, lib. i; Summary of Part ii; Aphorism 38 sq.

Idola tribus; Idols of the nation or tribe, to which, from certain common weaknesses of human nature, we are universally liable.

Idola specus; Idols of the den or cave, which, from the peculiar dispositions and circumstances of individuals, mislead them in different manners.

Idola fori; Idols of the forum, public assembly, or bar, arising from the current usage of words which represent things much otherwise than as they really are.

Idola theatri; Idols of the theatre, which false systems of philosophy and erroneous methods of reasoning have introduced.[2]

The intellect, therefore, may be perverted by mixing with pure reason our gregarious affections, or our individual propensities; the false suggestions involved in language, or the imposing delusions of received theories. Bacon declares that the doctrine concerning these Idols bears the same relation to the interpretation of nature as the doctrine concerning sophistical paralogisms bears to deductive Logic. Whewell, however, thinks that his precepts concerning these Idols "have little to do with Natural Philosophy."[3] And moreover the class *Idola fori*, the snares of language, corresponds pretty nearly with Aristotle's *Fallacies in dictione.*

§ 3. The next most notable attempt at a classification of error is that of Mill.[4] He uses the word "fallacies" to include all kinds of intellectual error, and discovers five genera:

1. Fallacies *a priori;*—Errors in simple inspection, arising from natural prejudices.

2. Fallacies of Observation;—Errors in the ground of induction, arising from either mal-observation or non-observation of the facts.

3. Fallacies of Generalization;—Errors in the process of induction, arising from a misconception of the legitimate mode of drawing conclusions from observed facts.

4. Fallacies of Ratiocination;—Errors in argumentation, provided against in the rules of the Syllogism.

5. Fallacies of Confusion;—Errors arising from evidence being conceived in so indistinct a manner as not to produce any clear consciousness of the means by which the conclusion is reached.

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[2] See Hallam's *Literature of Europe*, Part iii, ch. iii, §§ 58, 59. Read, also, the admirable chapter xx, Part 3d, of the *Port-Royal Logic*, on "Sophisms common in Civil Life."

[3] *Philosophy of Discovery*, ch. xv, § 20.

[4] *Logic*, bk. v, ch. ii.

Nos. 2 and 3 are Inductive Fallacies; No. 4, Deductive; No. 5, a kind of *omnium gatherum* of sorts and cases that do not come under one of the other heads. It occupies the whole ground included by Aristotle's *Fallacies in dictione*, and *extra dictionem*. It will appear, however, in the sequel, that these also are Deductive Fallacies, violating syllogistic rules.

§ 4. The arrangement adopted in most English manuals of Logic is that of Whately.[5] He rejects Aristotle's division as indistinct, and divides Fallacies into Logical or Formal, and Non-Logical or Material. The first class includes all cases "where the conclusion does not follow from the premises;" these violate the syllogistic rules. As Non-Logical, or Material, he reckons all cases "where the conclusion does follow from the premises;" but where either the premises are unduly assumed, or the conclusion is irrelevant to the point in dispute. Surely this passes beyond the sphere of Logic. It might, perhaps, be justified by an appeal to Aristotle, who in one place defines a fallacy as "a reasoning which, either in matter or form or both, appears to be that which it is not.[6] In apparent accord with this, to which, however, he makes no reference, Whately goes on to insert an intermediate class, the Semi-Logical Fallacies, which are described as those whereof "the fault lies partly in the form, and partly in the matter." I do not understand this. It would seem rather to be a double fault. Any error in form is of itself total and fatal. As for Non-Logical Fallacies, they are *ex vi termini* out of the pale. Hamilton, however, has adopted this distribution.[7]

With the matter of an argument, as to the truth or falsity of its premises, unless they be self-contradictory, Logic has nothing to do, but only with the validity of the conclusion from given premises. All that relates to the collection of true premises with respect to the vegetable world belongs to Botany; with respect to the heavenly bodies, to Astronomy; with respect to the relation of man to his creator, to Theology. Were it within the province of Logic, it would require the extent of an encyclopædia to enter upon questions connected with the matter of syllogisms. Thus Aristotle: "All the sources of fallacy could not be enumerated if we consider the truth of the premises. This would require omniscience, for the sources are possibly infinite, and every science has false principles peculiar to it. Our

[5] *Logic*, bk. iii, §§ 1–4. [6] *Topica*, i, 1, 3. [7] See *Logic*, Lect. xxiii.

present task, then, is to trace the fallacies common to every science. This we may do, for they are limited in number. The logician must investigate the common fallacies that belong to no particular sphere."[8] We shall accordingly limit our attention to formal fallacies; material fallacies are excluded. We shall consider matter only in so far as it may be needful to inspect it in order to discover a fault of form. But then, indeed, we shall undertake to show that nearly all the kinds of fallacies usually classed as material are at bottom formal, violating syllogistic rules, and we shall adopt the old Aristotelic and scholastic classification as sufficient to this end. All logical fallacies, properly speaking, are formal fallacies.

§ 5. A Fallacy is commonly described as "any unsound mode of arguing, which appears to demand our conviction, and to be decisive of the question in hand, when in fairness it is not." Says Kant: "A rational reasoning which is false in form while valid in appearance is a *fallacy*. Such a reasoning is a *paralogism* if we are ourselves deceived by it. It is a *sophism* if we seek to deceive others."[9] Let us define more widely, and say that any violation of logical law is a fallacy. This agrees with its etymology (*fallere, falsum*). We may have fallacious definitions and classifications as well as the *non sequitur*. As for Kant's subdivision, it is not logical, but psychological; one of not the least moment in Logic, and little used elsewhere. Although, by the influence of Hamilton, it has crept into our language, and is repeated by nearly all subsequent writers on Logic with humble deference to these great authorities, we shall make bold to discard it, and distinguish paralogisms and sophisms in a useful logical sense.

Fallacies, then, are of two kinds:

1st. Paralogisms; or those whose violation of logical law is manifest upon inspection of the form alone. This accords pretty nearly with the meaning of the word as used by Aristotle. It is so used by De Morgan, who says: "Paralogism, by its etymology, is best fitted to signify an offence against the formal rules of inference."[10] What here we call paralogisms are distinguished by Whately as "formal fallacies," and by Mill as "fallacies of ratiocination."

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[8] *De Sophistici Elenchi*, ch. ix. The full title of this treatise, which is the last of the series constituting the ORGANON, but printed by Waitz as the final section of the *Topica*, is as follows: Περὶ δὲ τῶν σοφιστικῶν ἐλέγχων καὶ τῶν φαινομένων μὲν ἐλέγχων ὄντων δὲ παραλογισμῶν ἀλλ' οὐκ ἐλέγχων.

[9] *Logik*, § 90.

[10] *Formal Logic*, p. 239.

2d. Sophisms; or those whose violation of logical law is not manifest upon inspection of the form alone, but requires a consideration of the language, or of the matter to discover it. These correspond in general to Whately's "material fallacies," and to Mill's "fallacies of confusion." "It answers the purpose of some persons," says Aristotle, "rather to seem to be philosophers and not to be, than to be and not to seem; for sophistry is seeming but unreal philosophy, and the sophist a person who uses the semblance of philosophy without the reality." That is to say, he is a counterfeit wise man (Σοφός, *clever, cunning*).[11]

Sophisms are, as indicated above, subdivided by Aristotle into two classes,[12] which, in the terminology of the Scholastics, are as follows:

(*a*) Those *in dictione*, or *in voce* (οἱ παρὰ τὴν λέξιν); the formal fault being concealed by ambiguity of language. Generally, therefore, they disappear by being translated from one language into another. They correspond to Bacon's *Idola fori*, and to Whately's "semi-logical fallacies." Of them Aristotle makes a selection rather than a division, for it is far from exhaustive, of *six* classes, which, subsequently, we treat in detail.

(*b*) Those *extra dictionem*, or *in re* (οἱ ἔξω τῆς λέξεως); the formal fault lying concealed in the subject-matter. Generally, therefore, as adhering to the thought, they persist, in whatever language expressed. They correspond to the "non-logical fallacies" of Whately. Of them Aristotle selects and treats *seven* kinds; subsequently considered.

It is needful to forewarn the reader that fallacies sometimes present a double or a manifold aspect, one view bringing them under one class, another under another. It becomes, in such case, a matter of doubt or of choice to which genus even a given species shall be referred. Very often the same individual fallacy may, with equal propriety, be referred to different species, and sometimes we can choose whether to regard it as a fallacy or not. For instance, if some one expatiates on the distress of a country, and hence argues that the government is tyrannical, we must suppose him to assume either that "Every coun-

[11] Sir Thomas More (*Works*, p. 475) thus caricatures him: "A Sophyster woulde, with a fonde argumente, prove unto a symple soule that two egges were three; because that ther is one, and that ther be twayne, and one and twayne make three. Yt symple unlearned man, though he lacke learnying to soyle hys fonde argumente, hath yet wit enough to laugh thereat, and to eat the two egges himselfe, and byd the Sophyster tak and eat the thyrde."

[12] *De Soph.* chs. iv, v.

try under a tyranny is distressed," which constitutes the fallacy of undistributed middle; or that "Every distressed country is under a tyranny," which, though materially a false premise, yields, nevertheless, a good argument, and is not a fallacy.

The foregoing distribution of fallacies, as well as the detailed statement hereafter, is substantially that of Aristotle. He has been followed closely by logicians for two thousand years, the only considerable modification being the scholastic terminology, which we adopt. Attempts at an improved classification have been made, but no one has been generally approved. Mill's arrangement is masterly, but in the department of deductive fallacy he adheres quite closely to Aristotle. We have herein, then, nothing new to present. In the special treatment, we hope to show, by a more thorough analysis, that the several classes are amenable to the laws of the syllogism, and hence are strictly formal fallacies. The classification and treatment are, however, far from exhaustive. The ground is boundless. No one can forecast the devious intricacies, the incoherences, the perplexities, the entanglements possible to the human understanding. "*On se fait une idée précise de l'ordre, mais non pas du désordre.*"

§ 6. **Paralogisms,** as we have termed them, were not treated as a class of fallacies either by Aristotle or by the scholastics. The master, and his devout disciples until very recent times, were so perfectly familiar with the laws of thought and their application, that the idea of an open offence against the formal structure of a proposition or syllogism being unconsciously committed and maintained seemed to them impossible and absurd. But it is different with us. Palpable violations of syllogistic laws, though they are all merely laws of common-sense, are as frequent as any other species of fallacy whatever. The slipshod judgments and crippled arguments that every-day talkers, and even legislators, preachers, and teachers, are sometimes content to use, unconscious of their utter inconsequence, greatly need to be brought into the sunlight and spread out in thin transparency. But one who has read the preceding pages, for him it were superfluous that we more than barely indicate these bald simplicities.

A paradox, in the logical sense, is a self-contradiction.[13] When this is manifestly equivalent to A=non-A, we have a formally fallacious

[13] This is the sense in which I understand Aristotle in general to use the word. See *De Soph.* ch. xii.

judgment or a contradictory attribute. Is such an error possible? When a speaker begins with "a preliminary remark," thus "referring to what he is about to say," we are reminded of a schoolboy "backward in his progress," and of the captain's "forward march to the rear." These, of course, are mere blunders. Fallacious definitions and divisions have been sufficiently illustrated under their topics.

Immediate inferences are sometimes fallaciously drawn. How often, in the silence of thought, if not orally, is this error committed: All A is B; therefore, all B is A! You agree with me that to possess a large amount of money is to be wealthy; then, in the haste of talk, I may afterwards say you just now admitted that to be wealthy is to possess a large amount of money, and, unchallenged, draw a false conclusion. The difficulty of determining whether a man is or is not good is a commonplace of moralists and satirists. Society, however, applies, without hesitation, a very simple rule. Since, beyond doubt, good men do good deeds, it concludes, quite satisfactorily to itself, that he who does good deeds is a good man; whereas selfish prudence dictates a virtuous course of action almost as imperatively as virtue itself. We are more liable to this error because so many universal affirmatives are, when we consider the matter, simply convertible; as, "Coin is metallic money." Moreover, though "All seed come from plants," it does not logically follow, however true it may be, that "All plants come from seed."

In logical opposition, the fallacy of using the contrary of a proposition, instead of its contradictory, has already been noticed.[14] Important practical errors may arise from this. When it is maintained, as in some popular creeds, that "Every dutiful act is meritorious," this should not be met by the moralists with "No dutiful act is meritorious,"—for of two contraries both may be false,—but with "Some dutiful acts are not so." This may be easily proved; not the other, at least not to popular apprehension. That a thing is not white does not prove it black. Nobody can commit this fallacy thus broadly stated; but in the intricacies of an argument, and in the confusion of many words, it often lies in wait and is fatal. Again, when I affirm that "Some are," my opponent ought not to triumph with "Some are not;" for, unless it be the same "Some," both may be true. Yet, if he artfully frames an extended reply, the people, the arbiters in all questions not strictly personal, will very likely give him the palm.

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[14] Part 3d, ii, § 8.

Paralogisms violating the law of syllogism have already been sufficiently illustrated in connection with the General Rules.[15] If the several propositions of a syllogism were fully stated, these paralogisms could hardly ever occur; but since almost always the expression is but partial, fallacy may lurk unseen in the unexpressed thought. The obvious remedy is complete statement.

Another paralogism is to regard the conclusion as false because a premise is false, or because the argument is unsound; also, to infer the truth of a premise from that of the conclusion. Thus, if some one argues for the existence of a God from its being universally believed, another might perhaps be able to refute the argument by producing an instance of some nation destitute of such belief, the contradictory of the minor premise; the argument ought then to go for nothing. But many might think otherwise, and consider that this refutation had disproved the existence of a God, in which they would be guilty of an illicit process of the major term; thus:

Whatever is universally believed must be true;
The existence of a God is not universally believed;
∴ The existence of a God is not true.

Others, again, from being already convinced of the truth of the first conclusion, the existence of a God, would infer the truth of the premise, which would be the fallacy of undistributed middle; thus:

What is universally believed is true;
The existence of a God is true;
∴ The existence of a God is universally believed.

If these two fallacies were put in hypothetical form, the one would proceed from the denial of the antecedent to the denial of the consequent, the other from affirming the consequent to the affirmation of the antecedent. These two conditional fallacies, which have been already pointed out under a previous topic, are, therefore, found to correspond respectively with those of illicit process and undistributed middle.[16]

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[15] See Part 4th, i, § 5.

[16] Whately, *Logic*, p. 191.

II. SOPHISMS IN DICTION.

§ 1. The *sophismæ in dictione* are those that require an inspection of the language in order to detect the formal logical fault. They all arise from ambiguities of expression. A term repeated ambiguously, though identical to eye and ear, must be counted twice, for it represents two different concepts. A syllogism containing such a term is, therefore, in thought, a *Quaternio terminorum*, or, as it has been derisively called, a logical quadruped, *animal quadrupes logicum* (see General Rules, No. 1). This, fundamentally, is the vice of all fallacies *in dictione*. When the ambiguity is in the middle term, the fallacy corresponds very nearly with that of undistributed middle; for while in ambiguous middle the extremes are compared with two different terms, in undistributed middle they are compared with two different parts of the same term.

We enter now upon the consecrated Aristotelic ground, and must adhere to the time-honored terminology. Aristotle enumerates and treats six kinds of these sophisms, of which we adopt the following scholastic designations.

§ 2. The first class, *Æquivocatio*, or *Homonymia* (ὁμωνυμία), is ambiguity in a single term, or the use of a word or words in two different senses. If this is the middle term, we have the sophism of ambiguous middle, formally a quaternion. For example,—

All criminal actions should be punished by law;
Prosecutions for theft are criminal actions;
∴ Prosecutions for theft should be punished by law.

The middle term is here doubly ambiguous, both "criminal" and "actions" being used in different senses. The phrase in one premise signifies highly injurious deeds; in the other, a legal process. Again:

Finis rei est illius perfectio;
Mors est finis vitæ;
∴ Mors est vitæ perfectio.

Here the ambiguity may be thrown either upon the *finis* or upon the *perfectio*. If upon the latter, we have ambiguous major. The follow-

ing example is one given by Aristotle (ch. iv), redressed by Poste. It is taken from the *Euthydemus* of Plato, § 12–18. The middle term, *γραμματικός*, is a schoolboy who has learned to spell. The minor term is ambiguous.

ὁ γραμματικὸς ἐπιστήμων·
ὁ μανθάνων γραμματικός·
∴ ὁ μανθάνων ἐπιστήμων.

Such obvious cases as these would of course deceive no one. The scorn with which logical examples are often treated overlooks, however, the fact that premises in actual discussions are often very wide apart,—one or the other, indeed, perhaps not stated at all,—and the conclusion also remote; and so an ambiguity may very well escape detection, and lead to error. Whenever we can bring together the premises and conclusion in the form of a compact syllogism, the sophism of equivocation is usually quite manifest. We must recollect, too, that a series of arguments is like a chain, which is not stronger than its weakest link. If an ambiguous term is lurking somewhere, the chain cannot be depended on. One may observe, "There is a great deal of truth in what has been said." Yes, maybe it is all true, except one essential point. The sophistry is most dangerous that lies hidden in minute neglected points. "Burglars do not, in general, come and batter down the front door; but climb in at some window whose fastenings have been neglected. An incendiary does not kindle a tar barrel in the middle of the hall, but leaves a lighted candle in the thatch or in a heap of shavings."

Perhaps no fallacy is so prolific of false doctrine as this. Are mere words, then, so dangerous? "Men imagine," says Bacon, "that their minds have the command of language; but it often happens that language bears rule over their minds." And this rule is often misrule. Living languages, especially, abound in ambiguities, and no procedure is safe that has not provided against them, and that does not keep close watch upon them. The only remedy is an exact definition and a consistent use of terms. Whoever would discuss a subject in writing or speech with scientific accuracy must set out with definitions, and often state the precise sense in which he uses common words. It is one criterion of an advanced science to have its terms accurately defined. The mathematical and physical sciences were the first to make progress in this direction, and only in recent times have the moral sciences thus attempted to escape vagueness and erroneous consequence.

It would, perhaps, be impossible to enumerate the sources or kinds of ambiguity in words, or the errors which are consequent upon it. Some select illustrations must suffice. A word used at one time in its etymological or primary sense, and at another in a secondary or acquired and perhaps more customary sense, yields of course a quaternion. Thus a "representative" being originally a mere spokesman, his constituents may mistake his proper function, and hold him a trust-breaker if he uses his own judgment about measures. They might as rightly insist that a sycophant is merely a fig-shower. So one might fancy himself safe from legal penalties for "publishing a libel," so long as he did not print it. Laws, however, do not travel in meaning with their words. The honor of a discovery is usually accorded to him who first publishes it. Hence M. Biot, against the decision of the Royal Society, claimed the priority in the discovery of fluxions for Leibnitz over Newton, because of a private letter on the subject written by the former to Oldenburg in 1676, which was prior, and, in the legal meaning of the term, a publication. Again, the word "to utter," meaning originally "to give out," "to issue," has changed its meaning. No one, however, under indictment for "the utterance of counterfeit coin" would be likely to plead in defence that nobody ever uttered coin except the princess in the fairy tale.[1]

More serious errors arise from the customary use of the same word in various senses. The word "nature" is quite ambiguous. Butler pointed out three meanings. Sir C. G. Lewis makes two general classes of its various meanings: 1st, a positive idea, expressing essence, quality, or disposition; 2d, a negative idea, excluding art, or human regulation or contrivance. The phrase "human nature" is used in the positive, "state of nature" in the negative sense. "Every man has a natural right to his liberty" is a jumble of uncertain sounds.

The word "moral" is variously used. It seems to have lost entirely its etymological sense (mos, *custom*), as has also the Greek synonym "ethical" (ἦθος, *custom*), but it has branched out into various meanings. It is opposed to physical in "the moral and physical sciences," and to demonstrative in "moral and demonstrative reasoning." Even in the specific sense of right and wrong its signification fluctuates. Accurately, its criterion is law; a moral act is one imposed by a superior. Hence when we speak of the moral governor of the universe, it must be understood to mean merely goodness or equity, which qualities

[1] De Morgan, p. 243.

may attach to a supreme legislator; but the sovereign has no moral duties; his enactments create these for his subjects.

The confusion of "law" in the juridical sense with "law" as a uniformity of nature is exemplified in Butler's chapter on "The Moral Government of God." He calls the course of nature a government merely on the ground that it induces precautions to avoid pain. But these precautions have nothing moral in them; they may be used for criminal ends. Guy Fawkes obeyed a law of nature when he arranged for firing his powder-mine with safety to himself.[2]

The several meanings in which the word "inconceivable" is used, and its confusion with "incredible," have obscured greatly, and needlessly extended, the controversy between the intuitional and empirical schools of philosophy. Antipodes were incredible to the ancients, but not properly inconceivable. Every child conceives clearly that "the cow jumped over the moon," and maybe believes it, or maybe not. Necessary truth is a thing conceivable, the contradictory of which is inconceivable, i. e., cannot be thought or imaged by the mind. This contradictory is incredible; but it does not follow that whatever is inconceivable is incredible. Two contradictories may be equally inconceivable, as finite and infinite space; but, being logical contradictories, one must be true. Again, before the coming of Christ, it was inconceivable that justice and mercy could consist, but not incredible; since then it has become clearly conceivable also. Now it is inconceivable that election and free-will can consist; but these, not being logical contradictories, are nevertheless found credible.[3]

The mercantile public frequently commit a fallacy by the ambiguity of the phrase "scarcity of money." In the language of commerce, "money" has two meanings,—currency, or the circulating medium, and capital seeking investment, especially investment on loan. In this last sense the word is used when the "money market" is spoken of, and when the value of money is said to be high or low, the rate of interest being meant. The consequence of this ambiguity is that as soon as the scarcity of money in this latter sense begins to be felt, as soon as there is a difficulty of obtaining loans, and the rate of interest is high, it is concluded that this must arise from causes acting upon

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[2] Bain's *Logic*, p. 617.

[3] The troublesome ambiguities of "inconceivable" are discussed by Mill in his *Examination of Hamilton*, ch. vi; and in his *Logic*, bk. ii, chs. v–vii. He argues, however, in the interest of empiricism, and has failed to dissipate the mists.

the quantity of money in the other and more popular sense; that the circulating medium must have diminished in quantity, or ought to have been increased. A cry then arises for more money, for more circulating medium, no increase of which can possibly relieve this pressure.[4]

When St. Paul concludes (Rom. iii, 28) that "A man is justified without the deeds of the law," he is using the word "justify" consistently throughout, as meaning "treated by God as free from guilt." When St. James says (Epist. ii, 24), "Ye see then how that by works a man is justified, and not by faith only," he too is using the word consistently, meaning "seen to be just before God," which, he says, requires the evidence of works. All candid minds will see and acknowledge that in such a case the two statements are not contradictory, and that both arguments are conclusive.[5]

The paronomasia, or pun, is generally the logical sophism of equivocation. Charles Lamb[6] quotes the following, taken from Swift's *Miscellanies:* "An Oxford scholar meeting a porter who was carrying a hare through the streets, accosts him with this extraordinary question: Prithee, friend, is that thine own hare or a wig?" Lamb comments on this, and analyzes the fun of it admirably. The Logic of it is quite plain. The enthymeme implied in the question expands thus:

A wig is not one's own hair;
Surely that is not your own hare;
∴ It must be a wig.

Here are two negative premises, or else undistributed middle, as well as ambiguous middle. Still we may say that a pun is quite generally a mock argument founded on a palpable equivocation of the middle term. As herein: "Two men ate oysters for a wager, one ate ninety-nine, but the other ate two more, for he ate a hundred and won." Here the reason is formally proposed. Virgil's famous line,[7]

"Mantua, væ miseræ nimium vicina Cremonæ!"

contains a double pun, as such untranslatable of course, but may be similarly analyzed.

It may be well to remark here, once for all, that most kinds of witty jests are mock logic of some sort. Humor seems to relate primarily to feeling, feeling exaggerated or misplaced. Wit relates

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[4] Mill's *Logic*, p. 564.
[5] McCosh's *Logic*, p. 176.
[6] *Essays of Elia*, "Popular Fallacies," No. ix.
[7] *Eclogue* ix, 28.

rather to cognition, is more intellectual in character, and often, from under a logical play of thought manifestly and even absurdly fallacious, lets fly a sharp dart of truth. Dr. Johnson's fishing-pole, "a rod with a worm at one end and a fool at the other," is a mock definition. Mr. Beecher's jest, "People are the good people, the bad people, and the Beechers," is a mock division. Artemus Ward, travelling on a railway-car, suddenly cries out in alarm, "Mister Conductor, you've put the cow-catcher on the wrong end of this 'ere train; there ar'nt nothing on airth to prevent a cow from coming right in behind here, and biting the folks." Here is a curious mixture of humor and sarcasm; humor in the affected alarm at the supposed mistaken arrangement, and the grotesque consequences apprehended; wit in the sly assumption "Your train runs slower than a cow," implied by the deduction through the ambiguous "cow-catcher." Even the most seriously intended sophism becomes, when reduced to strict logical form, so palpably a ludicrous sham that we wonder any one could be deceived by it. As majesty stripped of its externals becomes a jest, so many a grave argument may be exposed to laughter and contempt.

§ 3. The second class, *Fallacia amphiboliæ* (*ἀμφιβολία*), differs from the last in that the ambiguity lies in the construction of a sentence rather than in a term. E. g., How much is twice two and three? I will go and return to-morrow. I hope that you the enemy may slay. A member of the House of Commons, charged with having called another a liar, rose and said, "It is quite true, and I am sorry for it." An example of Aristotle's is:

τοῦτο ὃ ὁρᾷ τις ὁρᾷ·
ὁ κίων τοῦτο ὃ ὁρᾷ τις·
∴ ὁ κίων ὁρᾷ.

The major premise is ambiguous. Another example given by Aristotle he takes from the *Euthydemus*, § 67.[8] A disputant says, in reply to the question Is the speaking of the silent possible? that if we go by a factory at work, we shall find iron tools far from being silent things. This furnishes the syllogism,

The speaking of iron tools is possible;
The speaking of iron tools is the speaking of the silent;
∴ The speaking of the silent is possible. (Poste.)

In the Nicene Creed, the words "by whom all things were made" are

[8] See Jowett's *Plato*, vol. i, p. 205.

grammatically referable either to the Father or to the Son. In the Second Commandment, the clause "of them that hate me" is a genitive governed either by "children" or by "generation."[9]

When a sentence has thus two grammatical renderings, the hearer is likely to adopt that to which his preference inclines, and overlook the other. This was the habitual trick of the oracles. Thus the prophecy of the spirit in *Henry VI:*[10]

> The duke yet lives that Henry shall depose,
> But him outlive, and die a violent death.

But this, says York, is just the famous response of the oracle to Pyrrhus:

> Aio, te, Æacida, Romanos vincere posse;
> Ibis, redibis numquam in bello peribis.

§ 4. The third and fourth classes, *Fallacia compositionis* and *Fallacia divisionis* (*σύνθεσις* and *διαίρεσις*), arise from the confusion of a universal with a collective term. According to Whately, when a distributed term is afterwards used collectively, it is the fallacy of composition; when a collective term is afterwards used distributively, it is the fallacy of division. This is clear, but seems not to have been exactly the meaning of Aristotle, and the distinction is hardly worth preserving. Aristotle's example is as follows:

> Two and three (*distributively*) are even and odd;
> Two and three (*collectively*) are five;
> ∴ Five is even and odd.

The ambiguity of "all" has been repeatedly noticed. When taken at one time in its cumular, at another in its exemplar or distributive sense, it gives rise to this sophism. E. g.:

> All the angles of a triangle are equal to two right angles;
> A B C is an angle of a triangle:
> ∴ A B C is equal to two right angles.

So "All these trees make a thick shade" may mean either that all together do so, or that each does so. When a multitude of particulars are presented to the mind, many persons are too weak or too in-

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[9] One more notable amphiboly: "All the dormitories of this university shall be occupied by two students except nine, they being single." (*Old Regulations.*) "Two students shall occupy every room in this university except nine, and one student shall occupy these." (*Revised Code.*)

[10] Part 2, act i, sc. iv.

dolent to take a comprehensive view of them; but confine their attention to each by turns, infer, decide, and act accordingly. Thus, the debauchee destroys his health by successive acts of intemperance, because no one of these acts would of itself be sufficient to destroy it. Others reason thus: I am not bound to contribute to this charity, nor to that, nor to the other, drawing the practical conclusion that all charity may be neglected.[11] The Owenites are said to reason thus against the doctrine of human responsibility:

> He who necessarily goes or stays is not a free agent;
> But every one necessarily either goes or stays;
> ∴ No one is free.

All such reasonings are obviously quaternions.

We sometimes hear an argument to prove that the world could do very well without great men. If Columbus had never lived, America would still have been discovered, at most only a few years later; if Newton had never lived, some other person would have discovered the law of gravitation, etc. Granted, but probably not until some one arose having the qualities of Columbus and of Newton. Because any one great man might have had his place supplied by another great man, the argument concludes that all great men could be dispensed with. The term "great men" is distributive in the premises, and collective in the conclusion.[12]

§ 5. The fifth class is *Fallacia prosodiæ*, or *accentus* (προσῳδία). An example given by Aristotle is from Homer:

τὸ μὲν οὔ καταπύθεται ὄμβρῳ.[13]

Some critics, he says, emend this, speaking the οὔ more sharply (λέγοντες τὸ οὔ ὀξύτερον), changing affirmative to negative; instead of "part," saying "naught is rotten by the rain." He prefaces this by the remark that the ambiguity can hardly occur in speech, but only in writing. This is because in his time the written words of the Greeks were not marked with accents and breathings, and hence were sometimes ambiguous to the eye when not to the ear.

In like manner with us an ambiguity in a written word or phrase is resolved usually by a stress *in voce*. Thus, gal'lant, brave; and gallant', courteous. "Not the least difference" may mean either no difference at all, or a very considerable, perhaps the greatest, difference.

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[11] Whately, p. 217. [12] Mill, p. 570. [13] *Iliad*, 23, 328. Dindorf has οὐ.

If in reading "Thou shalt not bear false witness against thy neighbor," the last word is emphasized, we convey the meaning that perjury is not forbidden except against the neighbor. We read in the first book of Kings, xiii, 27, "And the prophet spake to his sons, saying, Saddle me the ass; and they saddled *him*." The italics indicate that the word was supplied by the translators; mistaking it for an emphatic word transfers the saddle. Jeremy Bentham, it is said, so feared being misled by false accent that the person employed to read for him was required to maintain a monotone.

The fashion of taking a Scripture text and drawing thence a series of doctrines by putting emphasis first on one word and then on another is very questionable, if not dangerous. A wrong emphasis may pervert and wholly confound the meaning. But, on the other hand, we may by admissible and various emphasis forcibly present different views of the same sentiment. Observe in what different lights the thought may be placed by changing the stress of voice on the words of our Saviour: Judas, betrayest thou the Son of man with a kiss!

Betrayest thou,—makes the reproach turn on the infamy of treachery.
Betrayest *thou*,—makes it rest upon Judas's connection with his Master.
Betrayest thou the *Son of man*,—rests it upon the Saviour's personal character.
Betrayest thou the Son of man *with a kiss!*—turns it upon his prostitution of the sign of friendship and peace to a mark of hate and ruin.

Any statement of something that has been said with a suppression of such tone as was meant to accompany it is the fallacy of accent. Gesture and manner may easily make all the difference between truth and falsehood. A person who quotes another, omitting anything which serves to show the *animus* of the meaning; or one who without notice puts any word of the author he cites in italics so as to alter its emphasis; or any one who attempts to heighten his own assertions, so as to make them imply more than he would openly avow, by italics, or notes of exclamation, or otherwise, is guilty of *F. accentus*. We have said that jests are generally fallacies. Sarcasm and irony may be referred to the fallacy of accent, perhaps cannot be assumed without it. Some one, it may be, declines a task as beyond his powers; and another assures him that his diffidence is highly commendable, and fully justified by the circumstances. Said Job to his friends, No doubt but ye are the people, and wisdom shall die with you;—meaning the contrary. The tones and inflections of his voice, we may feel sure, were those peculiar to irony. This is very effective, since it is hardly possible to frame a reply.

§ 6. The sixth class, *Fallacia figuræ dictionis* (σχῆμα λέξεως), was limited by Aristotle to the using of words having similar terminations; to cases wherein unlike things have names with like inflection. The name of what is not an action, he says, may terminate like the name of an action (e. g., ail*ing* and cutt*ing*), and give ground for sophistry. This, however, is hardly possible in uninflected languages, and so at present the species is commonly held to include any perversion of grammar, any solecism. For example:

Whatever a man walks on he tramples on;
This man walks on the whole day;
∴ He tramples on the day.

Very similar to this source of ambiguity is that arising from the use of paronyms, or conjugate words, such as a substantive, adjective, and verb coming from the same root. These have by no means similar meanings. E. g., "Artist, artisan, artful;" "Pity and pitiful;" "Presume and presumption;" "Project and projector;" What is "imaginary" is unreal, but an "image" formed of wood or stone is real; To "apprehend" is to lay hold on, or to come to a knowledge of, while "apprehension" often signifies fear or dread.

Designing persons are untrustworthy;
Everybody forms designs;
∴ Nobody can be trusted.

Are there people in the world foolish enough to think that strong drink, because it is strong, gives strength? Then they commit the double fallacy of ambiguous terms, and of supposing that an effect must be like its cause. They should try strong poison. Fallacies founded on such differences, says Whately, can hardly be more than jests. They are not named by Aristotle, because in Greek, a more regularly constructed language, the meaning of paronyms, with very few exceptions, does exactly correspond; and paronyms (*τὰ σύστοιχα*) were a locus of dialectic, i. e., of valid, reasoning.[14]

The literal construction of metaphors and other figures of speech is also to be included under *figura dictionis;* e. g.:

Herod is a fox;
A fox is a quadruped;
∴ Herod is a quadruped.

In giving this example, Hamilton's patience breaks down. Disgusted

[14] See *Topica*, ii, ch. ix. "Si l'on a démontré que l'un des conjugués est bon ou qu'il est mauvais, on aura démontré, par cela même, que tous les autres le sont également."—St.-Hilaire.

with these trifling distinctions, he says that *Sophismata equivocationis, amphiboliæ, et accentus* may easily be reduced to *Sophisma figuræ dictionis;* "they are only contemptible modifications of this contemptible fallacy." [15] But when we remember that figurative expressions are more natural and usual than literal speech, especially if the subject be important and interesting; that a matter entirely new can hardly be discussed or even spoken of except metaphorically; that the history of the moral sciences shows how difficult it is to avoid being misled by material conceptions, which are not even analogous, but only remotely comparative; and that in debate illustrations are constantly mistaken for arguments, and, if brilliant, dazzle the vision, and exert more convincing and persuasive power than the most solid logic; we may rightly conclude that the sophism *figura dictionis*, so far from being contemptible, is worthy of our closest and most watchful consideration.

The great stress laid by Aristotle and his early followers on so many different forms of verbal deception, what now we should call a mere quibble, may have arisen, says De Morgan,[16] from the tendency in early times to place undue force on the verbal form of engagements and admissions, independently of the understanding with which they were made. Jacob was allowed to keep the blessing which he obtained by a trick; Dido surrounded the site of Carthage with strips of the ox-hide; Lycurgus seemed fairly to have bound the Spartans to follow his laws until his return, though he intimated only a short absence, and made it eternal; and the Hindoo god, who, in the shape of a dwarf, begged a realm of three steps, and then, in shape of a giant, took earth, sea, sky, seems to have been considered as claiming no more than was granted. But, nowadays, one undertaking to cross a bridge in an incredibly short time, and then crossing it as we cross a street, would hardly be held as having fulfilled his engagement.

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[15] *Logic*, p. 327.

[16] *Logic*, pp. 244, 246.

III. SOPHISMS IN MATTER.

§ 1. The *Sophismata extra dictionem* are those in which we must go beyond the outer form and beyond the diction, and inspect the matter of thought, in order to discover the logical fault. They are commonly called "Material Fallacies," and described as those whose fault does not lie in form nor in language, but in the matter, meaning by this that the form is correct, but that the premises are false. If so, then they are logically faultless, and, as already said, their consideration does not belong to our subject. But it is not so; these sophisms are logically, formally faulty; only it is requisite that we examine the matter in order to discover this. Of this genus, Aristotle, and after him the Latin logicians, enumerated seven species,[1] as follows:

The first class, *Fallacia accidentis* (παρὰ τὸ συμβεβηκός), arises, says Aristotle, from the equation of subject and accident, or whenever it is assumed that subject and accident have all their attributes in common. By "accident" here (συμβεβηκός as opposed to οὐσία) Aristotle means, not merely what is usually called the accident in Logic, but any subordinate part of a general notion. Every species and individual is to be regarded as an accident of its genus in this sense.[2] For example, "All men (*subject*) are mortal; but Every horse is (*an accident of*) mortal; hence (*equating subject and accident*), Every horse is a man, and Every man is a horse." But it does not follow that "man" and "horse" have all their attributes in common. An example from the text is: "Since Coriscus is not Socrates, and Socrates is a man, it does not follow that Coriscus is not a man, because Socrates, who is denied of Coriscus, is merely an accident of man." Obviously these examples are, the one undistributed middle, the other illicit major; but as illustrations of the present sophism we must take a different view of them. Either premise of the first and the major of the second are supposed to be converted simply, instead of *per accidens*.

[1] *De Soph.* ch. v. Aristotle does not consider these sophisms as having false premises, but exposes in detail their formal faults. He repeatedly excludes from Logic the consideration of matter as true or false.

[2] See *De Soph.* ch. xxiv, where *Accidens* is discussed at greater length.

This, if legitimate, would give Barbara and Camestres, but, being illegitimate, gives rise to the *F. accidentis.* Another example from the text is as follows:

> You do not know what I am going to ask you about;
> I am going to ask you about the nature of the *summum bonum;*
> ∴ You do not know the nature of the *summum bonum.*

Here the subject (*unknown*) of the genus (*about to be asked*) is equated with its accident (*summum bonum*). The example may be viewed as undistributed middle, or still more properly as an amphiboly.

We are now enabled to classify certain sophisms which have long been lying loose in our Logics. The standard example is:

> He who calls you a man speaks truly;
> He who calls you a knave calls you a man;
> ∴ He who calls you a knave speaks truly.

Here is inferred of a subject naming a species (*knave*) what is premised of a subject naming a genus (*man*). This is the best solution I have seen, but it is not thereby brought under any Aristotelic class. De Morgan confesses it troublesome, and concludes it is best considered Equivocation.[3] But it is clearly Aristotle's *F. accidentis.* Thus: "You (*subject*) are a man (*genus*); but A knave is (*an accident of*) a man; therefore (*equating subject and accident*) You are a knave." Or else, evidently, undistributed middle.

The name given to the legitimate conversion of A by Boethius[4] confirms this explanation of Aristotle's meaning. He has been very generally and very variously misunderstood, so that practically this species of sophism has long since dropped out of the list. Indeed, there are very few logicians who treat it correctly, or seem even to understand it. Errors arising from this malconversion have already been indicated in i, § 6, on Paralogisms.

§ 2. The second class, *Fallacia a dicto secundum quid ad dictum simpliciter* (*τὸ ἁπλῶς ἢ μὴ ἁπλῶς ἀλλὰ πῇ ἢ ποῦ ἢ ποτὲ ἢ πρός τι λέγεσθαι*), arises from the confusion of an absolute statement with a statement limited in manner, place, time, or relation. It is obvious that this includes the correlative *Fallacia a dicto simpliciter ad dictum secundum quid.* This, beyond question, was the intent of Aristotle; but Whately, followed by De Morgan, Mill, Bain, and their seconda-

[3] *Logic*, p. 242.

[4] Part 3d, ii, § 7.

ries, identifies the latter with *F. accidentis*, which, in the Aristotelic sense, is ignored. It is needless to make separate species of these correlatives.[5]

The first infers from a statement made under a restriction (*secundum quid*) to one made without restriction (*simpliciter*). E. g.:

> Whatever is pernicious ought to be forbidden;
> The use of wine is pernicious;
> ∴ The use of wine ought to be forbidden.

Here the minor premise refers to wine used immoderately; the conclusion, to wine, however used. This is the time-honored sophism of arguing against a thing from the abuse of it.

The second infers from a statement made without limitation to one limited, proceeding from what is essential, it may be, to what is accidental.[6] The old standard example is:

> What you bought yesterday you ate to day;
> You bought raw meat yesterday;
> ∴ You ate raw meat to-day.[7]

Here is inferred, in the conclusion, of meat with the accidental quality of rawness added, what in the major is said of it simply; i. e., of the essential substance, without regard to its accidental qualities.

The first of these cases, when we look into the matter, may evidently be construed as illicit minor; for what is premised of some, a certain use of wine, is concluded of all use of wine. The second case is plainly a quaternion, having an ambiguous middle; for "What you bought yesterday" is used in two different senses,—first simply or essentially only, secondly with its accident.

Under this class of sophisms might be included one to be called *F. a dicto secundum quid ad dictum secundum alterum quid.* When it is asserted that the desire of a sportsman to take life is cruel and despicable, to answer that those, also, who eat flesh from which life has been taken by others have therefore cruel and despicable desires is to infer from one special case to another special case, and is the sophism named.[8]

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[5] See *De Soph.* ch. xxv. [6] Hence, perhaps, the confusion with *F. accidentis.*

[7] "This piece of raw meat has remained uncooked, as fresh as ever, a prodigious time. It was raw when Reisch mentioned it in the *Margurita Philosophica*, in 1496; and Whately found it in just the same state in 1826."—De Morgan, p. 251.

[8] De Morgan, p. 265.

Perhaps the commonest and most dangerous sophisms of the species now before us are those which do not lie in a single syllogism, but slip in when passing from one syllogism to another in a chain of argument, and are thus committed by changing the premises. One of the conditions oftenest changed is the qualification of time. It is a principle in political economy that prices, profits, wages, etc., "always find their level." This is often interpreted as if it meant that they are most generally *at* their level, while the truth is they rarely are, but, as Coleridge expresses it, "they are always *finding* their level," which might be taken as a paraphrase or an ironical definition of a storm.

It is a very good rule not to encourage beggars, but we should not infer of all who solicit alms what is true only of professional beggars. So, also, it is a good general rule to avoid lawsuits, but sometimes circumstances make an appeal to law a duty. These may be taken as instances of the error vulgarly called the misapplication of abstract truth; that is, where a principle, true in the abstract, is applied to concrete cases, and reasoned on as if it were true absolutely, and no modifying circumstances could ever by possibility exist. This is to reason *a dicto simpliciter ad dictum secundum quid.* It is an error very common and very fatal in politics and society.[9]

It is by this fallacy that orators and devotees deceive others, and are themselves deceived, while they use the words loyalty, authority, liberty, faith, religion. The essence of these noble qualities is confounded with their accidents. Men commend a loyalty to a person which is disloyalty to a nation; obedience to a power which has no rightful authority; a liberty which is licentiousness; a faith which is mere credulity; a religion which is superstition.[10]

The gods, say the Epicureans, must be invested with human form, because that form is most beautiful, and everything beautiful must be found in them. But as the human form is not absolutely beautiful, but only in relation to other bodies, it does not follow that it must be in God, who is beautiful absolutely.[11]

The law, especially in criminal cases, requires a degree of accuracy in stating the *secundum quid* which to many persons seems absurd. A man indicted for stealing a ham was acquitted on the ground that the evidence showed only that he had stolen a part of a ham. Another being convicted of perjury committed "in the year 1846," the

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[9] Mill's *Logic*, p. 562. [10] McCosh's *Logic*, p. 128. [11] Arnauld, p. 262.

judge entertained the objection of the counsel that it ought to have read "in the year of our Lord 1846."[12] Such minutiæ are denounced as "the quibbles and quirks of the law;" but abundant experience has shown that the most minute caution is requisite not to commit injustice through the fallacy of *secundum quid.*

We recur again to the statement that jests are usually palpable fallacies. Boccaccio tells the following story: "A servant who was roasting a stork for his master was prevailed upon by his sweetheart to cut off a leg for her to eat. When the bird came upon the table, the master desired to know what was become of the other leg. The man answered that storks never had but one leg. The master, very angry, but determined to strike his servant dumb before he punished him, took him the next day into the fields, where they saw storks standing each on one leg, as storks do. The servant turned triumphantly to his master, on which the latter shouted, and the birds put down their other legs, and flew away. 'Ah, sir,' said the servant, 'but you did not shout to the stork at dinner yesterday; if you had done so, he would have shown his other leg too.'" The gist of this is in the assumption that what can be predicated of storks in general can be predicated of roasted storks; *a dicto simpliciter ad dictum secundum quid.* And so when the calculating boy, Zerah Colburn, was asked how many black beans it would take to make ten white ones, he promptly replied, "Ten, if you skin 'em." A worthy reply. A bean stripped of its accidents is still a bean.

§ 3. The third class, *Ignoratio elenchi* (*τὸ παρὰ τὴν τοῦ ἐλέγχου ἄγνοιαν*), is ignorance of the refutation, answering to the wrong point, proving something not the contradictory (*elenchus*) of the thesis which one intends to overthrow. This supposes a disputant, an attempt at confutation, and is the view to which Aristotle limited his treatment. It is usual now to take a wider view, and under the more general title, proposed by Whately, of Irrelevant Conclusion, or mistaking the issue, to include all cases where the attempt is to establish a thesis by a proof of something not sustaining it, or of something which may be mistaken for it. This latter might well be termed *Ignoratio* or *Mutatio conclusionis.* Formally the fault is either in establishing something that is not the required contradictory of the thesis, or else establishing something that is not the required thesis.

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[12] For a discussion of these two cases, see De Morgan, p. 252 sq.

If I argue the general utility of some proposed measure, and my opponent offers, in confutation, proof that we are not specially interested in it, he ignores the true elenchus, and his conclusion is irrelevant. If, in support of my thesis, I show that it is the proper consequence of previous legislation, I ignore the true conclusion, and my conclusion is irrelevant. If it be affirmed that a man has a right to dispose of his property as he thinks best, and you attempt to refute by showing that the way he has adopted is not the best; if one party vindicates, on the ground of general expediency, a particular instance of resistance to government, and you oppose that we ought not to do evil that good may come, you are guilty in each case of *ignoratio elenchi.* Again, if, instead of proving that the prisoner has committed an atrocious crime, you prove that the crime of which he is accused is atrocious; if, instead of proving that the poor ought to be relieved in this way rather than that, you prove that the poor ought certainly to be relieved, you are guilty in each case of *ignoratio conclusionis.* The special pleadings, technically so called, in our courts of law previous to trial are intended to produce, out of the varieties of statement made by the parties, the real points at issue, so that the case may not be *ignoratio conclusionis*, nor the defence *ignoratio elenchi.* "A demurrer" is about equivalent to the remark "Well, what of that?" That is, granting the statement in question, it may, perhaps, be no ground of action, and, if so, is irrelevant.

Nothing can be more important in the construction and prosecution of an argument than a clear and adequate conception of the precise point to be proved or disproved. In the speech of Diodotus[13] in answer to Cleon, who had argued that it would be just to put the Mitylenians to death, he reminds him that the question was not that, but whether it would be *expedient* for the Athenians to execute them. So Canning, in a speech in the House of Commons in reply to Mr. Perceval, says, "The question is not, as assumed by my opponent, whether we shall continue the war in the Peninsula, but whether it is essential to our success in the war that our present system of currency remain unchanged." Thus it is not unusual, after a protracted debate, for the cooler thinkers to preface their remarks with reminding the audience of the real nature of the point on which issue is joined; and the longer and more heated the discussion, the greater the need for these monitory exordiums. For, especially when the field of debate is large, the

[13] *Thucydides*, bk. iii, year 5.

combatants often join issue on the wrong points, or do not join issue at all. One goes to the east, another to the west; one loses the proposition in question, and wanders amidst a crowd of irrelevant details; another mistakes contraries for contradictories, or universals for particulars; and, after some hours of storm, they know not what they have been discussing. One has made out a case which his adversary admits, the more readily as it has not the least bearing on the question; another, having overthrown a similar collateral proposition, makes his pretended triumph resound over the field; yet another, having been rather shattered by reasons, appeals to the prejudices of his auditory, and, overwhelming his more rational antagonist with ridicule and abuse, comes off the apparent and acknowledged victor in the contest.[14]

And this reminds us that the *ignoratio* or *mutatio* often takes the form of personalities. We dispute with warmth, and without understanding one another. Passion or bad faith leads us to attribute to our adversary what is far from his meaning, in order to carry on the contest to greater advantage. It is a sign both of weakness and depravity that in almost every dispute the debaters ignore the question, and aim their tongues or their pens at their antagonists. In all the controversies that have shaken the opinions of mankind, this tendency is visible. In politics, the epithets radical and rebel, tyrants and traitors, have for ages been watchwords and weapons. In philosophy, the terms materialist, sensualist, idealist, transcendentalist, are, in different mouths, terms of admiration or contempt. In religion, the names Quaker and Methodist are memorials of scorn in the past; and "heretics," "bigots," "fanatics" are plentiful in the present. We rush at the throat of our antagonist, and the world, delighting in a display of pugnacity, crowns the fiercer and more vituperative combatant. But argument, not abuse; reason, not ridicule, is the touchstone of truth. What if Luther did and wrote many absurd things? This does not prove the authority of the Roman Church. What if Calvin did burn Servetus? This does not prove Calvinism to be fanaticism. The success of Pascal's vituperative *Provincial Letters* is very little to the honor of their author, for it indicates at once the weakness of those he attacked and of those whom he thus aroused to join in his hostility. The satirists of all ages have done as little for truth as Juvenal did for the morality of Rome.

[14] Sydney Smith's well-known *jeu d'esprit*, "The Noodle's Oration," furnishes some amusing examples of the Irrelevant Conclusion.

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Again, the *ignoratio* is often a mere dodge. Instead of even a pretended confutation, something is offered which answers practically. A sophist defending one who has been guilty of peculation, which he wishes to extenuate, but cannot disprove, may succeed by making the jury laugh. On the other hand, the prosecutor, if extenuating circumstances have been proved, may dodge the question, and practically attain his end by exciting the disgust of the jury, saying, "Well, but, after all, the fellow is a thief, and that is the end of the matter," which, however, not being denied, is not the question. Here the fallacy appears as an abuse of the *argumentum ad populum*. Emotion succeeds where reason fails. Likewise the *argumentum ad hominem*, an appeal to personal opinion, and the *argumentum ad verecundiam*, an appeal to respected authority, and other modes of arguing, in themselves legitimate, may be abused to establish irrelevant conclusions.

Another form is to prove or disprove a part of what is required, and to dwell on that, suppressing the rest. This is the dodge of prejudiced book-reviewers. Its frequent success shows the danger of bringing in bad arguments to support a good cause. Many a guilty prisoner has been acquitted, because some one witness against him has been caught lying. Vulnerable points should not be exposed. Achilles would have been alive now had he never shown a clean pair of heels.

Yet another form consists in showing that there are objections to the proposition, and thence inferring that it should be rejected, when it ought to be proved that the objections against receiving it are weightier than the reasons for it. Objections can be raised against any reform, and even against Christianity itself. "There are objections," said Dr. Johnson, "against a plenum, and also against a vacuum; but one or the other must be true." To suspend judgment until all objections are removed is practically to decide in favor of the existing state of things. "Not to resolve is to resolve," says Bacon.

Let us remark, in closing, that the fallacy of irrelevant conclusion is greatly aided by the adroit practice of suppressing the statement of the conclusion, and leaving it to be supplied by the hearer, who then is less likely to perceive whether it be the proper one or not.[15]

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[15] See Whately's *Logic*, pp. 240–249. De Morgan classifies under *I. elenchi* any attempt to transfer the *onus probandi* to the wrong side. The burden of proof always lies properly on the party making an assertion, whether positive or negative. If he shifts this burden onto his disputant, demanding a disproof of his bare assertion, there is a *mutatio* which may fairly be referred to this sophism.

§ 4. The fourth class, *Fallacia consequentis* (τὸ παρὰ τὸ ἑπόμενον), gives rise to fallacy, says Aristotle, "because the consecution of antecedent and consequent seems reciprocal. If B follows from A, we imagine that A must follow from B. Because whatever is generated has a beginning, it need not be that whatever has a beginning is generated. Because every man in a fever is hot, it does not follow that every man who is hot is in a fever."[16] These examples, at first glance, seem to be merely the fallacy of converting simply a universal affirmative. This cannot be Aristotle's meaning. Let us examine further. Subsequently he says,[17] "In another mode of this falsely inferred consequence, the relation of the contradictories of the antecedent and consequent is supposed to correspond directly to the relation of the antecedent and consequent. If B follows from A, it is falsely assumed that non-B follows from non-A. So in Melissus's argument, if the generated is limited, the ungenerated is unlimited; so that if the heavens are uncreated, they are boundless." This makes it sufficiently plain that Aristotle's *F. consequentis* is to infer the truth of the antecedent from the truth of a consequent, and to infer the falsity of the consequent from the falsity of an antecedent. When it is admitted, If A is, then B is, we cannot say, But B is, and therefore A is; nor can we say, But A is not, and therefore B is not.[18]

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[16] *De Soph.* ch. v. [17] Id. ch. xxviii.

[18] De Morgan states the *F. consequentis* to be simply the affirmation of a conclusion which does not logically follow from the premises, a mere *non sequitur*. His example is:

> Episcopacy is of Scripture origin;
> The Church of England is the only episcopal church in England;
> ∴ The church established is the church that should be supported.

The maintenance of the logic of this, he says, as "consecutive and without flaw," was recently imputed by an English newspaper to the clergy; which, he adds, was hard on the clergy. Truly, for, being sexipedalian, it is merely a logical insect. But De Morgan's definition will apply equally well to any and every fallacy; is, in fact, a proper definition of logical fallacy in general. This, then, could not have been the meaning of Aristotle, nor of the schoolmen, his studiously passive followers, who surely meant to be specific. Neither De Morgan nor Hamilton, who omits all mention of this sophism in his *Lecture* xxiii, seems to have looked into the treatise *De Sophistici Elenchi*. The former apparently draws from Aldrich, who misses the point entirely. Nor is Aldrich corrected by Mansel in his notes. Bain views the examples as merely erroneous conversions (p. 675). No recent writer seems properly to apprehend the scope of this species; and the false reasoning duly included by it, if treated at all, is treated entirely out of place.

This inconsequence has already been noticed under Paralogisms, where the formal fault is pointed out. But the fallacy is often concealed by the matter, and beclouded by feeling. People continually think and express themselves as if they believed that the premises cannot be false if the conclusion is true. The truth, or supposed truth, of the inferences which follow from a doctrine often enables it to find acceptance in spite of its gross absurdity. How many philosophical systems which had scarcely any intrinsic recommendation have been received by thoughtful men because they were supposed to lend additional support to religion, morality, some favorite view in politics, or some other cherished persuasion; not merely because their wishes were thereby enlisted on its side, but because its leading to what they deemed sound conclusions appeared to them a strong presumption in favor of its truth![19]

And, on the other hand, a good cause supported by false premises or a bad argument falls into disrepute. A notable instance is the cause of Temperance. Its warm and extreme advocates adduce in its favor an appalling amount of misstatement and of distorted and disproportioned facts; and, again, from unquestionable facts they sometimes reach their conclusions by a startling logic unknown to Aristotle and his slow-gaited followers. Now the argument for this good cause is very simple and impregnable; but, unfortunately, it does not furnish material enough for the popular oratory of the day, which, therefore, soars untethered by fact or logic. The revulsions the cause has suffered ought to teach its advocates that a bad argument is worse than no argument. For when people discover the fallacy, they instantly commit the counter-fallacy, and conclude that because a premise is false, or the argument illogical, therefore the conclusion is false; and so the last state of that cause is worse than the first. Whoever would think truly should hold steadily to the principle that in such case the conclusion is not disproved, but merely unproven. An indictment fails, and the prisoner is declared "Not guilty," which, I take it, is an abbreviation for "not proved guilty." But the people conclude he has been "found innocent." True, he is to be presumed innocent until found guilty; but presumption is not proof. The more deliberate and skilful the criminal, the more likely is he to win this verdict. The vast remove between unproved guilt and innocence ought to be clearly marked.

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[19] Mill's *Logic*, p. 560.

§ 5. The fifth class is *Petitio principii* (τὸ παρὰ τὸ ἐν ἀρχῇ λαμβάνειν ἢ αἰτεῖσθαι). Says Aristotle, "Petition (αἴτησις) is an assumption opposed to the belief of the hearer; or, still wider, a proposition requiring proof assumed without proof."[20] Elsewhere he says that the *Petitio quæsiti*, as this sophism may more correctly be called,[21] or begging the question, "appears to occur in five ways. The first and most manifest way is when the very thing that should be proved is assumed. This cannot easily pass undetected when the terms are the same; but when synonyms are used, or a name and its definition or a circumlocution, it may escape detection. A second way is when a particular is to be proved, and the universal is assumed; as, for instance, if we have to prove that contraries are objects of a single science, and assume that opposites, their genus, are objects of a single science. It appears that what should be proved alone is assumed in company with other propositions. A third way is when a universal is to be proved, and the particular is assumed; as when what ought to be proved of all contraries is assumed of some. Here it appears that what is to be proved in company with other propositions is assumed alone. A fourth way is when we divide the question to be proved, and assume it in detail; as when we have to prove that medicine is the science of health and disease, and successively assume it to be the science of each. A fifth way is when two facts are reciprocally involved, and we assume the one to prove the other; as when we

[20] *Anal. Post.* i, 10.

[21] *Petitio principii* is rather a blundering translation of the Aristotelic phrase, though of universal acceptance. In his *Metaphysics*, iv, i, 3, Aristotle defines "principle," in general, as "that from which anything exists, is produced, or is known." It is always and properly used for that on which something else depends; and thus both for an original *law* and for an original *element*. Cf. Hamilton's *Reid*, p. 761. The fallacy before us is the assumption, not of the *principle* properly so called, but, in some form or other, of the *question* originally proposed for proof. Pacius, in his *Commentarius in Organon* (in *Anal. Prior.* ii, 16), says, "Non est petitio τῆς ἀρχῆς, id est, principii, vel ἐν τῇ ἀρχῇ, id est, in principio; sed τοῦ ἐν ἀρχῇ προκειμένου, id est, ejus problematis, quod initio fuit propositum et in disquisitionem vocatum." See also Hamilton's *Logic*, p. 369; and Mansel's *Aldrich, Appendix*, note E.

We have rather a startling etymology of the phrase furnished us by Du Marsias, *Logique*, p. 81, which is worth preserving for its own sake: "Ce mot s'appelle *pétition de principe*, du mot grec πέτομαι, qui signifie *voler vers quelque chose*, et du mot latin *principium*, qui veut dire *commencement;* ainsi faire une *pétition de principe*, c'est recourir en d'autres termes à la même chose que ce qui a d'abord été mis en question."

assume that the side of a square is incommensurate with the diagonal, when we have to prove that the diagonal is incommensurate with the side."[22] The first two of these five modes, they being the most important, we will now proceed to illustrate at some length.

The first mode of this sophism occurs when a premise is either the same in sense as the conclusion, or else actually proved from it. This indicates two varieties, named the Hysteron proteron, and the Circle.

The former (*ὕστερον πρότερον*), wherein the conclusion and a premise are in sense the same, does not extend beyond a single proposition or syllogism; e. g., "The doctrine is heretical, for it has wrought a schism in the church." A proposition which is thus a corollary from itself would not, by any person in his senses, be considered as therein proved, were it not expressed in language which makes it seem to be two. It is not uncommon that a proposition expressed in abstract terms is offered as proof of the same proposition expressed in concrete terms. Pretended proof and pretended explanation both take this form; e. g., The loadstone attracts iron because of its magnetic power. This is burlesqued by Molière in the speech of Bachelierus:[23]

"Mihi a docto doctore
Demandatur causam et rationem quare
Opium facit dormire.
A quoi respondeo:
Quia est in eo
Virtus dormitiva,
Cujus est natura
Sensus assoupire."

The English language, being compounded of several languages, is peculiarly well fitted for this form of *petitio principii*. We make an affirmation in words of Saxon origin, and offer as a reason or explanation the same in words of Norman origin, and *vice versa;* e. g., The bill before the House is well calculated to elevate the character of education in the country, for the general standard of instruction in all the schools will be raised by it. These are "ladies' reasons." It is so. Why? Because it is so. The propositions are merely equipollent, and should be distinguished from immediate inferences.

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[22] *Topica*, viii, 13. Aristotle then proceeds to distinguish five modes also of *Petitio contrariorum*. In *petitio principii* the wrong procedure has reference to and affects the conclusion; in *petitio contrariorum* it affects only the contrary propositions themselves and the relation subsisting between them. For a paraphrase of these five modes, see Grote's *Aristotle*, vol. ii, p. 62.

[23] *Le Malade Imaginaire:* Troisième Intermède.

This fallacy does not, however, require a proposition, but occurs in what Bentham calls "question-begging appellatives;" meaning, names which beg the question under guise of stating it. The names of political parties, as Democratic, Republican, Liberal, Conservative, are much used in this way; e. g., "Those who favor the preservation of the fundamental principles of our government should of course act with the Conservative party." These are potent when laudatory, but even more so when vituperative; as, Radicals, Rebels, and most political catchwords. The word "innovation" having acquired a bad sense, the admission, which is unavoidable, that a new measure is an innovation is always construed to its disadvantage.

Galileo has accused Aristotle himself of being guilty of *petitio principii* in the following argument:

> The nature of heavy things is to tend to the centre of the universe, and of light things to fly from it;
> Now experience proves that heavy things tend towards the centre of the earth, and that light things fly from it;
> ∴ The centre of the earth is the centre of the universe.

How could Aristotle say in the major that heavy things tend to the centre of the universe, except by assuming that the two centres are identical, which is what he undertakes to prove.[24]

Plato, in the *Sophistes*, attempts to prove that things may exist which are incorporeal, by the argument that wisdom and justice are incorporeal, and wisdom and justice must be something. Here, if by "something" be meant, as Plato did in fact mean, a thing capable of existing in and by itself, and not as the quality of some other thing, he begs the question; if he means anything else, the conclusion does not follow. This fallacy might also be classed as ambiguous middle; "something" in the one premise meaning some substance, in the other, some object of thought, whether substance or attribute.

It was once an argument for the infinite divisibility of matter, that every portion of matter, however small, must have an upper and an under surface. Those using this argument did not see that it assumed the very point in dispute, the impossibility of arriving at a minimum of thickness; for if there be a minimum, its upper and under surface will of course be one; it will be a surface, and nothing more. The argument is very plausible because the premise seems more obvious than the conclusion, though really identical with it.[25]

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[24] Arnauld, p. 249.

[25] Mill's *Logic*, p. 574.

The formal fault of Hysteron proteron is that it is a pretended syllogism of two terms only,—a logical biped. This is disguised by the usual enthymemic mode of stating but two of the propositions, and by giving them in different words. The forms are these,—

A is B;	A is B;
A is B;	A is A;
∴ A is B.	∴ A is B.

There is no step forward here; it is merely "marking time."

We can now understand why Aristotle, in the passage quoted, distinctly condemns the premising of definitions as this mode of *petitio principii*. Let us consider the following:

Every rectilinear figure of three sides has its angles equal to two right angles;
Every triangle is a rectilinear figure of three sides;
∴ Every triangle has its angles equal to two right angles.

Here the minor premise is a definition. Now the subject and predicate of a defining proposition are identical in thought, the latter merely being explicit. The point to be proved, in the above example, is that the three-sided figure has its angles equal to two right angles, whether it be called a triangle or not. This is assumed in the major premise, and reiterated in the conclusion. The example is obviously in the second of the two preceding forms.[26] Whenever either extreme of an apparent syllogism is identical in thought with the middle term, there are of course but two terms, however much the phraseology may change. Such a pseudo-syllogism involves mere iteration, and no progress of thought; the conclusion has already been stated in a premise, and nothing is proved. It is merely the replacement of a term by its definition, or the reverse; as in the following: "The effect of the proposed measure will be to depress wages and to oppress all needy persons, since lower rates of payment for labor will be caused by it, and a cruel, unjust burden laid upon the poor."[27]

The use of a proposition to prove that on which it is itself dependent for proof by no means implies the degree of mental imbecility

[26] See also the "Demonstratio potissima" in Part 4th, iii, § 1.

[27] I am strongly inclined to the opinion that this view might be extended to the analytic and synthetic judgments of Kant (see Part 3d, i, § 12). Perhaps it would be correct to say that any syllogism having either premise a mere analytical judgment, unfolding what is contained in a name, is *petitio principii*, and actually proves nothing; and that only those whose premises are synthetical judgments, a conjunction of distinct facts, amount to actual proof. If so, this would modify the defence of the syllogism (Part 4th, ii, § 8) and facilitate it.

which might be supposed. The difficulty of comprehending how this sophism can possibly be committed disappears when we reflect that all persons, even the instructed, hold a great number of opinions without exactly recollecting how they came by them. Hence they may easily be betrayed into deducing them alternately from one another. A person may at one time insist on the divine origin of the Scriptures because they contain certain sublime doctrines which could not be discovered by the natural sagacity of the writers; at another time he may insist that these doctrines are true because found in the Scriptures, which, being of divine origin, are to be wholly accepted. So Plato, says Hamilton,[28] in his *Phædo*, demonstrates the immortality of the soul from its simplicity; and, in the *Republic*, demonstrates its simplicity from its immortality.

When a premise and conclusion which are actually the same are thus somewhat remote from each other, this variety of the first mode of *petitio principii* is called "Reasoning in a Circle," *Orbis vel circulus in demonstrando, vel diallelus* (δι' ἀλλήλων). The form may be represented as a pro- and epi-syllogism, thus:

A is B;		C is B;
C is A;	then—	A is C;
∴ C is B.		∴ A is B.

Of course any number of syllogisms may intervene, and the greater the number of intermediate steps, the more likely is the sophism to escape detection. A man walking around a hill is fully conscious of his circular movement; not so when he walks along a meridian line. Hence, to expose this fallacy, we have only to narrow the circuit by casting out intermediate steps, and exhibit the proposition, when it comes round again, in the same words.

The following example of reasoning in a circle is from Whately:[29]

Every particle of matter gravitates equally.
Why? What reason have you for that?
Because those bodies which contain more particles ever gravitate more strongly; that is, are heavier.
But those which are heavier are not always more bulky.
No, but still they contain more particles, though more closely condensed.
How do you know that?
Because they are heavier.
How does that prove it?
Because, all particles of matter gravitating equally, that mass which is specifically the heavier must needs have the more of them in the same space.

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[28] *Logic*, p. 372. [29] *Logic*, p. 221.

On this Mill remarks that such a process, wherein there is an actual attempt to prove two propositions reciprocally from one another, is seldom resorted to, at least in express terms, by any person in his own speculations, but is more likely to be committed by one who, being hard pressed by an adversary, is forced into giving reasons for an opinion of which, when he began to argue, he had not sufficiently considered the grounds. Hence another way to expose a Diallelon: challenge the reasoner to prove his premise, which if he undertakes to do, his whirl is evolved.[30]

A notable example of reasoning in a circle is the argument of Edwards and other metaphysicians for a necessitated will. The will, they affirm, must be subject to the law of necessity, because its determinations are always, as a matter of fact, in accordance with the strongest motive, the greatest apparent good. The strongest motive determines the choice, hence the will is necessitated. But what do you mean by the strongest motive? It is, of course, the motive that prevails. We know that it is the strongest because it does prevail. If it were not the strongest, the will would not have followed it; and being the strongest, the will must follow it. Then that is to say, the will must follow the strongest motive, because the strongest motive is the one the will must follow.

The second mode of *petitio principii* is that in which a universal is assumed to prove a particular. For example: "Is William, King of Germany, in any respect tyrannical? Of course he is; for all men possessing power are more or less tyrannical."

It is remarkable that this does not differ in form from the legitimate syllogism. It seems to give new ground for the charge, already discussed,[31] that the Aristotelic syllogism is essentially *petitio principii*. But observe that the fault here indicated is not a formal fault; it does not lie within the syllogism itself, but precedes it. It lies in the assumption of a principle by the reasoner, from which the conclusion truly follows, but which stands in need of proof as much or even more than the conclusion itself, and therefore cannot establish it, the whole

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[30] *Logic*, p. 571. That every particle of matter gravitates equally will not be granted by those who accept the atomic theory, according to which the particles have different specific combining weights. It is true, however, that these particles, though they may be real *minima* for the purposes of chemical combination, may not be the ultimate particles of the substance; and this doubt renders the hypothesis of equal weights admissible as an hypothesis.

[31] Part 4th, ii, § 8.

question being still afloat. This, then, is not at all a formal fallacy. Its fault lies solely in taking that for granted which is not granted. It would be *petitio principii* to prove to a Mohammedan the divinity of Christ from texts in the New Testament, for he does not admit the authority of the Bible; but it would be a valid *argumentum ad hominem* to prove to him from the Koran the prophetic mission of Jesus, for the authority of the Koran he acknowledges.

The phrase *petitio principii*, the unwarranted assumption of a principle, or the begging the question, is properly and specifically applied to designate this second mode of the sophism. It is not, however, to be understood as if every probation in which anything is presupposed and not proved were at once to be rejected as worthless. If so, it would be necessary in every case to ascend to the ultimate principles of human knowledge, and these themselves, being incapable of proof, might be rejected as unwarranted assumptions. Were this the meaning, there could be no probation whatever.[32] A probation is guilty of this sophism only when a proposition which may be doubted on the ground on which the thesis itself is doubted is assumed as a principle of proof, and we thus attempt to prove the uncertain by the equally uncertain. Sound probation must depart from such principles as are either immediately given as ultimate, or mediately admit of proof from other sources than the proposition itself in question.[33] "It is allowed," says Aristotle, "that when assumptions are closely connected with the issue, we may deny them, and refuse them as premises, on the plea that they beg the question."[34]

Among the schoolmen this second mode of the sophism was of peculiar interest. The philosophy of their time consisted largely of certain general propositions (*principia*) established by authority, and supposed to be ultimately derived from intrinsic evidence. Among these tenets were the doctrines of Aristotle, which were regarded with a reverence due only to inspired Scriptures. *Stultum est dicere Aristotelem errare.* Others were propositions which were considered as

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[32] "The main principles of reason are in themselves apparent. For to make nothing evident of itself to man's understanding were to take away all possibility of knowing anything. And herein that of Theophrastus is true, 'They that seek a reason of all things do utterly overthrow reason.'"—Hooker, *Eccl. Pol.* i, 8, 5.

[33] Hamilton's *Logic*, p. 371. He further observes that a *saltus* in probation is a special case of *petitio*; for, by an ellipsis of an intermediate link, we use a proposition which is actually without its proof.

[34] *De Soph.* xvii.

having been fully established by demonstrations as rigorous as those of Euclid. None were ever questioned; except, perhaps, in rare cases, when, consequently, as in the nominalist controversy, society was shaken to its foundations by a moral earthquake. These *principia*, being universally admitted, were at the command of every disputant. The syllogism in Barbara had properly a *principium* for its sumption, and an *exemplum* for its subsumption. The *petitio principii* occurred when any one, to prove his case, made it an example under a principle which was not among those received, and which was assumed without offering to bring it under their logical empire. Thus, were one to argue from "Every being void of reason must perish" that therefore the brutes perish, it would be denounced as *petitio principii*, this sumption not being found among the acknowledged *principia*. Again, suppose one to argue that since "Entire liberty is essential to well-being and happiness," civil law, being an abridgment of liberty, is therefore detrimental and should be abolished. To this would be replied, Of course, if your major is true; but unless you offer preliminary proof, you beg the question. We may illustrate further by the reply of Cardinal Richelieu to an applicant for clemency who thought to reason the matter, saying, "Mais, monsieur, il faut vivre." Said his Grace, "Je n'en vois pas la nécessité." There is, perhaps, a breath of inhumanity in this, but logically it means that the postulate was not among the principles admitted by him as Cardinal, and that one might reasonably beg his life, but not the question.

The third mode of *petitio principii* assumes the particular to prove the universal. Aristotle himself seems to be guilty of this when he maintains that slavery is in accord with natural law, on the ground that the neighboring barbarians, being inferior in intellect, are the born bondsmen of the Greeks.[35]

The fourth and fifth modes need no special illustration. Concerning the latter, however, we will remark how easy it is to frame propositions apparently different by the use of opposed or correlative terms. For example, "Everywhere the light of life and truth was lacking, for darkness covered the land, and gross darkness the people." Again, "Alexander was the son of Philip; therefore Philip was the father of Alexander." The last example is cited by Dr. Reid as a case of "simple reasoning" for which Logic does not provide. Truly so; but, on the other hand, Logic has been careful to provide against it.

[35] *Politica*, i, 2.

§ 6. The sixth class is *Non causa pro causa* (τὸ μὴ αἴτιον ὡς αἴτιον τιθέναι). "We mistake," says Aristotle,[36] "for a cause what is not a cause [meaning, a reason for what is not a reason] when an irrelevant proposition has been foisted into an argument as if it were one of the necessary premises." His example is a *reductio ad impossibile* to prove that "Life and the soul are not identical;" thus:

We assume that the opposite of destruction is generation;
Therefore the opposite of a particular destruction is a particular generation.
But death is a particular destruction, and its opposite is life;
Life, therefore, is generation, and to live is to be generated.—*This is absurd.*
Therefore life and the soul are not identical.—Q. E. D.

The absurd conclusion may be a proper sequence, and its absurdity justify the contradiction of a premise. But here an unexpressed premise, that "Life and the soul are identical," is mentally foisted into the train, and its contradictory stated as the Q. E. D. It is treated as if *it* were the cause of the absurd conclusion, which it is not, and so we have the fallacy of false cause, or *non causa pro causa.* Aristotle afterwards says[37] that to detect this fallacy we must examine whether the suppression of this premise would interrupt the sequence. If it does not, then we know that it is a superfluous proposition foisted in and treated as the cause of the absurd conclusion; and this is the fallacy in question. In the *Prior Analytics,* he says, "The most obvious case of the irrelevance of the thesis to the conclusion is when the thesis is not connected by any middle term with the conclusion, as was said in the *Topica* when discussing the sophism of *non causa pro causa.* We should exemplify this if, to disprove the commensurateness of the side of the square to the diagonal, we appended an argument for Zeno's theorem that there is no such thing as locomotion, pretending thereby to establish a *reductio ad absurdum.*"[38]

It is clear that Aristotle intended to designate by *non causa pro causa* the pretence that the proposition we wish to refute is the cause, in a *reductio ad impossibile,* of the false conclusion which in fact flows from other premises; that is, the sophism consists in maintaining that the conclusion is false because *that particular premise* is false. It is a case of sheer impertinence. It arises in dialectic disputation from the practice of asking the opponent to grant certain premises. An unnecessary proposition is asked and granted among the rest, and afterwards *it* is selected as the false assumption.[39]

[36] *De Soph.* v. [37] Id. ch. xxix. [38] *Anal. Prior.* ii, 19.
[39] See Mansel, in notes on Aldrich, Appendix, § 4, 4.

Aristotle does not, however, limit the sophism of false cause to cases of *reductio ad impossibile*, but includes under it all cases wherein a conclusion is declared to exist by virtue of a premise that does not necessitate it. He himself is not guiltless of this vice. For instance, he insists that there are three kinds of simple motion, because body has three dimensions, but hardly makes it clear how the one follows from the other, i. e., gives us no middle term to connect these propositions. He would prove also that the heavens are unalterable and incorruptible, because they have a circular motion, and there is no motion contrary to circular motion. But what has the contrariety of motion to do with the corruption or alteration of body? And is not rectilinear motion contrary to circular?

This sophism has been misunderstood, or at least misstated, by perhaps all recent writers on Logic. We have already noticed several common misapprehensions, deviations from the Aristotelic sense more or less grave. In this case the error is of sufficient importance to require that the common view be set aside and the original one restored. It is needful to explain the deviation and to justify this statement.

Let us first note a distinction drawn by the old logicians. The *Causa essendi* is that which determines the existence of a fact. When rain falls upon the ground, the ground is wet; the rain is the cause of the ground's being wet. The cause of there being an eclipse of the sun is that the moon interposes between it and the earth. The *Causa cognoscendi* is the cause of our knowing a fact. It has rained, therefore I know that the ground is wet. Here the same thing is the cause both of the existence of the fact and of my knowing the fact. But what is effect in the first sense may be cause in the other. E. g., The ground is wet, therefore I know it has rained. There is an eclipse of the sun, hence the moon must be between it and the earth.[40] The *causa cognoscendi*, then, is the logical ground; it is the cause determining, not the fact, but the judgment. This we now commonly call the reason for, or sign of, a thing, and use the word cause only in the specific sense of *causa essendi*.[41]

There can be no doubt that Aristotle, in the title of the sophism under consideration, intended exclusively the *causa cognoscendi*, or rea-

[40] In this inversion, reasoning from effect to cause, we should note that we are liable to the fallacy of Plurality of Causes. An effect may be due to a variety of causes, perhaps to a cause other than any that have been observed.

[41] The illative "because" is still used generically.

son; and that his followers, ancient and mediæval, so understood him, and intended the same limitation.[42] In recent times, the word cause becoming used almost exclusively for the *causa essendi*, logicians have commonly mistaken his meaning and wrongly interpreted this sophism.

They define the fallacy to be the assumption without sufficient ground that one thing is the cause (*causa essendi*) of another. Thus, that a change in the moon is the cause of a change in the weather;—thirteen at table brings bad luck;—the dog-star, Sirius, causes the heat that prevails during his ascension.[43] Whitefield once attributed his being overtaken by a hail-storm to his not having preached at the last town. Since many a nation having a heavy debt has prospered, therefore a national debt is a national blessing. These are clearly instances of the fallacy *Post hoc ergo propter hoc*, or of *Cum hoc ergo propter hoc*.[44] This fallacy is merely a case of bad generalization or bad induction, and therefore, however important it may be, has no proper place in Deductive Logic. But by our recent writers it is declared to be strictly the *non causa pro causa*, and is introduced and exclusively discussed in this place and under this title. Now it is not only an entire deviation from the meaning of Aristotle and the scholastics thus to interpret the *non causa pro causa*, but also a logical blunder to include the inductive *post hoc* among the deductive fallacies. On the other hand, however lightly Aristotle's *non causa pro causa* may be esteemed, it clearly belongs to the deductive fallacies; its formal vice, since it has no middle term, being that it is *quaternio terminorum*.

Next to the restriction of the word cause in *usus loquendi*, the error was probably due secondarily to the influence of Arnauld and Aldrich, or, at least, was thereby confirmed. The former says, "The *non causa pro causa* is very common, and we fall into it through ignorance of the true causes of things. It is in this way that philosophers have attributed a thousand effects to nature's abhorrence of a vacuum; for instance, that vessels full of water break when it freezes, because the water then contracts, and thus leaves a vacuum, which nature cannot endure;" and so on, through a variety of illustrations.[45]

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[42] *αἴτιον* is fairly rendered "cause," but has the general sense of "that which is chargeable with a thing;" mostly the bad sense of "something blamable."

[43] See Virgil, *Æn.* x, 273.

[44] Says Cicero, "Causa ea est quæ id efficit cujus est causa. Non sic causa intelligi debet, ut, quod cuique antecedat, id ei causa sit, sed quod cuique efficienter antecedat."

[45] *Port-Royal Logic*, pp. 251–56.

Aldrich designates it "Fallacia a non causa pro causa; sive sit a non vera pro vera; sive a non tali pro tali: ut, Cometa fulsit; ergo Bellum erit. Nullo modo; nam si fuerit, aliis de causis futurum est. Hæc fallacia bene solvitur negando causam falsam; melius adducendo germanam."[46] Whately, under the influence mainly of Aldrich, is evidently at fault. He first accepts his mistaken view, and illustrates it. Then, dissatisfied, he guesses correctly the blunder, that logicians were confounding cause and reason; and proposes to substitute the title "Fallacy of Undue Assumption," remarking that the varieties of this are infinite.[47] Verily; for this is merely to reason from a false premise, suppressed or disguised in any way. But such is not a logical fallacy at all, for Logic has nothing to do with the falsity of the premises. De Morgan treats the *non causa pro causa* very gingerly. He says, "It is the mistake of imagining necessary connection where there is none, in the way of *cause*, considered in the widest sense of the word."[48] This is wide enough, truly, and might include both the right and the wrong. But his examples show that he takes the wrong view only. For instance, he quotes the statement that Saunderson had such a profound knowledge of music that he could distinguish the fifth part of a note; and then remarks, "The one who made this statement did not know, first, that any person who cannot distinguish less than the fifth part of a note to begin with, if he exhibit the least intention of learning any musical instrument in which intonation depends upon the ear, should be promptly bound over to keep the peace; and, secondly, that if Saunderson were not so gifted by nature, knowledge of music would no more have supplied the defect than knowledge of optics would give him sight." These remarks show that he had only the *causa essendi* in mind; for he therein denies the assumption that knowledge of music was the efficient cause of the discrimination. And so our recent English logicians generally.[49]

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[46] *Logic*, Appendix, § 4, 4.

[47] Whately's *Logic*, pp. 223–33.

[48] *Formal Logic*, p. 268.

[49] Bain makes the mistake (*Logic*, p. 626 and p. 675). Hamilton, following Krug, misstates the meaning of *non causa*, and treats the mistaken view as a deductive fallacy. He also wrongly puts *post hoc* among the deductive fallacies (*Logic*, pp. 237–39). Mill does not use the title *non causa pro causa*, and omits to notice the Aristotelic species. He puts the *post hoc* in its appropriate place among false inductions. (See *Logic*, bk. v, ch. v, on "Fallacies of Generalization.") Minor writers, all that I have examined, and they are many, blunder along with passive sequacity.

§ 7. The seventh class is *Plurium interrogationum* (τὸ τὰ πλείω ἐρωτήματα ἓν ποιεῖν), the sophism of many questions. It is the effort to get a single answer to several questions asked in one. E. g., Was Pisistratus the tyrant and scourge of Athens? As he was the one, but not the other, either a yea or a nay would commit the respondent to a false position. A variation is to ask a single question, indeed, but so stated or compounded that a simple answer will assert or deny some other implied proposition. E. g., Did you take anything when you broke into my house last night? Are you the only rogue in your family? Have you quit drinking?[50] Have you cast your horns? From this last ancient example, the sophism is sometimes called the Cornutus. "Several questions put as one should be met at once by the decomposition of the compound question into its elements."[51] Obviously; as in the following example,[52] which has long served as the standard illustration: "Menedemus, Alexino rogante, Numquid patrem verberare desiisset? inquit, Nec verberavi, nec desii." So the Royal Society savans at last solved the waggish query of Charles II: Why does not a live fish add to the weight of a bowl of water, as a dead one does? This implies two questions, which for a time the puzzled philosophers overlooked, viz. 1st, *An sit?* 2d, *Cur sit?*[53]

All this seems quite frivolous. The occasion for noting the sophism is to be found in the eristic method of dialectic disputation among the Greeks, which proceeds usually by question and answer, the answers being conventionally yea or nay,[54]—a method familiar to readers of Plato's Dialogues. The effort of the Sophist is to entrap his unwary respondent into an admission which can be turned against him as paradoxical. The following example, borrowed from Fries,[55] is attributed, in its original form, by Diogenes Laertius (vii, § 196), to Eubulides the Megarian as the inventor:

Have you lost ten counters?—No.
Must you not have lost what you had at the beginning of the game and have not now?—Yes.
Have you ten counters now?—No.
Then you have lost ten counters, and have contradicted yourself.

But he had lost only two of the ten counters, and still had eight.

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[50] See Part 3d, i, § 12.
[51] *De Soph.* xxx.
[52] Originally from Diogenes Laertius, ii, 135.
[53] See the hackneyed story at length in Hamilton's *Metaphysics*, p. 118.
[54] See *De Soph.* xvii; and Diog. Laert. bk. ii, ch. 18, § 135.
[55] *Logik*, § 109. It is cited also in *De Soph.* xxii.

It is perhaps worthy of remark that lawyers sometimes nowadays badger unsophisticated witnesses in this way. To some compound question they demand what they call "a categorical answer," by which they mean a simple yea or nay, when either answer will entrap the witness in a self-contradiction or in other falsity. To deny the possession of a whole is not to deny the possession of a part, as in the above example and in the case of the stolen ham. To admit the existence of a certain motive (e. g., one mercenary) for an action still leaves the question undecided as to the concurrence of perhaps many other motives, and says nothing of their comparative strength.

Every question containing an ambiguous term may be viewed as double. Cicero is much puzzled to answer the question whether anything vicious is expedient.[56] Expedient may be understood either as conducive to temporal welfare or as conducive to ultimate welfare. If the answer, in view of the latter meaning be Nay, an opponent may confute with the former meaning, saying, "But theft is certainly vicious, yet, as it may conduce to temporal welfare, it is sometimes expedient." Or if the answer, in view of the former meaning, be Yea, he may object, "But no vice can ever conduce to ultimate good, therefore nothing vicious is ever expedient."

The double question may often be construed as an incomplete, and hence false, disjunction. Thus the Cornutus may be stated, "Either you have cast your horns, or you have them still; which?" But there is a third horn omitted, i. e., "or you have never had horns at all." In this form it is merely a case of a false premise.

The thirteen Aristotelic sophisms are comprised in the following mnemonic hexameters:

> Æquivocat. Amphi. Componit, Dividit, Acc. Fi.
> Acci. Quid, Ignorans, Non Causa, Con. Petit. Interr.

The *non causa* is displaced here from the original order which is the one we have followed.

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[56] *De Off.* bk. iii.

IV. EXAMPLES.

§ 1. Logic, from the time of Aristotle, became among the Greeks a profession. The acute and fun-loving Athenians especially busied themselves to invent puzzles with which to entangle and deride the stately professors; and these worthies themselves used the same means to discredit their rivals. Many of these puzzles, together with similar inventions by the scholastic logicians, have been handed down the centuries to us, discussed at every turn. As satisfactory solutions were rare, they received the title of "Inexplicabiles Rationes." They were collected, mostly from Diogenes Laertius, by Gassendi, in his *Liber de Origine et Varietate Logicæ*, and are analytically reviewed by Hegel.[1] Appearing generally to be a mere play of wit and acuteness, we marvel at the interest they have excited, at their celebrity, and at the importance attached to them by some of the most distinguished thinkers of antiquity. They certainly have an historical interest; and as literature makes frequent references to them, the student of Logic cannot neglect to make their acquaintance.

The disguises which sophistry may assume are innumerable. It seems to lurk most securely in the conditional forms, for these, being often very intricate, are confusing. Perhaps the most complete disguise is the dilemma, which, from its great capacity for entangled statement, was the favorite form of the Sophists, and hence is always regarded with suspicion and distrust. In some cases, however, very simple forms have proved very troublesome. We will select and examine a few of the most noted of the Inexplicables. They are known by specific names derived generally from the matter to which they were originally applied.

§ 2. The *Achilles* was proposed by Zeno the Eleatic, to support the leading tenet of Parmenides, the unity of all things, by showing that the identity of rest and motion is a necessary result of the contrary opinion. Probably, however, he was not serious in this argument, but intended it to retort the ridicule which had been thrown on

[1] See *Gesch. der Philos.*, *Werke*, xvi, p. 119 sq.

the doctrine of his master by involving his opponents in the same absurdities that they professed to find in his theory.[2]

The sophism runs thus: Suppose that Achilles runs ten times as fast as a tortoise that is one mile in advance. Now, when Achilles has run this mile, the tortoise has advanced $\frac{1}{10}$ of a mile beyond. When his pursuer has run this $\frac{1}{10}$, the tortoise has advanced $\frac{1}{100}$ of a mile farther; and then $\frac{1}{1000}$ of a mile; and so on, *ad infinitum*. Hence Achilles can never overtake the tortoise.

Hamilton pronounces this a sound argument, though leading to palpable falsehood. Whately says the pretended demonstration cannot possibly be exhibited in syllogistic form.[3] This confession, says Mansel, is a surrender of the syllogistic criterion. But nothing is easier. Thus:

Any space equal to $\frac{m}{10} + \frac{m}{100} + \frac{m}{1000} + \text{- - -}$ is infinite, being the sum of an infinite series;

The space to be passed over by Achilles is equal to this sum;

$\therefore$ This space is infinite.

The whole mystery of this famous sophism lies in this: The major premise is false. The sum of an infinite series may be, and in this case is, finite. The premise is equally false, whether space is or is not divisible *ad infinitum*. This is the solution given by Descartes.[4] The solution attempted by Coleridge[5] is refuted by Herbart. Mill says[6] the fallacy lies in the ambiguity of the word "infinite," in the tacit and false assumption, as Hobbes hinted, that whatever is infinitely divisible is infinite. The argument proves that to pass through a finite space requires a time that is infinitely divisible, but not an infinite time. This is tantamount to the solution of Descartes. Viewed as having a false premise, it is not a logical fallacy. Viewed as involving an ambiguous term, it is a quaternion.

Aldrich says, "Solvitur ambulando, quod fecit Diogenes." This reminds us that Dr. Johnson, in like view, thought he refuted Berkeley's idealism by kicking a stone. Zeno and Berkeley affirm that reason contradicts sense. Diogenes and Johnson reply, practically, that sense contradicts reason; which is *ignoratio elenchi*.[7]

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[2] Mansel's note in Aldrich, Appendix, § 5, 1. Cf. Plato, *Parmenides*, p. 128; Aristotle, *De Soph.* x, 2, and xxxiii, 4; and Cousin, *Nouveaux Fragmens, Zénon d'Elée.*

[3] *Logic*, p. 411.

[4] *Epist.* pt. i, Ep. 118.

[5] *Friend*, vol. iii, p. 93.

[6] *Logic*, p. 168.

[7] The four principal arguments with which Zeno proposed to disprove the reality of motion are as follows:

§ 3. The *Diodorus Cronus* is so called from the name of its inventor.[8] It also professes to demonstrate the impossibility of motion. It ranks high among the Inexplicables, and has probably been more discussed than any other puzzle on record. It is as follows:

If motion is possible, a body moves either in the place where it is, or in the place where it is not;

But it cannot move in the place where it is, for there is not room; nor in the place where it is not, for it is not there to move, and nothing can act or suffer where it is not;

∴ Motion is impossible.

The story goes that Diodorus had reason to lament this brilliant invention. He sent for a surgeon to reset his dislocated shoulder, who, instead of setting it, set himself to prove by this same irrefutable logic that dislocation was impossible.

Formally the reasoning is quite correct. It is a conjunctivo-disjunctive syllogism, treated conjunctively in the tollent mood. But the major premise is false.

First, the disjunction is not contradictory. "The place where a body is" is contradictory of "the place where it is not;" but "moves in a place where it is" is not contradicted by "moves in a place where it is not," but rather, as it should be, by "does not move in a place where it is." If so stated, the same conclusion could not be formally drawn, for then the consequent could not be totally denied on the grounds adduced in the minor.

Secondly, we cannot view the disjunction as merely incomplete, requiring a *tertium quid* to complete it, and therefore inept; for the second member, "moves in a place where it is not," cannot be accepted at all; it is a self-contradiction, or a mere jumble of words, a bit of sheer nonsense.

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1. Motion cannot begin, because a body in motion cannot arrive at another place until it has passed through an unlimited number of intermediate places.

2. Achilles cannot overtake the tortoise, because as often as he reaches the place occupied by the tortoise at a previous moment, the latter has already left it.

3. The flying arrow is at rest; for it is at every moment in only one place.

4. The half of a division of time is equal to the whole; for the same point, moving with the same velocity, traverses an equal distance (i. e., when compared, in the one case, with a point at rest, in the other with a point in motion), in the one case, in half of a given time, in the other in the whole of that time.

For interesting historical notices concerning these famous arguments, see Ueberweg's *Hist. of Phil.* § 20.

[8] Diog. Laert. ii, 112.

Thirdly, the first disjunct member is rendered absurd by an inaccurate use of the preposition "in." A body cannot be thought as moving "in a place," *in situ*. This, also, is essentially a self-contradiction, an incongruous use of words. A body can only be thought as moving *from* the place in which it was, *through* the place in which it is, *into* the place where it is about to be. This objection was raised by Gassendi, and is repeated by De Morgan.[9]

Mansel considers the disjunction as incomplete, as omitting a third horn, the possibility of "moving partly in a place where it is, and partly in a place where it is not;" and therefore he rejects the major premise. The same solution, substantially, is given by Hobbes.[10] But I cannot clearly understand what is meant by a body's "moving partly in a place where it is not." Hobbes, however, undertakes to prove with a diagram that a body, *quantulumcunque sit*, however small it may be, "cannot all at once so leave the whole of its former place that a part of it shall not be in that portion which is common to the two places, namely, the one which is left and the other which is reached." This is merely an evasion; for a part of a body is itself a body, to which the sophism still applies. Or it may be considered as an attempt to solve the difficulty metaphysically, involving the question concerning the infinite divisibility of matter.

Bowen refers the sophism to *F. accidentis*,[11] and supplies the omitted limitation thus: "A moving body, *at any one indivisible moment*, must be either where it is or where it is not. Hence, *in any one indivisible moment* motion is impossible, for motion requires time as well as space. When the proviso here italicized is expressed, the proposition is true, the reasoning is sound, and the conclusion correct." I am still partially in the dark. What does he mean by the second member of the disjunction, "or it must be where it is not?"

§ 4. The *Litigiosus*, or *Reciprocus*, is a noted dilemma of which we have two accounts, one Greek[12] and one Roman.[13] The latter tells it of Protagoras, the prince of sophists, and Euathlus, his pupil in the law. Euathlus had contracted to pay his tuition fee when he gained

[9] *Formal Logic*, p. 260.

[10] *Philosophia Prima*, pt. ii, ch. viii, § 11.

[11] *Logic*, p. 298.

[12] By Suidas, in Waltz's *Rhetores Græci*, vol. iv, p. 13, where it is told of Corax and Tisias; and thence is said to have originated the proverb, Κακοῦ κόρακος κακὸν ὠόν, which in part still survives among the vulgar of to-day.

[13] Aulus Gellius, bk. v, ch. x.

his first case. But not having any case, he was finally sued for the fee by Protagoras, who, in court, addressed him thus:

"Learn, most foolish of young men, that, however matters may turn out, pay me my demand you must. For if the judgment be against you, I shall obtain the fee by decree of the court; and if in your favor, by the terms of our contract, for then you shall have gained your first case."

To this Euathlus, proving at least that he was an apt pupil, replied in corresponding terms, as follows:

"Most sapient of masters, learn from your own argument that, whatever may be the finding of the court, absolved I must be from any claim of yours. For if the decree be in my favor, I shall accordingly pay nothing; and if adverse, I shall pay nothing by virtue of the contract, for I shall not have gained my first case."

The perplexed judges, unable to find a *ratio decidendi*, adjourned the case *sine die*.

The dilemmas are the same. The disjunction is incomplete. The omitted member is "no decree at all." Protagoras had no ground for suit, and the judges should have quashed the case with a *nolle pros*. Practically this was the result.

§ 5. The *Mentiens*, classed of old among the *Insolubilia*, and known to the Greeks by the title Ψευδόμενος, was also invented by Eublides. Chrysippus, the Stoic, wrote six treatises on it, and Philetas of Cos, it is said, studied himself to death in the vain attempt to solve it.[14] Cicero states it thus:

"If you say that you lie, and say so truly, then you do lie; but if you say so falsely, then you speak the truth. The same assertion, therefore, is at once false and true."[15]

"The solution," says Mansel,[16] "is very obvious. No one can lie without lying about something. The statement 'I lie,' taken alone, is senseless." But it seems we are to understand that "I lie in this very statement that I lie." Then it would be more formally logical to say that this statement, being, like all other assertions, primarily offered as true, is a logical paradox, a self-contradiction, destroying itself, and therefore null. Gassendi puts the sophism thus: "Qui jurat se falsum jurare et falsum jurat, vere jurat."

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[14] Diog. Laert. 7, § 196.

[15] *Acad. Quæst.* iv, 30.

[16] Note in Aldrich, Appendix, § 6, 4.

Let us take this occasion to speak once again, and more generally, of the logical paradox.[17] A self-contradiction *in terms* is, of course, a blunder *in dictione*, as in the following examples:

Human thought is bounded only by the infinite.
Let us compel them to volunteer.
The crime of suicide deserves capital punishment.
There are many kinds of individuals.
It is better not to know so much than to know so many things that aren't so.

But sometimes the paradox is *extra dictionem*, and not quite so obvious. Suppose we say of a man that he is always a liar. If this be true, then he can never say or imply "I lie," for this would be telling the truth. But since we must think that any man may say this, it follows that "a man always a liar" is an impossible conception, it is a self-contradiction. "*Hoc unum scio, quod nihil scio*," Socrates is reported to have said. "It is certain that there is nothing certain," said the paradoxical philosophers of the Middle Academy. This saying Pyrrho, the Sceptic, disputed thus: "Everything is so uncertain that it is even doubtful whether there be nothing certain." But this absolute scepticism of "those who doubt that doubt itself is doubting" involves also a self-contradiction; it professes a belief that there is no belief. Of all universal propositions, it has been said, one only is allowable, *In generalibus latet error:* this denies all others; and then, when closely looked at, it too commits suicide.

Recurring once more to the remark that jests are generally fallacies, we add that the essence of the "Irish bull" is self-contradiction. Though perhaps not half the lies they tell of the Irish are true, yet bulling seems a natural art with them, and not intentional mistake. Bulls are jests in earnest. The Paddy who said he was not dead, but only speechless, was a living *felo de se*, and typical of the isle whose overflowing cup of woe is not yet full, asking only for non-interference, and for but little of that.

§ 6. The *Sorites* (σωρός), *a heap*, is attributed by Persius[18] to Chrysippus as the inventor, but Diogenes Laertius[19] attributes it to Eubulides. It is defined by Ulpian as a sophism in which by very small degrees the respondent is brought from the evidently true to the evidently false. For example, I ask, Does one grain of corn make a heap? No. Do two grains make a heap? No. Do three grains? No. And

[17] See Part 1st, ii, § 3. [18] *Satires*, vi, 80. [19] Bk. ii, § 108.

so on, adding each time a single grain, until at last the respondent is forced to say that the total reached does make a heap. I then charge him with saying that a single grain makes all the difference between what is and what is not a heap, which is absurd.

This reasoning, as applied to various objects, is called by various names. Besides *Sorites*, which Cicero[20] renders by *Acervalis*, we have the *Crescens*, the *Superpositus*, the *Calvus*, and others. This last name is derived from the exemplum of Eublides, wherein the series of questions begins with asking whether pulling one hair from a man's head makes him bald. The sophism is used by Horace[21] to ridicule the fashion of valuing authors merely for their antiquity. The name *Sorites* does not occur in Aristotle. After him, it was used by the ancients, but only as a designation of the above sophism. About the middle of the fifteenth century it came to be applied also to the chain syllogism,[22] between which and this ancient sophism there exists no analogy whatever.[23]

In explanation of the sophism, Krug says,[24] "It attempts, from the impossibility of assigning the limit of a relative notion, to show, by continued interrogation, the impossibility of its determination at all. There are certain notions which are conceived only as relative, as proportional, and whose limits we cannot, therefore, assign by the gradual addition or detraction of one denomination. But it does not follow that, if a notion cannot be determined in this manner, it is incapable of any determination, and therefore null." This explanation is adopted by Hamilton.[25] The sophism, in this view, is evidently, as to form, a fallacy of definition.

§ 7. The *Ignava ratio* (ἀργὸς λόγος) is commonly attributed to the Stoics, by whom it was employed in support of their doctrine of fate.[26] It is propounded by Cicero[27] in the form of a complex constructive dilemma; thus:

> If it be fated that you recover from your present disease,
> then, whether you call in a doctor or not, you will recover;
> and if it be fated that you do not recover,
> then, whether you call in a doctor or not, you will not recover.
> But it is fated either that you recover, or that you do not recover.
> ∴ It is useless to call in a doctor.

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[20] *De Divinatione*, ii, 4. [21] *Epist.* ii, 1, 43 sq. [22] Part 4th, iv, § 3.
[23] See Hamilton's *Logic*, pp. 267–69. [24] *Logik*, § 177. [25] *Logic*, p. 332.
[26] This origin is questioned by Hamilton, *Logic*, p. 331. [27] *De Fato*, ch. xii.

The strictly logical conclusion drawn from these premises would be,—

∴ Whether you call in a doctor or not, you will, or you will not, recover.

This amounts to nothing. The dilemma is badly formulated. Let us redress it, and in more general terms; thus:

If an event is fated to be, my effort is useless;
and if it is fated not to be, my effort is useless.
But it is fated either to be or not to be.
∴ My effort is useless.

This is, in form, a simple constructive dilemma, and logically sound.

The ancient idea of fate is the ground of this argument for inaction. It considered all future events as pre-established, fixed by an inevitable necessity, by a destiny originating independently of divine will and beyond divine control, so that not only nature and man, but the gods themselves, were subject to its unalterable decrees. An event that is fated to be will inevitably be, regardless of any second causes that may concur to counteract or modify its order. "If this doctrine were true," says Cicero, "life would be reduced to a state of hopeless inactivity, and the above argument would prove the inutility of any endeavor to bring about a desirable result or to avert a threatened calamity." Fate was personified by the Greeks in the Parcæ; the impersonal doctrine prevailed in the rest of the ancient world. It has barely survived with the Turks, who, as fatalists, will not take precautions against pestilence, and who have suffered Constantinople to be repeatedly destroyed by fire without an effort to stay the conflagration. It need hardly be said that there is no such thing as fate, personal or impersonal. It is a vain and vague imagining, without ground in fact or reason.

Necessitarians of the present day do not argue in the above form. Their doctrine admits the contingency of second causes; but these are determined and determining. "There is no thing produced, no event happening, in the known universe which is not connected by a uniformity or inevitable sequence with some one or more of the phenomena which preceded it. These antecedent phenomena, again, were connected in a similar manner with some that preceded them, and so on. All the phenomena of nature, then, are the necessary, or, in other words, the unconditional, consequences of some former collocation of causes. The state of the whole universe, at any instant, is the consequence of its state at the previous instant. If one knew all the agents which exist at the present moment, and the laws of their

agency, he might predict the whole future history of the universe."[28] This doctrine excludes human liberty, and, if pushed to its logical results, does not differ essentially from fatalism.

The controversy between Liberty and Necessity has continually agitated the world, in one form or another, from most ancient times. No controversy is more ancient, none more universal, none has more keenly excited the minds of men, none has exerted a greater influence on morals; it has divided not only schools, but nations, and has modified not only their opinions, but their manners, customs, religion, and government. Under its influence the *Ignava ratio* has taken many forms, been applied to various matter, and received a variety of names. Among these are: *De fato*, *Metens* (the reaper), *De possibilibus*, *De libero arbitrio*, *De providentia*, *De divinis decretis*, *De futuris contingentibus*, *De physica prædeterminatione*, etc. We are here concerned with the argument only in the form given above.

The subsumption is false. Let us examine it. What is meant by "an event fated to be?" The essential idea of "fated" is "inevitable," that is, not modified by any precedent or concurrent events, substantially and personally expressed by "regardless of my effort." So our personal argument reduces to:

> If an event is, regardless of my effort, to be,
> my effort is useless;
> and if it is, regardless of my effort, not to be,
> my effort is useless.
> But every event is, regardless of my effort,
> either to be or not to be.
> ∴ All my effort is useless.

Now, consider this subsumption. Who can affirm it? What ground has it? It may be true of some event, as an eclipse, that I can neither effect it nor affect it; and of it we may say that it must either take place from other causes or not at all. But many events depend wholly or in part upon my effort as a *conditio sine qua non*. The only real fate is a concurrence of causes, an assemblage of conditions. Supply a new cause, take away one of the necessary conditions, and the result will be different, though still, if you please to call it so, a fated or necessary result. Fate changes then her decree. Sending for a doctor introduces a new cause; neglecting to send may be the omission of a condition necessary to recovery.

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[28] Mill's *Logic*, p. 250.

If the necessitarian objects that my will is itself determined by prior causes, I reply that then, it may be, I am fated to send for a doctor, and so to recover; or may be fated not to send, and, as a consequence of this neglect, fated to die. So my effort is not useless, not inconsequent. Zeno, the Stoic, who adopted the argument, once, it is said, conceded this. He undertook to flog his slave for theft, who aptly pleaded that he was fated to steal. "And I to flog you," was the reply.

§ 8. **Praxis.** Among the following miscellaneous examples are some cases of good reasoning from true premises, and others from false premises, as well as fallacies proper. If the reasoning is sound, give the mood; if not, analyze the thought, indicate the logical defect, and name the species of fallacy.

1. A legitimate argument may fail to win assent;
   No fallacy is a legitimate argument;
   ∴ No fallacy can fail to win assent.
2. Whatever represses the liberties of mankind ought to be resisted;
   Among those things that do so, there are governments;
   ∴ Governments ought to be resisted.
3. Every one desires happiness;
   Virtue secures happiness;
   ∴ Every one desires virtue.
4. Idolatry is wicked;
   Wickedness should be punished by law;
   ∴ Law should punish idolatry.
5. Christianity cannot be proven true by its success,
   for Mohammedanism has succeeded;
   Nor can it be proven by its alleged miracles,
   for Buddhism has its alleged miracles;
   ∴ Christianity cannot be proven to be true.
6. We ought to give one day in seven to religious duties,
   if the fourth commandment is obligatory upon us;
   But we ought thus to devote one day in seven;
   ∴ The fourth commandment is obligatory upon us.
7. We are forbidden to kill;
   Inflicting capital punishment is killing;
   ∴ We are forbidden to inflict capital punishment.
8. A king is a man. Then it immediately follows, by added determinants, that a good king is a good man.

9. No moral principle is an animal impulse;
   But some animal impulses are principles of action;
   ∴ Some principles of action are not moral principles.
10. The papists would be aggrieved if the penal laws against them were enforced;
    But these are not, and hence they have no reason to complain.
11. Nothing is better than wisdom;
    Dry bread is better than nothing;
    ∴ Dry bread is better than wisdom.
12. No person destitute of imagination is a true poet;
    Some who are destitute of imagination are good logicians.
    ∴ No true poet is a good logician.
13. Some practically virtuous men are necessitarians;
    But all necessitarians speculatively subvert the distinction between vice and virtue;
    ∴ Some who deny the distinction are practically virtuous.
14. Interference with another man's business is illegal;
    Underselling interferes with another's business;
    ∴ Underselling is illegal.
15. Pestilence, being "a visitation of God," its event is not determined by physical causes.
16. No one desires to do wrong if cognizant of its nature, but only in consequence of ignorance; and, therefore, virtue is knowledge, and is to be attained by education; for no one desires evil knowing it as such, and to do wrong is evil.—From *Plato's Gorgias.*
17. His imbecility of character might have been inferred from his proneness to favorites; for all weak princes have this failing.
18. Quod tangitur a Socrate illud sentit;
    Columna tangitur a Socrate;
    Ergo, Columna sentit.
19. The right of the government is unquestionable; therefore we ought to obey it.
20. Every visible object that does not decompose light is seen by white light, and is therefore white;
    A black-board is a visible object that does not decompose light.
    ∴ A black-board is white.
21. Any form of government that excludes the people from political power is subject to violent revolutions;
    A democracy does not so exclude the people, and therefore is not subject to violent revolutions.

22. The planets are seven;
Mercury and Venus are planets;
∴ They are seven.

23. No cat has two tails;
Every cat has one tail more than no cat;
∴ Every cat has three tails.

24. To allow every man freedom of speech must always be, on the whole, for the good of the state; for it is highly conducive to the interests of the community that each individual should enjoy a liberty, perfectly unlimited, of expressing his sentiments on its affairs.

25. Let $x=2$, and $y=3$. Take the self-evident equations $ax=ax$ and $ay=ay$, add them together and transpose terms; this will give $ax-ax=ay-ay$. Dividing by $a-a$, we obtain $x=y$, or $2=3$.

26. Mus syllaba est;
Mus caseum rodit;
Ergo, syllaba caseum rodit.—*Seneca, Epist.* 48.

27. There are many men that reason exceeding clear and rightly, who know not how to make a syllogism; therefore, Logic is useless.—*Locke.*

28. If all testimony to miracles is to be admitted, the popish legends are to be believed;
But they are not to be believed;
∴ No testimony to miracles is to be admitted.

29. None but whites are civilized;
The ancient Germans were whites;
∴ They were civilized.

30. Only give me the luxuries of life, and I will dispense with the necessaries.

31. Unless Logic professes to be an instrument of invention, the reproach that it discovers nothing is unfounded;
But it does not make this profession;
∴ The reproach is unfounded.

32. *Teacher*,—"What is conscience?"
Smart boy,—"Don't know" (="unprepared").
Teacher,—"Why, conscience is something within you that tells you when you have done wrong."
Boy,—"Oh, yes; I had it once, and they had to send for the doctor."

33. All these exercises will fatigue me;
This itself is one of them;
∴ It will fatigue me.

34. A grain of corn does not make a heap;
A hundred (99+1) is made by one grain;
∴ A hundred is not a heap.

35. What is no uncommon occurrence may reasonably be expected;
To gain a high prize in a lottery is no uncommon occurrence;
∴ To gain a high prize may reasonably be expected.

36. My hand touches the pen;
The pen touches the paper;
∴ My hand touches the paper.

37. The *minimum visibile* is the least magnitude that can be seen. No part of it alone is visible, and all parts must affect the mind in order that it may be visible. Hence every part, though invisible, must affect the mind.—See *Hamilton's Metaphysics*, p. 243.

38. Improbable events happen every day;
But what happens every day must be a very probable event;
∴ Improbable events are very probable.

39. Omnis equus est bestia;
Omnis justus est æquus;
∴ Omnis justus est bestia.—*Burgersdyck.*

40. Nuisances are punishable by law;
A noisy dog is a nuisance;
∴ A noisy dog is punishable by law.

41. *Ham.* There's ne'er a villain dwelling in all Denmark,
But he's an arrant knave.
Hor. There needs no ghost, my lord, come from the grave
To tell us this.—*Hamlet*, act i, sc. v.

42. All that glitters is not gold;
Tinsel glitters;
∴ Tinsel is not gold.

43. *Bishop* (inspecting the chancel) to Curate (ritualistically inclined), —"I am sorry to see that you have placed a cross over the altar" (pointing to the sign deeply cut into the wood-work).
Curate,—"But please observe we have not placed one there, but, on the contrary, have taken one away."—*Punch.*

44. Tu es qui es;
Quies est requies;
Ergo, Tu es requies.

45. What would be the consequence should an irresistible force encounter an insurmountable obstacle?

Ans.—Compound stationary motion.

46. Is Patrocles your brother, Socrates?

Yes, he is my half-brother, the son of my mother, but not of my father.

Then he is, and is not, your brother.

Not by the same father, good man; for Chæredemus was his father, and mine was Sophroniscus.

Chæredemus, then, was other than a father?

Yes, than mine.

But can he who is other than a father be a father? or, are you the same as a stone?

I am afraid you will prove me so.

Are you not, indeed, other than a stone?

I am.

And, being other than a stone, you are not a stone; and being other than gold, you are not gold.

Very true.

And so Chæredemus, being other than a father, is not a father.

It seems he is not a father.

At least, if Chæredemus is a father, then Sophroniscus, being other than a father, you, my Socrates, are fatherless.—*Plato's Euthydemus*, § 26.

47. Who is most hungry eats most;

Who eats least is most hungry;

∴ Who eats least eats most.

48. A professional school ought to be zealously attended, for in it we specially prepare for our vocation;

This school is not professional, for in it we do not specially prepare for business;

∴ This school need not be zealously attended.

49. There is no rule without exceptions;

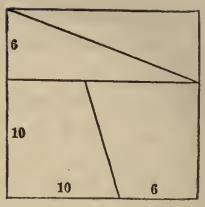
This statement is itself a rule;

∴ This statement has exceptions;

i. e., There are rules without exceptions.

50. Now is no part of time, for a whole is composed of its parts, and time is not composed of nows. Time is either past or future. The former no longer exists; the latter not yet exists. Therefore time has no existence.—*Aristotle's Physica*, iv, 10.

51. The annexed figure is a square, say 16 inches; therefore containing $16 \times 16 = 256$ square inches. Cut it in pieces by the dividing lines, and place the parts in position so as to make the rectangle whose base is 26 inches and altitude 10 inches. This rectangle contains $10 \times 26 = 260$ square inches.



$\therefore 256 = 260$.

52. The man who is walking away from me does not grow smaller;
But what I *see* grows smaller;
$\therefore$ What I *see* is not the man.—*Hume.*

53. If the blest in heaven have no desires,
they will be perfectly content;
and so will they if their desires are fully gratified.
But either they will have no desires,
or they will have them fully gratified.
$\therefore$ They will be perfectly content.

54. Knowledge is power;
Power is desirable;
$\therefore$ Knowledge is desirable.

55. Qui sunt domini sui sunt sui juris;
Servi sunt domini sui;
Ergo, Servi sunt sui juris.

56. The acute metaphysician Bishop Berkeley boasted that he had forever put an end to "scepticism, atheism, and irreligion" by the following demonstration of the existence of an Eternal Mind, of God:
Ideas cannot exist without a mind in which they inhere.
I have the same idea to-day that I had yesterday.
But this would be impossible unless there were a mind in which it existed during the interval.
Hence there must exist a Universal Mind in which all ideas have their permanent residence during the interval of their conscious presence in our own minds.

57. Whoever thrusts a knife into another person commits an injury.
A surgeon does this; therefore he commits an injury.

58. Ein Weib nur zu besitzen ist seiner Leidenschaft Ziel.—*Fries*, § 109.

59. A magistrate is justified in using his official power to forward his religious views, because every man has a right to inculcate his own opinions.

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60. Quod est bonum, omne laudibile est;
Quod autem laudibile est, omne honestum est;
Bonum igitur quod est, honestum est.[29]

61. Prayer may be regarded as useful, if, indeed, we can regard our prayers as announcing to Deity what he does not know, or as changing his purposes.
But we cannot tell the Omniscient what he does not already know, nor change his eternal purposes.
∴ Prayer is useless.

62. Do you know your own father? Yes.
Do you know this man who is veiled? No.
Then you do not know your father; for it is he.[30]

63. "The Cretians are alway liars."—*Epistle to Titus*, i, 12.
Epimenides, who said this, was himself a Cretian, and "this witness is true."—*Ibid.*
If so, Epimenides himself was always a liar, and this witness is not true.

64. Quand la terre est humide, il se forme de la vapeur, et par suite de la vapeur, un nuage; et par suite du nuage, de la pluie; par suite de la pluie, l'humidité de la terre: mais ceci est précisément le point de départ, et l'on y est revenu circulairement (*κύκλῳ περιελήλυθεν*).—*Saint-Hilaire, Comment.* t. i, p. 524.

65. Predestination makes man immoral; for if a man be an heir of grace, his exertions are useless; if of wrath, unavailing.
But, according to predestination, a man is an heir either of grace or of wrath;
Therefore, according to predestination, his exertions must be useless.
But he who believes his exertions to be useless must be immoral;
Therefore predestination makes man immoral.—*Macaulay.*

66. The gods, said the Epicureans, are very happy;
None can be happy without virtue;
There is no virtue without reason;
Reason is found nowhere except in human form;
Therefore, the gods have human form.

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[29] A Stoical argument, from Cicero's *De Finibus*, bk. iii.

[30] *Obvelatus*, or *Occult*, by Eublides of Megara. — Diog. Laert. ii, § 108. Also called *Electra*, from Sophocles, *Elect.* 1222. See *De Soph.* 24, 2; and a cursory review of Aristotle's solution in Grote's *Aristotle*, vol. ii, p. 119 sq.

67. There is no such thing as a void; for in a void there could be no difference of up and down; for as in nothing there are no differences, so there are none in privation or negation. But a void is merely a privation or negation of matter. Therefore, in a void, bodies could not move up and down, which it is in their nature to do.—*Aristotle's Physica.*

68. The doctor who attended Pat's wife in her last illness afterwards had to sue him for the fee. The court gave Pat permission to question him. "Docthor, didn't ye agree that, 'kill or cure,' ye would charge me only a guinea?" "Yes." "Well, docthor, did ye cure her?" "No, she's dead." "Well, docthor, did ye kill her?" "No, surely." "Thin, docthor, if ye did *naither*, how can ye ask a fee?"

69. Themistoclis filius nec Græcis imperat, nec de imperando cogitat. Verum imperat matri, quæ imperat Themistocli, qui Græcis imperat. Dominatur itaque Græcis, et non-dominatur.[31]

70. If the wife you espouse be beautiful, she excites jealousy;
If she be ugly, she disgusts;
Therefore it is best not to marry.—*Bias, quoted by Aulus Gellius.*

71. A sailor is not a board, nor is a sailor a shore;
But always he is either aboard or ashore;
∴ A sailor is not a sailor.

72. A desire to gain by another's loss is a violation of the tenth commandment. All gaming, therefore, since it implies a desire to profit at the expense of another, breaks this commandment.

73. He that is of God heareth God's words; ye, therefore, hear them not, because ye are not of God.—*John* viii, 47.

74. If Abraham were justified by works, then he had whereof to glory (before God). But not (any one can have whereof to glory) before God (therefore, Abraham was not justified by works).—*Romans*, iv, 2.

75. What is of less frequent occurrence than the falsity of testimony cannot be established by testimony;
Miracles are of less frequent occurrence than the falsity of testimony;
∴ Miracles cannot be established by testimony.—*Hume.*

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[31] This famous "Inexplicable" is called *Dominans*, or Κυριεύων. It is mentioned by Plutarch, Arrian, Lucian, Gellius, and others, but not fully explained by any. For a discussion of it, see Butler's *Lectures on Ancient Philosophy*, i, p. 414; and Mansel in Aldrich, Appendix, § 9.

76. Exemption from punishment is due to the innocent; therefore, as you maintain that the prisoner at the bar ought not to be punished, it appears that you maintain his innocence.
77. In Platon's Euthydemos kommt dieses vor: Hast du einen Hund? Ja. Hat er Junge? Ja. Ist er der Vater der Jungen? Ja. Also ist dein Hund ein Vater, und folglich dein Vater ein Hund. —*Fries*, § 109. Cited also in *De Soph.* xxiv.
78. If men are not likely to be influenced in the performance of a known duty by taking an oath to perform it, the oaths commonly administered are superfluous; if men are likely to be so influenced, every one should be made to take an oath to behave rightly throughout his life. But one or the other of these must be the case. Therefore, either the oaths commonly administered are superfluous, or every man should be required to take an oath to behave rightly throughout his life.—*Whately.*
79. The principles of justice being variable, and the appointments of nature invariable, it follows that the principles of justice are no appointment of nature.—*Aristotle's Ethics*, bk. iii.
80. If the favor of God is not bestowed at random, on no principle at all, it must be bestowed either with respect to men's persons or with respect to their conduct. But "God is no respecter of persons." Therefore his favor must be bestowed with reference to men's conduct.—*Sumner's Apostolical Preaching.*
81. Jupiter was the son of Saturn, therefore the son of Jupiter was the grandson of Saturn.
82. Two straight lines cannot enclose a space (Axiom x). But two parallel straight lines of infinite length do enclose an infinite space. Moreover, if a third line parallel to these be drawn midway between them, it will divide this infinite space into two equal parts, each of which is one half of infinity.
83. Opium is poison. But physicians advise some of their patients to take opium. Therefore, physicians advise some of their patients to take poison.
84. "The knowledge of relatives is one." I cannot be conscious of a mental act without being conscious of the object to which that act relates. But the object of memory confessedly lies in the past, and has ceased to be. Therefore, in memory I am conscious of an object in the past, and am conscious of what does not exist.—See *Hamilton's Metaphysics*, p. 146 sq.

85. Animal food may be dispensed with, for the vegetarians do not use it; and vegetable food may be dispensed with, for the Esquimaux do not use it.

But all food is either animal or vegetable.

∴ All food may be dispensed with.

86. No soldiers should be brought into the field but those who are well qualified to perform their part;

None but veterans are well qualified to perform their part;

∴ Veterans only should be brought into the field.

87. We can attend to a plurality of objects at once. For if attention be nothing but the concentration of consciousness on a smaller number of objects than constitute its widest compass of simultaneous knowledge, it is evident that, unless this widest compass of consciousness be limited to only two objects, we do attend when we converge consciousness on any smaller number than that total complement of objects which it can embrace at once. —*Hamilton's Metaphysics*, p. 165.

88. In what and how many phrases does Hamilton, in his "immediate demonstration" that sight is cognizant of extension, assume the point to be proved?—*Ibid.*, p. 385.

89. It would be bad reasoning in an Anabaptist to prove against the Catholics that they are wrong in believing that infants are capable of baptism, since nothing is said of it in the Scripture; because this proof would assume that we ought to believe only what is in the Scripture, which is denied by the Catholics.—*Arnauld*, p. 251.

90. How can we conceive God, since we can attribute no virtue to him? For shall we say that he has prudence? But since prudence consists in the choice between good and evil, what need can God have of this choice, not being capable of any evil? Shall we say that he has intelligence and reason? But intelligence and reason serve to discover to us that which is unknown from that which is known; now there can be nothing unknown to God. Neither can justice be in God, because this relates only to the intercourse of men; nor temperance, since he has no desires to moderate; nor strength, since he is susceptible of neither pain nor labor, and is not exposed to any danger. How therefore, can that be a god which has neither intelligence nor virtue?[32]

[32] Cotta, quoted by Cicero, *De Natura Deorum*, bk. iii.

91. That which has the use of reason is better than that which has not; There is nothing better than the universe;
∴ The universe has the use of reason.—*The Stoics.*

92. Why does a ball, when dropped from the mast-head of a ship in full sail, not fall exactly at the foot of the mast, but nearer to the stern of the vessel?

93. Gold and silver are wealth; therefore the diminution of the gold and silver of the country by exportation is a diminution of the wealth of the country.

94. If man be not a necessary agent determined by pleasure and pain, there is no foundation for rewards and punishments. These would be useless unless men were necessary agents, and were determined by pleasure and pain; because if men were free and indifferent to pleasure and pain, pain could be no motive to cause them to observe the law.

95. It is certain that drunkenness is a vice odious to God and man. It is equally certain that alcoholic drinks are destructive to the moral, intellectual, and physical energies of him who habitually makes use of them. I claim, therefore, not only that it is the duty of every man to abstain totally from their use, but, as a good citizen and philanthropist, to exert all his influence to obtain and enforce a law prohibiting the sale of any kind of intoxicating beverages.

96. (Analyze the argument of Krug and the reply of Biunde, as stated in Hamilton's *Metaphysics*, pp. 565–66.)

97. Minus multiplied by minus cannot give minus; for minus multiplied by plus gives minus, and minus multiplied by minus cannot give the same product as minus multipled by plus.—*Euler's Algebra.* See Mill's *Logic*, p. 575.

98. "Either God wills to remove evils and cannot; or he can and will not; or he neither will nor can; or he both will and can. If he will and cannot, then he is weak, which is not true of God. If he can and will not, then he is malicious, which also is foreign to the nature of God. If he neither will nor can, then he is both malicious and weak, and therefore cannot be God. If he both can and will, which alone is consistent with the nature of God, then whence are evils, or why does he not remove them?"[33]

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[33] Epicurus, quoted by Lactantius, *De Ira Dei*, xiii.

99. The doctrine of an omnipresent divine power and agency in the operations of Nature is as contrary to the Scriptures as it is to sound philosophy; for the Scriptures say expressly, "The earth bringeth forth fruit *of herself*."—*Mark*, iv, 28.

100. If a reconciliation between the ancient historical records and modern culture be sought by means of interpretation, it will be attempted to prove either that the divine did not manifest itself in the manner related, which is to deny the historical validity of the ancient Scriptures; or that the actual occurrences were not divine, which is to explain away the absolute contents of these books.[34]

101. Do but let the Bible tell its own story; grant, for the sake of argument, the truth of the dogmas which it asserts throughout, and it becomes a consistent whole. When a man begins, as Strauss does, by assuming the falsity of its conclusions, no wonder he finds its premises a fragmentary chaos of contradictions.[35]

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[34] D. F. Strauss, in *Leben Jesu*, Int. § 1.

[35] Dean Alton Locke, *Works*, vol. i, ch. xxxviii.

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